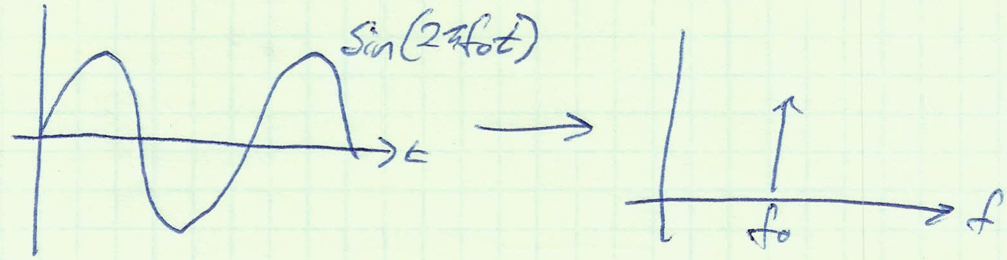
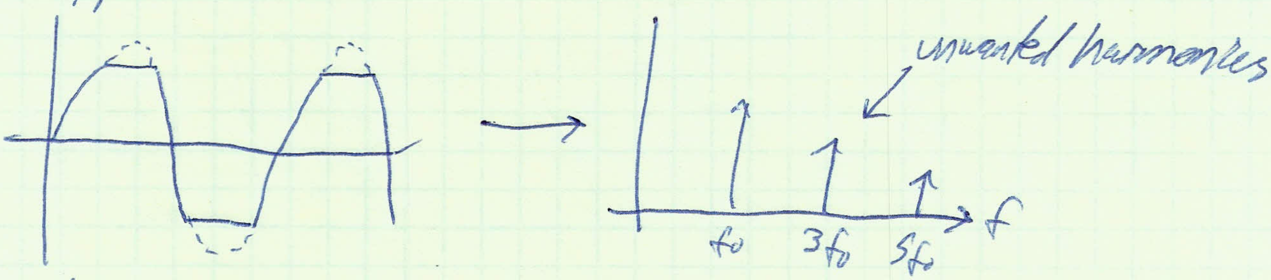
More on lab #5 - Design of an oscillator

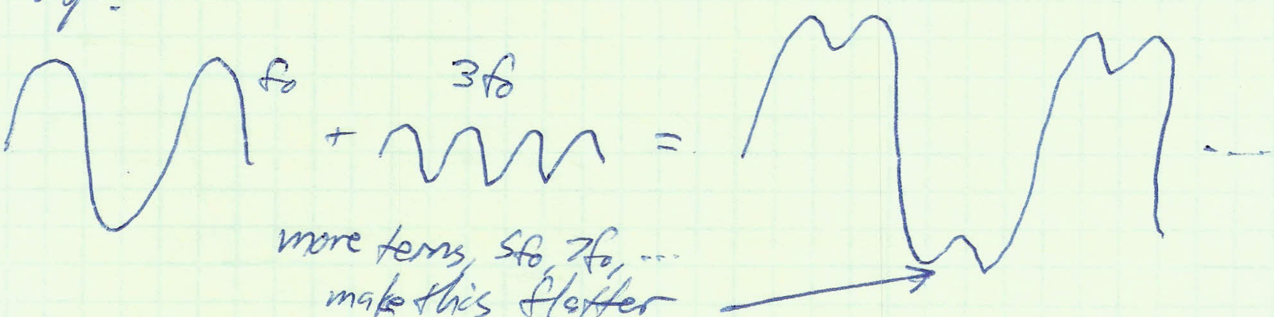
Effect of clipping




Clipped sine wave




Why?

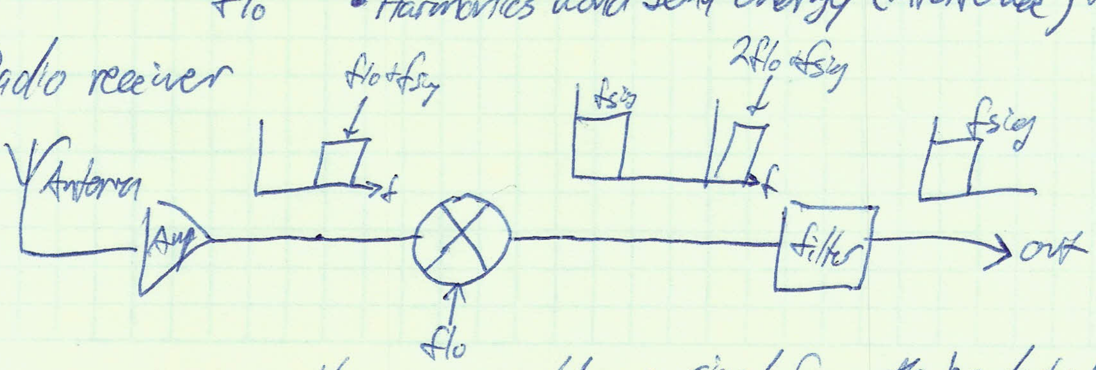



Why is this important?

Radio transmitter


- Harmonics would send energy (interference) to other bands

Radio receiver

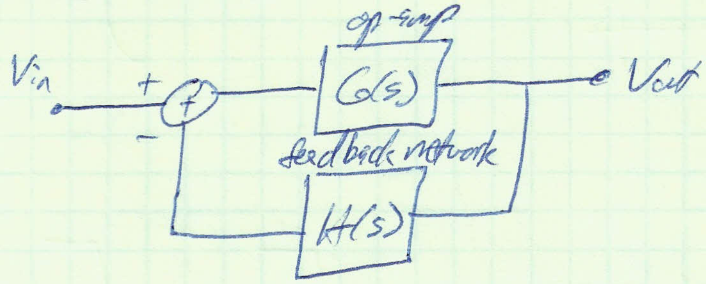



- Harmonics would cause signals from other bands to be mixed in

## Control systems

So far we have only looked at circuit feedback, but it has many applications, e.g. controlling a physical system.

### Circuit feedback

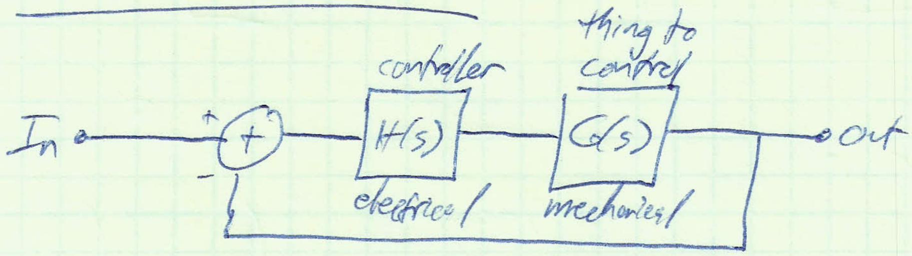



$$V_{out} = G(s)\left(V_{in} - H(s)V_{out}\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{G(s)}{1+G(s)H(s)} = \frac{1}{H(s)} \cdot \frac{G(s)H(s)}{1+G(s)H(s)}$$

if $G(s)H(s) \gg 1$, $\frac{V_{out}}{V_{in}} \simeq \frac{1}{H(s)}$

### Feedback control system




$$out = G(s)H(s)\left[In - out\right]$$

$$\frac{out}{In} = \frac{G(s)H(s)}{1+G(s)H(s)}$$

In both cases, the poles of $\frac{out}{In}$ are the same.

System will be stable only if there are no poles in the right half plane

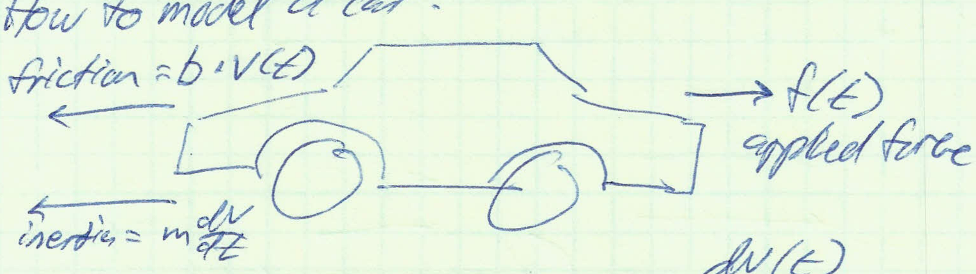
what is instability for a mechanical system controller?

e.g. car cruise controller

Instability could be oscillation – constantly changing speed 


or accelerator full on 


How to model a car:



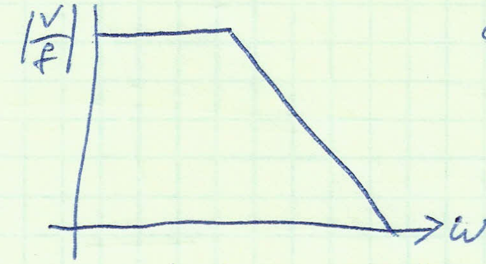

V = velocity

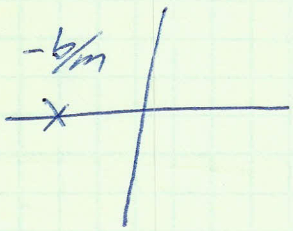
$$f(t) = b\,V(t) + m\frac{dV(t)}{dt}$$

laplace transform $F(s) = b\,V(s) + m\,s\,V(s)$

$$\frac{V(s)}{f(s)} = \frac{1}{b+ms} = \frac{1/m}{s+b/m}$$

one pole



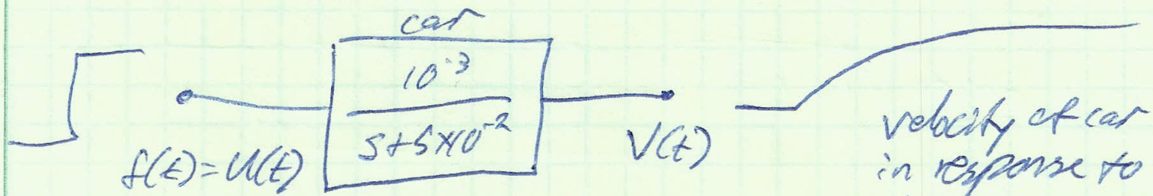




car behaves as a low-pass filter

It is stable

<u>example:</u>

$$m = 1000\ kg \qquad b = 50\ \frac{Ns}{m} \longrightarrow \frac{V(s)}{F(s)} = \frac{10^{-3}}{s+5\times10^{-2}}$$




velocity of car in response to step input of 1 Newton

$F(s) = \frac{1}{s}$

$$V(s) = \frac{1}{s} \cdot \frac{10^{-3}}{s+5\times10^{-2}} = \frac{(2\times10^{-2})(5\times10^{-2})}{s(s+5\times10^{-2})}$$

$$\frac{a}{s(s+a)} \longleftrightarrow u(t)\left[1-e^{-at}\right]$$

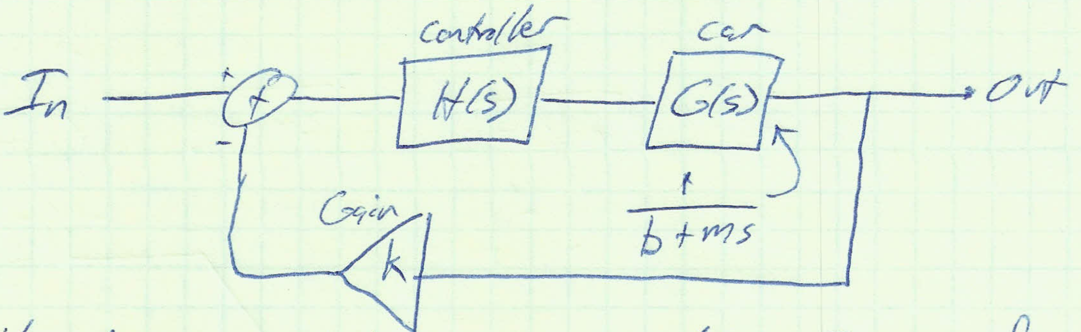
$$V(t) = 2\times10^{-2}\,u(t)\left[1-e^{-5\times10^{-2}t}\right]$$

One newton force from the engine will increase speed by 0.02 m/s

How fast? $e^{-5\times10^{-2}t} = 0.1$ (90% rise time)

$t \simeq 50$ seconds

How do we make the car reach its final speed more quickly?




How does the controller improve acceleration?

for example, low frequency response

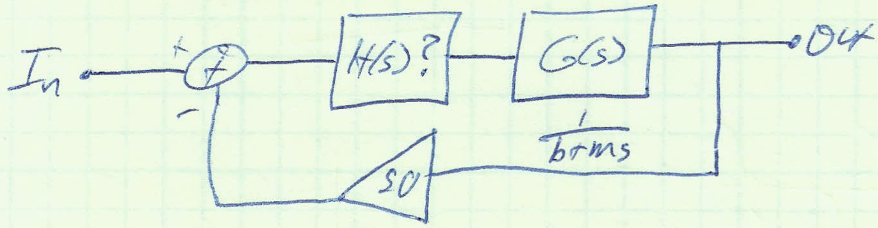
$$Out = G(s)H(s)\left[In - K\,Out\right]$$

$$\frac{Out}{In} = \frac{G(s)H(s)}{1+G(s)H(s)K} \simeq \frac{1}{K} \quad \text{for } G(s)H(s) \gg 1$$

The controller should not change the final velocity,
Just how fast we get there. → low frequency gain should be the same

For the car <u>by itself</u>, the DC gain is $\frac{1}{b}$

the new system should have the same DC gain → $\frac{1}{K} = \frac{1}{b}$, $K = b = 50$




what kind of controller should we use?

Assume a simple gain stage: $H(s) = h$

$$\frac{Out}{In} = \frac{G(s)H(s)}{1+G(s)H(s)K} = \frac{G(s)h}{1+G(s)hK}$$

poles of the transfer function: $1 + G(s)hK = 0$

$$1 + \frac{10^{-3}\,h\,50}{s+5\times10^{-2}} = 0$$

$$s + 5\times10^{-2} + 5\times10^{-2}h = 0$$

$$s + (1+h)5\times10^{-2} = 0$$ → pole has moved

However, the system is still stable.

<u>Before feedback:</u>

pole at $s = -5\times10^{-2}$

<u>After feedback</u>

pole at $s = -(1+h)5\times10^{-2}$

## Step response:

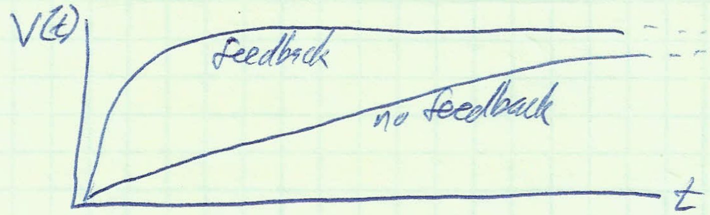
$$2\times10^{-2}\,u(t)\left[1-e^{-(1+h)5\times10^{-2}t}\right]$$

h sets the step response

e.g. $h=100$

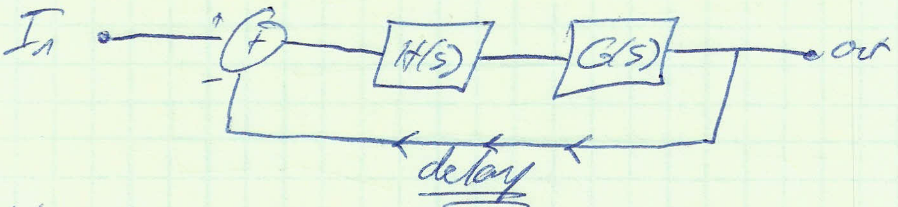
$$V(t)=2\times10^{-2}\,u(t)\left[1-e^{-5t}\right]$$

time to reach 90% of final velocity is ~0.5 seconds

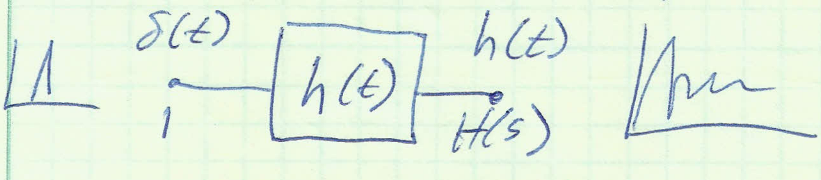


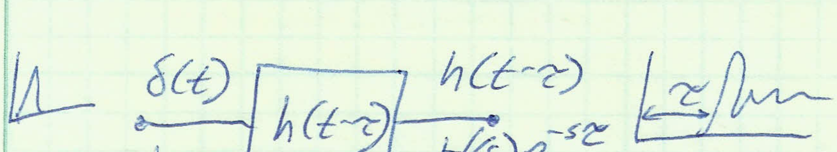

## Effect of delay in feedback loop

can't instantaneously correct for an error in the output

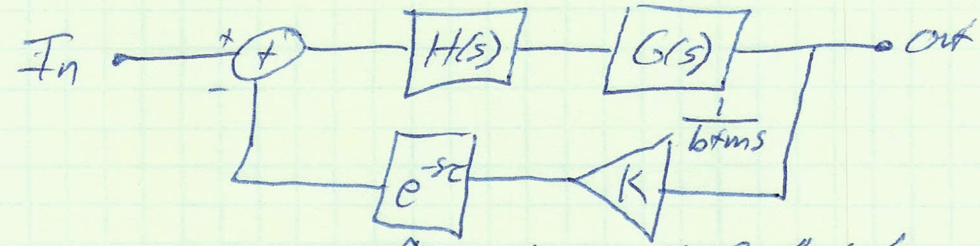



How do we model the time delay?




A delay of $\tau$ in the time domain
corresponds to $e^{-s\tau}$ in the frequency domain




delay in the feedback loop

$$\frac{Out}{In} = \frac{G(s)H(s)}{1+G(s)H(s)Ke^{-s\tau}}$$

poles: $1+G(s)H(s)Ke^{-s\tau} = 0$

not a simple polynomial

Use polynomial approximation for $e^{-s\tau}$

$e^{-s\tau} \simeq 1 + k_1 s + k_2 s^2 + k_3 s^3 + \ldots$ solve for $k_1, k_2, k_3, \ldots$

taylor series for $e^{x} = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots$ $\underline{x = -s\tau}$

$$e^{-s\tau} \simeq 1 - s\tau + \frac{s^2\tau^2}{2} - \frac{s^3\tau^3}{6} + \ldots$$

However, $|e^{-s\tau}| = 1$ always.

but the Taylor series does not have constant magnitude

We need a polynomial approximation of $e^{-s\tau}$ with magnitude = 1 for all values of $\omega$

Remember, $\left|\frac{a-jb}{a+jb}\right| = 1$ always

this is called an "all-pass" response

(for next lecture)