ECE 100 **Lab 4** **Design of a Differentiator** **Fall 2011**

The goal of this lab is to design a signal processing circuit that will output the time derivative of the input scaled by the appropriate factor. The specification for this *differentiator* is that it should convert a triangle wave of 1v peak to peak at 1 KHz into a square wave of ±1 v (at 1 KHz of course). The polarity of the square wave is not important, but the 10% to 90% rise time must be less than 20 μs and the output voltage must not overshoot more than 15%, i.e. 0.30 v.

The circuit can be based on the operational amplifier inverter topology. If the op amp is ideal this simply requires an input capacitor and a feedback resistor. Then the ideal transfer function is $H_D(s) = -sRC$, which (apart from a phase shift of 180deg) is what we need. Unfortunately the simple design will be either marginally stable or completely unstable, and some *compensation* will be required. In this case "unstable" means that the circuit will oscillate with no input present, and thus is useless as a differentiator. The nature of the compensation is very much like the way the follower was modified to drive a large capacitive load. However in this assignment we will study how to do this compensation theoretically.

The transfer function H(s) for the inverter, including the opamp gain A(s) is:

$H(s) = V_0(s)/V_I(s) = -(Z_F(s)/Z_I(s))*\{T(s)/(1 + T(s))\}$ where $T(s) = A(s) * Z_I(s)/(Z_I(s) + Z_F(s))$ is the loop gain.

When $|T| >> 1$ $H(s) \to -Z_F(s)/Z_I(s)$ which we can call the "ideal" transfer function, because it would apply if the opamp were ideal. The factor {T/(1+T)} gives the deviation from ideal behavior. All feedback circuits have a factor like this in their transfer function. It is the possibility of a zero in the term (1+T) that causes instability.

1. System Level Design: Here $Z_I(s)=1/sC$ and $Z_F(s)=R$, so $H_{IDEAL}(s) = -sRC$ and $V_0(t) = -RC\, dV_{IN}/dt$. We must choose the RC product necessary to meet the spec. We need $V_0(t) = \pm 1$v for the given triangle input. All we need is dV_{IN}/dt for the triangle wave and we can find RC. A reasonable choice is R = 100K, so we can then find C.

(a) Check that R and C have been calculated correctly by simulating the circuit with an ideal opamp. Use a VCVS with a gain of 1E10. Use VPWL as the input, and set the parameters to provide a few cycles of the triangle wave. Make sure that the output voltage is the desired square wave.

(b) We can think of the differentiator as a product of H_{IDEAL} which converts the triangle wave into the desired square wave, and a second filter T/(1+T) which turns out to be a second-order low-pass filter. This low pass filter will have a finite bandwidth and thus rise time. We will have to make sure that the bandwidth is broad enough that the rise time will meet the spec. Write the expression for T/(1+T) in normalized form assuming A(s) = G/s. You will see that it is a quadratic low pass filter. Find expressions for ω_0 and ζ in terms of G and τ = RC. For a second order circuit the rise time depends on both ω_0 and ζ, however for a first order circuit $\tau_R = 2.2/\omega_0$. Using this limit, what is the minimum value of G (convert the units to Hz) needed for the circuit? Will the LF411 be satisfactory?

(c) Estimate ζ using the known value of G for the LF411. What overshoot would you expect? Use %os = $100*\exp(-\pi\zeta/\text{sqrt}(1-\zeta^2)$. Clearly this will not meet the spec. The problem is that the phase margin of the loop gain T is rather small. We will need to increase the phase margin to at least 45^0. Make a Bode plot of T using Matlab, find the unity gain frequency, and then read off the phase margin. The easiest way to do this is:

```
s = tf('s');   %defines s as a variable of transfer function type
G = 2*pi*3e6; r = 10e3; c = 0.5e-7;    %set the constants
tau = r*c;
T = G/(s*(1+s*tau)); %so T is also a transfer function variable
bode(T,{100,1e6});          %does the entire Bode plot for 100<ω<1e6
```

While you are in Matlab you can easily confirm that the system would have too much overshoot.

```
H = T/(1+T);   %calculate the closed loop transfer function
```

```
step(H, 1e-4); %plot the step response
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(d) We can improve the phase margin by modifying the loop gain. In such cases adding a zero near the unity gain frequency can be very helpful. You can do this by putting a compensation resistor R_C in series with the input C. Write the modified expression for T(s). You will see that the addition of R_C adds a zero at $1/R_C C$. Find the value of R_C that puts this zero at the unity gain frequency. Make a Bode plot of the compensated loop gain $T_C(s)$ and find the new phase margin. Show that you can obtain exactly the same effect by putting C_C in parallel with R, instead of R_C in series with C.

(e) We need to check the overshoot with the compensation resistor. The overshoot should be much smaller, but may still be over the spec. Increase R_C until the overshoot just meets the spec. Check the phase margin with this R_C.

2. Circuit Level Simulation: The system level design was done using a rather simple model for the opamp. The opamp model is ideal except for its gain, which is A(s) = G/s. In fact the input and output impedances may have an effect, and the gain model may to too simple to get the behavior exactly correct. So we'll need to simulate the circuit using a macromodel for the opamp.

(a) Simulate the original differentiator (with $R_C = 0$) using the specified input triangle wave and the macromodel for the LF411. Measure the rise time and the overshoot.

(b) Repeat the simulation with the calculated value of R_C. Measure the rise time and overshoot. Does it meet the spec? It should be close. If it does not meet the spec, adjust R_C until it does meet the spec.

(c) Find the C_C that should put a zero in exactly the same place as R_C and confirm that the rise time and overshoot are the same.

3. Measurement: It is essential to test real systems where stability is an issue, because small variations in the circuit may be quite important. In this case the real opamp has significant differences with the macromodel, which can impact stability. Furthermore the actual circuit as laid out on a proto-board has significant "stray" capacitance at the circuit nodes, and this can be important too.

Before you start, check the calibration on your scope probes and adjust if necessary.

Would you expect slew rate limiting to be a factor with this circuit?

(a) Setup and measure the original differentiator (with $R_C = 0$) using the specified input triangle wave. Measure the rise time and the overshoot. Make a hard copy.

(b) Repeat the test with the calculated value of R_C. Measure the rise time and overshoot. Does it meet the spec? It should be close. If it does not meet the spec, adjust R_C until it does meet the spec. Make a hard copy.

(c) Try the test with the equivalent value of C_C instead of R_C and see if the overshoot and rise time are the same in real-life.

Don't forget to photograph your circuit, and include it with your report.