Homework I

Due Jan 19, 2018.

1 [Acheson 6.1] Shows that the expression

$$t_i = -pn_i + \mu n_j \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right)$$

may be written in vector form as

$$\boldsymbol{t} = -p\boldsymbol{n} + \mu[2(\boldsymbol{n} \cdot \boldsymbol{\nabla})\boldsymbol{u} + \boldsymbol{n} \times (\nabla \times \boldsymbol{u})]$$

by rewriting the vector expression in suffices.

2 Poiseuille flow through a pipe with an elliptic cross-section has the velocity field $\boldsymbol{u} = (u, 0, 0)$ with $u = U(1 - y^2/a^2 - z^2/b^2)$. Compute the shear stress $f_i = \tau_{ij} n_j$ acting on the plane $x = 0$, where

$$\tau_{ij} = \mu \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right).$$

Where is the stress largest in magnitude and where is it smallest?

3 Show that the two-dimensional velocity field $\boldsymbol{u}(r, \theta)$ given by $\boldsymbol{u} = \nabla\phi$, where $\phi = U(r + a^2/r)\cos\theta + \Gamma\theta/(2\pi)$, satisfies $\boldsymbol{u} \cdot \boldsymbol{n} = 0$ on the circle $r = a$. Compute $-\int_S p \, \mathrm{d}\boldsymbol{S}$ and $-\boldsymbol{k} \cdot \int_S p\boldsymbol{x} \times \mathrm{d}\boldsymbol{S}$ over the circle $r = a$, where $p = -\frac{1}{2}\rho|\boldsymbol{u}|^2$.

4 [Kundu & Cohen 2.6] For the velocity field

$$u = \frac{x}{1+t}, \qquad v = \frac{2y}{2+t},$$

find the streamlines, the particle path starting at $(1, 1)$ at $t = 0$, and the streakline passing through $(1, 1)$ starting at $t = 0$.