Homework V

Due Feb 23, 2018.

1 Show that the Bernoulli equation for an isentropic perfect gas for which $p/p_0 = (\rho/\rho_0)^\gamma$, with γ the ratio of specific heats, can be written in the form

$$\frac{\partial \phi}{\partial t} + \frac{1}{2}|\boldsymbol{u}|^2 + \frac{c^2}{\gamma - 1} + \Omega = B(t),$$

where $c = \sqrt{\gamma p/\rho}$ is the speed of sound in the gas.

2 [Kundu 4.10] A hemispherical vessel of radius R has a small rounded orifice of area A at the bottom. Show that the time required to lower the level from h_1 to h_2 is given by

$$t = \frac{2\pi}{A\sqrt{2g}}\left[\frac{2}{3}R(h_1^{3/2} - h_2^{3/2}) - \frac{1}{5}(h_1^{5/2} - h_2^{5/2})\right].$$

3 [Kundu 4.14] Water flows down out of a vertical pipe in the presence of gravity. Neglecting the effects of viscosity and surface tension, solve the appropriate conservation equations for the variation of cross-sectional area $A(z)$ of the fluid stream. The coordinate z points up and $z = 0$ is the base of the pipe, where the velocity v_0 is uniform and the area is A_0.

4 A spherical bubble of gas of radius $R(t)$ is initially at rest in an infinite extent of inviscid fluid.
(i) Assuming the motion remains radially symmetric, show that the velocity potential of the flow outside the bubble is given by

$$\phi = -\frac{R^2\dot{R}}{r},$$

where r is the usual spherical coordinate.
(ii) Show that the pressure in the fluid is given by

$$p = p_\infty + \rho\left(\frac{R^2\ddot{R}}{r} + \frac{2R\dot{R}}{r} - \frac{R^4\dot{R}^2}{2r^4}\right),$$

where p_∞ is the pressure far from the bubble.

(iii) If the pressure is neglected inside the bubble, find the equation

$$R^3 \dot{R}^2 = \frac{2p_\infty}{3\rho}(R_0^3 - R_3)$$

for the evolution of the bubble surface. Show that the bubble collapses in a time

$$t_c = \sqrt{\frac{3\rho}{2p_\infty}} \int_0^{R_0} [(R_0/R)^3 - 1]^{-1/2} \, \mathrm{d}R.$$