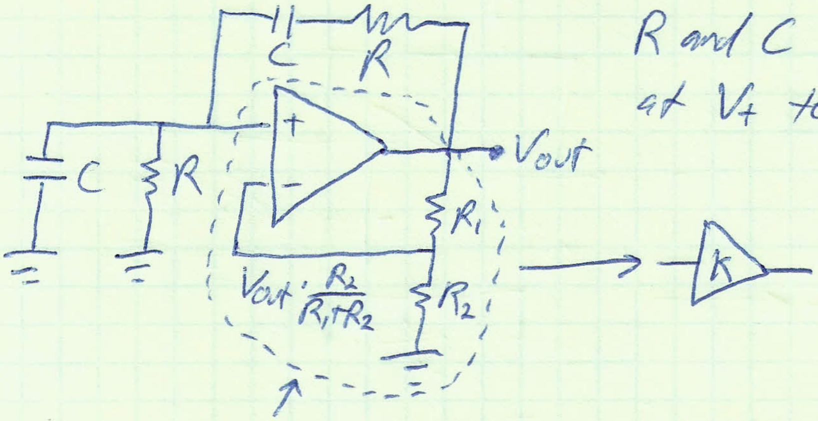
Lab #5 Design of a Wien-Bridge Oscillator

- First product sold by the Hewlett-Packard company
- Derived from William Hewlett's 1939 Stanford MS Thesis



R and C in series and parallel at V_+ terminal have same values

$$K = \frac{R_1 + R_2}{R_2}$$

Consider this part of the circuit as an amplifier with gain K

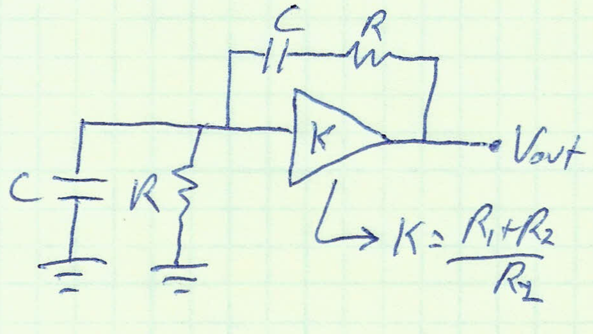
why? Op-amp drives output until $V_- = V_+$

Consider V_+ as V_{in}

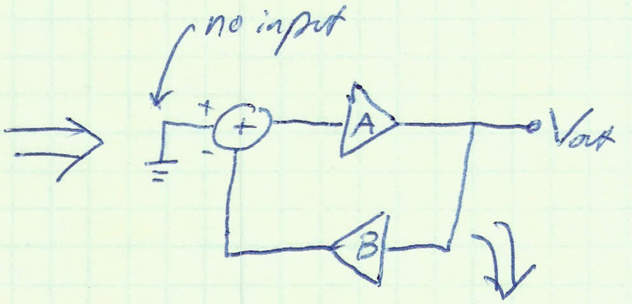
and $V_- = V_{out} \frac{R_2}{R_1+R_2}$

$$\rightarrow V_{out} = \frac{R_1+R_2}{R_2} V_{in}$$

Now the circuit looks like this:



$\Longrightarrow$

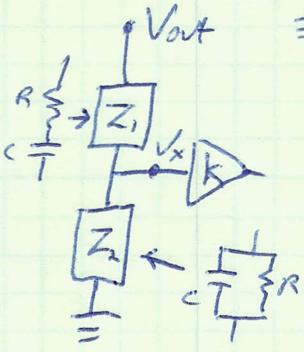


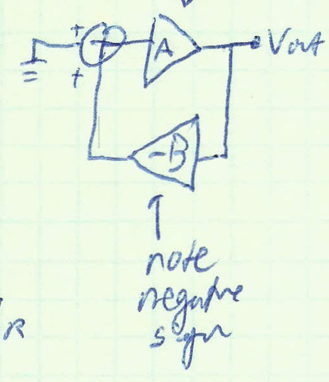
calculate the loop gain AB

$A = K$

$$V_x = \frac{Z_2}{Z_1+Z_2} V_{out}$$

$$B = \frac{-V_x}{V_{out}} = \frac{Z_2}{Z_1+Z_2}$$





negative sign because attached to V_+, not V_- !

$$Z_1 = \frac{1}{sC} + R = \frac{1+sCR}{sC}$$

$$Z_2 = \frac{\frac{1}{sC}\cdot R}{\frac{1}{sC}+R} = \frac{R}{1+sCR}$$

$$B = \frac{-Z_2}{Z_1+Z_2} = \frac{-\frac{R}{1+sCR}}{\frac{1+sCR}{sC}+\frac{R}{1+sCR}} = \frac{-R}{R+\frac{(1+sCR)^2}{sC}}$$

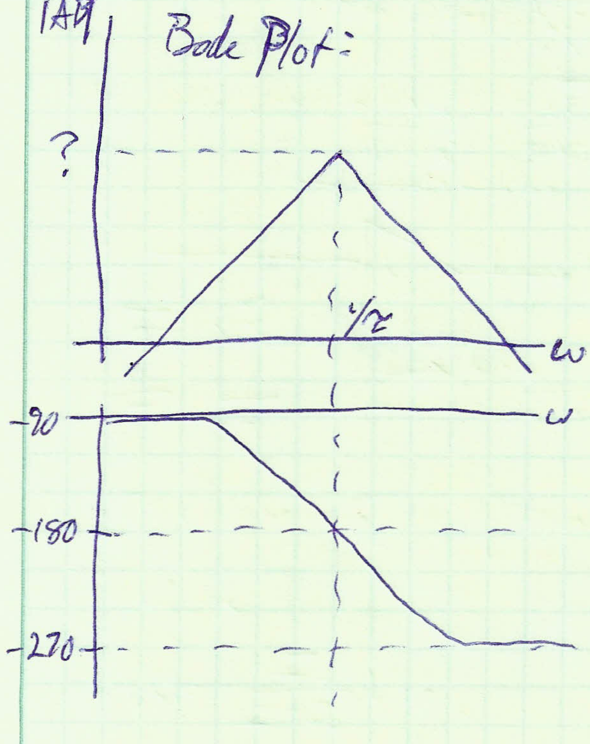
$$B = \frac{-sCR}{sCR+(1+sCR)^2} = \frac{-sCR}{1+3sCR+s^2C^2R^2}$$

$$AB = \frac{-sKCR}{1+3sCR+s^2C^2R^2}$$ define $\tau = CR$

$$AB = \frac{-sK\tau}{1+3s\tau+s^2\tau^2}$$

Zero at $\omega = 0$
Two poles at $\omega_0 = \frac{1}{\tau}$

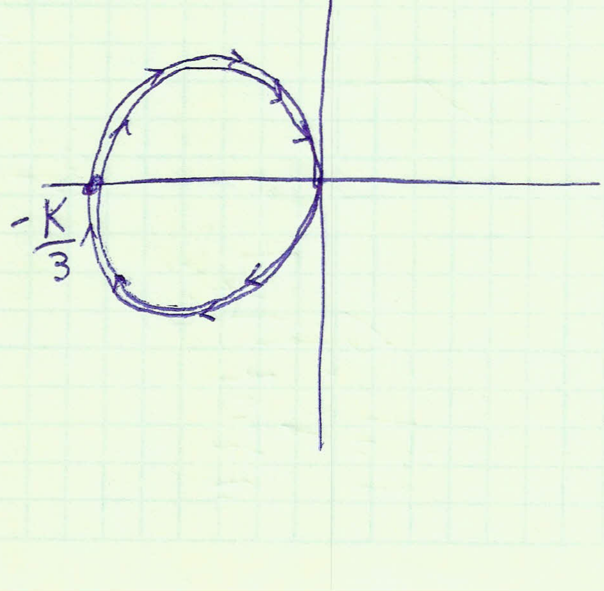
Bode Plot:



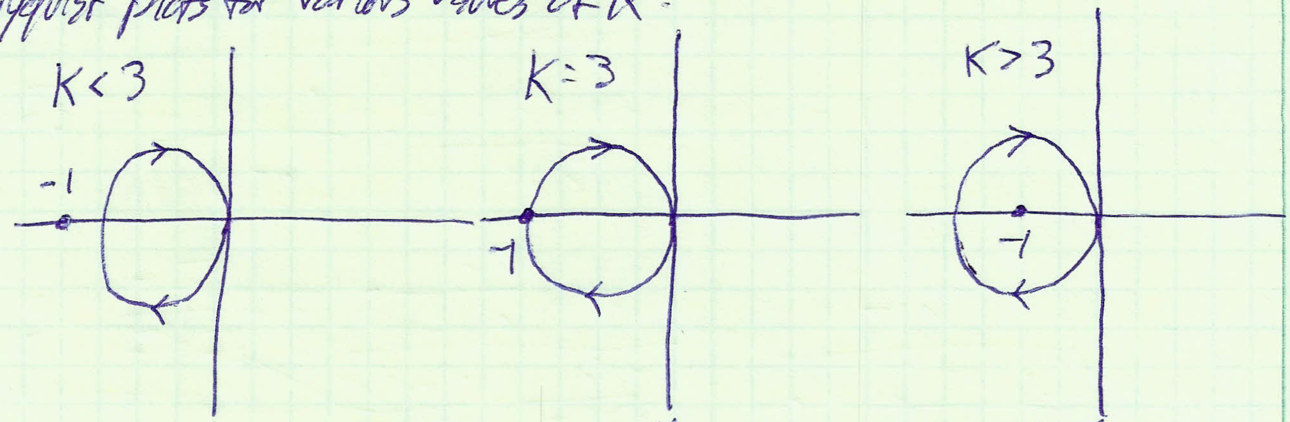
What is the magnitude at $\omega = \omega_0$?

$$|AB| = \left|\frac{-jK}{1+j3-1}\right| = \frac{K}{3}$$

Nyquist Plot:

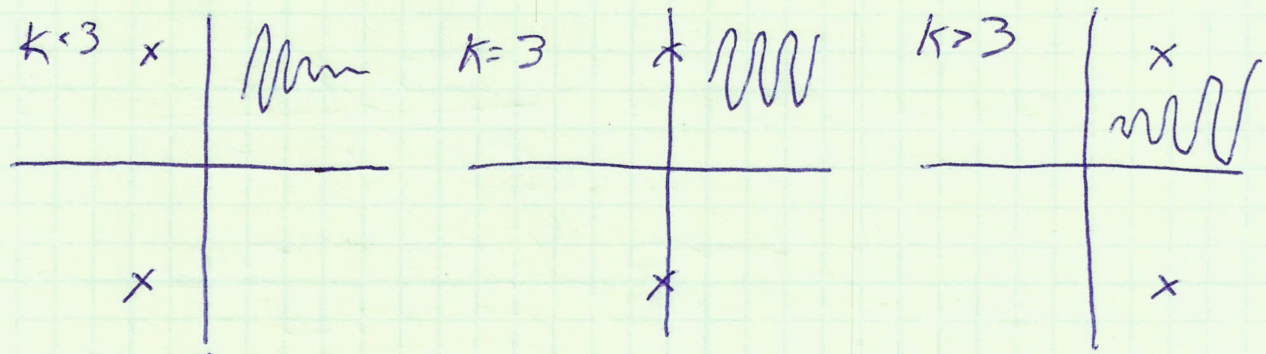


Nyquist plots for various values of K:



Stable - not an oscillator | oscillator | exponentially growing sinusoidal signal

Pole-zero diagrams (poles only)



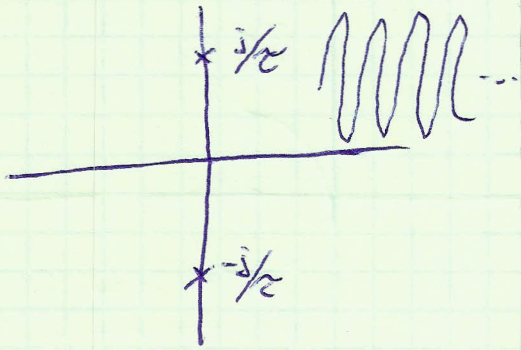
For an oscillator, we don't want the poles too far in the RHP.
Ideally they are right on the imaginary axis
Otherwise, the signal grows until nonlinearities generate harmonics.

Closed loop gain:

$$H(s) = \frac{V_{out}}{V_{in}} = \frac{A}{1+AB}, \quad A = 3 \text{ for oscillator}$$

$$AB = \frac{-3s\tau}{1+3s\tau+s^2\tau^2}$$

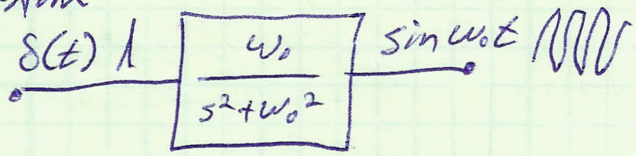
$$H(s) = \frac{3}{1 - \frac{3s\tau}{1+3s\tau+s^2\tau^2}}$$



$$= \frac{3(1+3s\tau+s^2\tau^2)}{(1+3s\tau+s^2\tau^2)-3s\tau}$$

$$= \frac{3(1+3s\tau+s^2\tau^2)}{1+s^2\tau^2} \rightarrow \text{poles at } \pm j/\tau$$

Laplace transform:



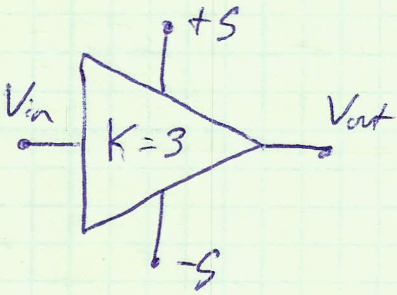
What about the s and s^2 terms?

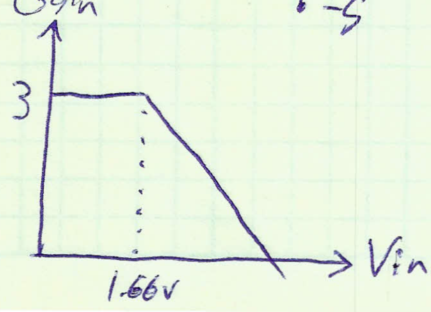
remember : $f'(t) \longleftrightarrow sF(s) - f(0)$

these just add more cos and sin terms (at same frequency)
this is equivalent to a phase shift, depends on magnitude of these terms

- How do we get $K = 3.000...$ for a stable output signal?

V_{in}	V_{out}	Gain
±1	±3	3
±1.5v	±4.5v	3
±1.66v	±5v	3
±2v	±5v	2.5
±2.5v	±5v	2





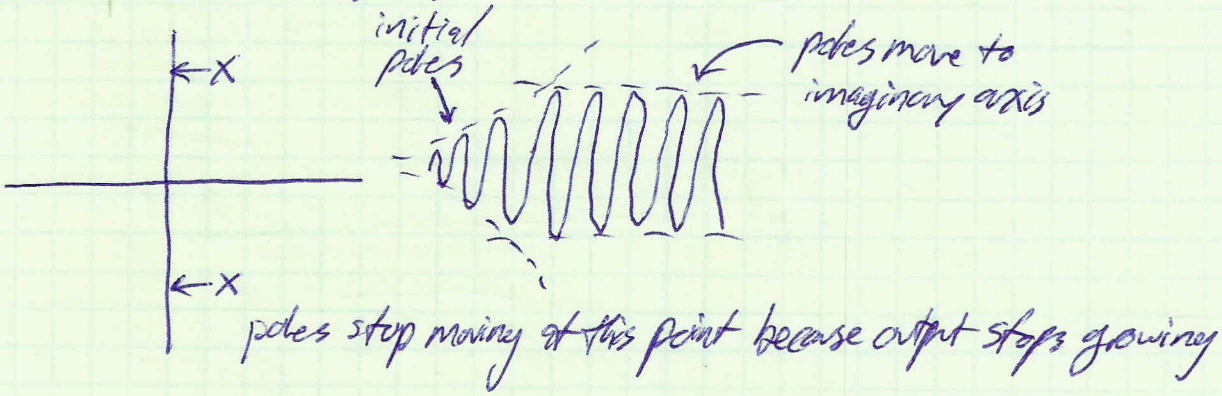
The gain of <u>any</u> amplifier is decreased beyond a certain input voltage (gain compression)

We can use this to limit the output: set gain to ~3.2-3.5

This places the poles slightly into the RHP

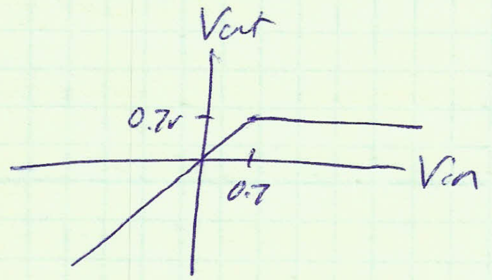
Also helps the oscillation build up from noise

Poles move to imaginary axis as output increases.

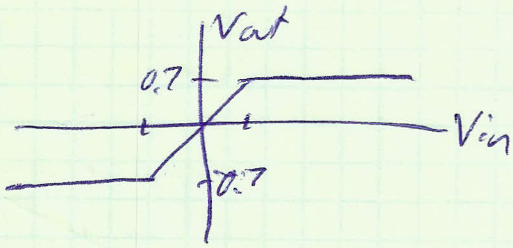


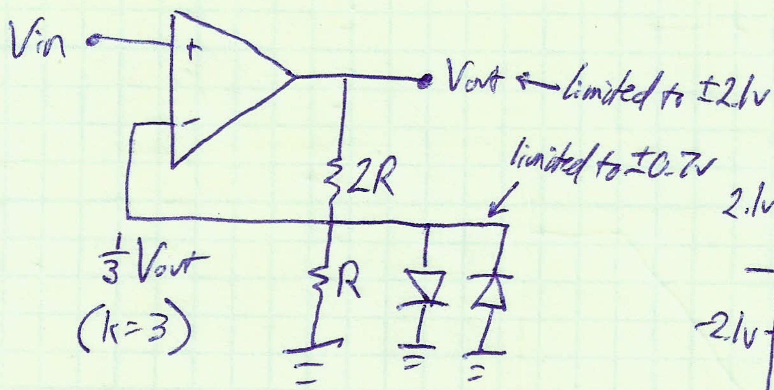
Other ways to limit the output:

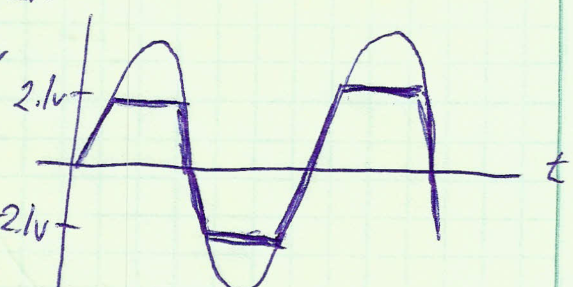
Diode limiter:



Need full limiter for + and − voltage:

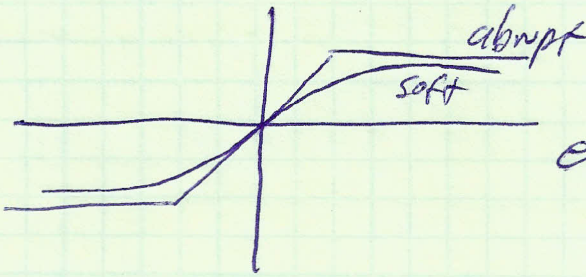




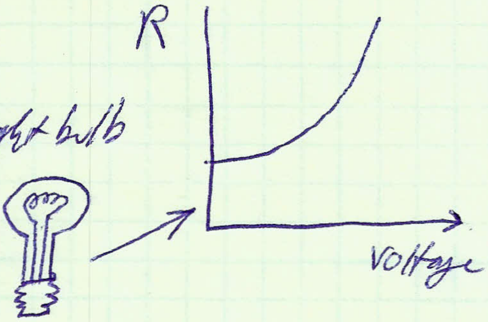


This will generate unwanted harmonics

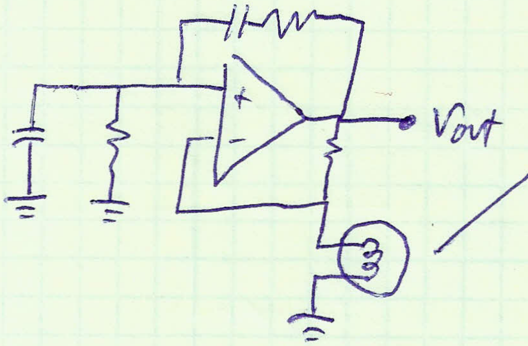
Prefer to have a soft limiter: reduce harmonics



e.g. light bulb

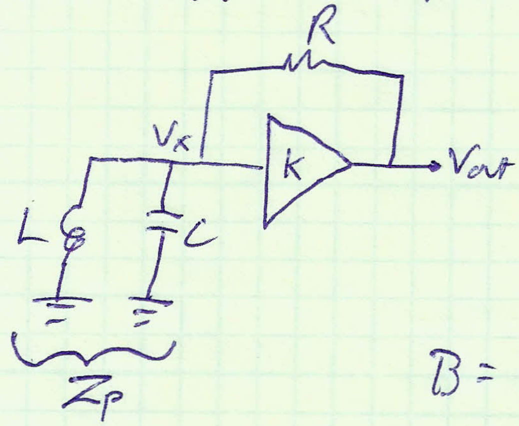


Light bulb used in original HP oscillator



- Allows higher initial gain so signal can build up from noise
- Stabilizes output amplitude
- Provides low harmonics

Other oscillator circuits:



$$B = \frac{-V_x}{V_{out}} = \frac{-Z_p}{R + Z_p}$$

$$Z_p = \frac{\frac{1}{sC} \cdot sL}{\frac{1}{sC} + sL} = \frac{sL}{1 + s^2LC}$$

$$B = \frac{\frac{-sL}{1+s^2LC}}{R + \frac{sL}{1+s^2LC}} = \frac{-sL}{R + sL + s^2LC}$$

K can be high-freq. amp.

Zp can be:

- LC circuit
- Microstrip resonator
- Cavity resonator (including tunable)
- ...

$$B = \frac{-sL/R}{1 + sL/R + s^2LC}, \quad AB = \frac{-sKL/R}{1 + sL/R + s^2LC}$$

notice, when $\omega = \omega_0 = \frac{1}{\sqrt{LC}}$

then $AB = -K$

so we want $K = 1$