MAE210A: Fluid Mechanics I Winter Quarter 2018

http://web.eng.ucsd.edu/~sgls/MAE210A_2018

Midterm

This is a 50 minute closed-book closed-note exam. You may use a sheet of notes that you have prepared. No calculators.

1 (10 points) For the velocity field

$$\boldsymbol{u} = (\cos t, x, -z),$$

compute the particle path starting from $(0,0,1)$ at $t = 0$, the streamlines at $t = 0$ and the streakline at $t = \pi$ made up of dye released from $(0,0,1)$ during $0 \leq t \leq \pi$. Results can be left in parametric form.

2 (15 points) Consider the velocity field $\boldsymbol{u} = u_\theta(r)\hat{\boldsymbol{\theta}}$ in cylindrical polar coordinates (r, θ, z) given by

$$u_\theta(r) = \begin{cases} \Omega r & \text{for } 0 < r < a, \\ \Omega a^2/r & \text{for } a < r. \end{cases}$$

(i) Calculate the vorticity field.
(ii) Calculate the circulation around a circle of radius R centered at the origin.
(iii) The density is constant, the fluid is inviscid and gravity points in the negative z-direction. Compute the pressure field by integrating the radial and vertical equations of motion. Sketch lines of constant pressure in the (r, z) plane.

3 (15 points) Fluid flows between two parallel plates at $y = -h$ and $y = h$.
(i) Show that the velocity field $\boldsymbol{u} = A(y^2 - h^2, 0, 0)$ satisfies the incompressible Navier–Stokes equations.
(ii) What is the volume flux Q in terms of A and h? Take $Q > 0$; what is the sign of A?
(ii) What is the pressure gradient? What are the rate of strain and the shear stress?
(iii) Calculate the force exerted on each boundary per unit width over the region $0 < x < L$. In what direction does it act?

For reference Vorticity in cylindrical polar coordinates (r, θ, z):

$$\boldsymbol{\omega} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\hat{\mathbf{r}} + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\hat{\boldsymbol{\theta}} + \left(\frac{1}{r}\frac{\partial (r u_\theta)}{\partial r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)\hat{\mathbf{z}}.$$

Radial component of the Euler equation in cylindrical polar coordinates (r, θ, z):

$$\frac{\partial u_r}{\partial t} + (\boldsymbol{u} \cdot \boldsymbol{\nabla}) u_r - \frac{u_\theta^2}{r} = -\frac{1}{\rho}\frac{\partial p}{\partial r}.$$