MAE210A: Fluid Mechanics I Winter Quarter 2018
http://web.eng.ucsd.edu/ sgls/MAE210A_2018

Homework IV

Due Feb 9, 2018.

1 This question treats two-dimensional incompressible flow of a Newtonian fluid.
(i) Show that the velocity can be written as $u_i = \epsilon_{ij3}\partial\psi/\partial x_j$ where ψ is the streamfunction.
(ii) Show that the vorticity out of the plane is given by $\omega = -\nabla^2\psi$.
(iii) For the streamfunction $\psi = a_{ij}x_i x_j$ where $a_{i3} = a_{3j} = 0$, calculate the velocity and vorticity. What is the condition on a for the flow to be irrotational?
(iv) What is the viscous term in the Navier–Stokes equation for this flow?
(v) What is the dissipation rate ϕ?

2 [Kundu 4.8] Show that the thrust developed by a stationary rocket motor is $F = \rho A U^2 + A(p - p_{atm})$, where p_{atm} is the atmospheric pressure, and p, ρ, A, and U are, respectively, the pressure, density, area, and velocity of the fluid at the nozzle exit.

3 Consider a fluid at rest with variable density. Explain why the total stress tensor is $\tau_{ij} = -p\delta_{ij}$. Write down the momentum equation with a gravity field (force/unit area) written as $\boldsymbol{\nabla}\phi$. Derive the relation $\boldsymbol{\nabla}\rho \times \boldsymbol{\nabla}\phi = \mathbf{0}$.

4 Consider incompressible flow with constant density through an axisymmetric nozzle of length L with varying cross-section. Let x the coordinate along the axis of the nozzle and z be the radial coordinate. The local radius of the nozzle is $R(x)$. Assume that the fluid velocity in the nozzle can be expressed as

$$\boldsymbol{u}(\boldsymbol{x}) = v_x(x)\boldsymbol{e}_x - \alpha r \boldsymbol{e}_r,$$

where α is a constant.
(i) Use the continuity equation to solve for $u_x(x)$ with the condition $u_x(0) = U_0$ at the entrance of the nozzle.
(ii) Knowing that the velocity at the nozzle wall is tangential to the nozzle, obtain an expression for the shape $R(x)$ of the nozzle in terms of α, U_0 and the entrance radius $R_0 = R(0)$. Realizing that the nozzle shape must be independent of U_0, explain why α and U_0 must be proportional.
(iii) Calculate the flow rates through the entrance and exit of the nozzle located at $x = 0$ and $x = L$. The two flow rates should be equal. Discuss.