# Homework 0

For practice. Do not hand in. Stars indicate more challenging problems.

**1** Find a vector $\boldsymbol{x}$ for which $\boldsymbol{u}(\boldsymbol{x} \cdot \boldsymbol{u}) + \boldsymbol{x} = \boldsymbol{v}$ where $\boldsymbol{u}$ and $\boldsymbol{v}$ are given vectors.

**2** Find the locations of the minima of the function $f(x,y) = 2x^2 + y^2 - 3y + 2$ in the plane. Now do the same in the circle of unit radius centered at the origin.

**3** Calculate

- $\nabla^2 r^{-n}$
- $\nabla(\Omega \cdot \boldsymbol{x})$ where $\Omega$ is a constant vector
- $\nabla f$, where $f$ is the scalar function $f \equiv A_{ij} x_i x_j + B_j x_j + C$. What happens if $A$ is a symmetric matrix?

**4** Compute the surface integral $-\int_S p \, \mathrm{d}\boldsymbol{S}$ where the integrand is given by $p(z) = p_0 + N^2 z^2 / 2g$, where $S$ is a closed surface enclosing a volume $V$. [Hint: think about the location of center of mass of the volume.]

**5*** If the vector field $\boldsymbol{u}(\boldsymbol{x})$ is irrotational, show that for a closed volume

$$\int_V |\boldsymbol{u}|^2 \, \mathrm{d}\boldsymbol{x} = \int_S \phi \frac{\partial \phi}{\partial n} \, \mathrm{d}S.$$

What is $\phi$ in this expression? [Optional: what happens if $V$ is unbounded?]

**6*** Calculate the integral

$$\int_V x_i x_j x_k \, \mathrm{d}V$$

over a sphere of radius $a$. [Hint: think about isotropic third-rank tensors.]

**7** Find the principal axes of the tensor field $t_{ij} = x_i x_j - \delta_{ij} r$ at the point $(2, 1, -1)$.