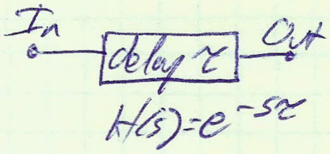
More on time delay



model $e^{-s\tau}$ by $H(s) = \dfrac{a_0 + a_1 s + a_2 s^2 + \dots}{b_0 + b_1 s + b_2 s^2 + \dots}$

we also want $|H(s)| = \left|\dfrac{A-jB}{A+jB}\right| = 1 \rightarrow$ constant amplitude

start with $e^{-s\tau} \simeq \dfrac{1-ks}{1+ks}$, what is k?

remember, $e^{-s\tau} = 1 - s\tau + \dfrac{(s\tau)^2}{2} - \dots$ Taylor series

$$\frac{1-ks}{1+ks} = 1 - s\tau + \frac{s^2\tau^2}{2}$$

$$1-ks = \left(1 - s\tau + \frac{s^2\tau^2}{2}\right)(1+ks)$$

$$1-ks = 1 - s\tau + \frac{s^2\tau^2}{2} + ks - k\tau s^2 + \frac{k\tau^2 s^3}{2}$$

s terms: $2k - \tau = 0 \rightarrow k = \tau/2$

s^2 terms: $\dfrac{\tau^2}{2} - k\tau = 0 \rightarrow k = \tau/2$

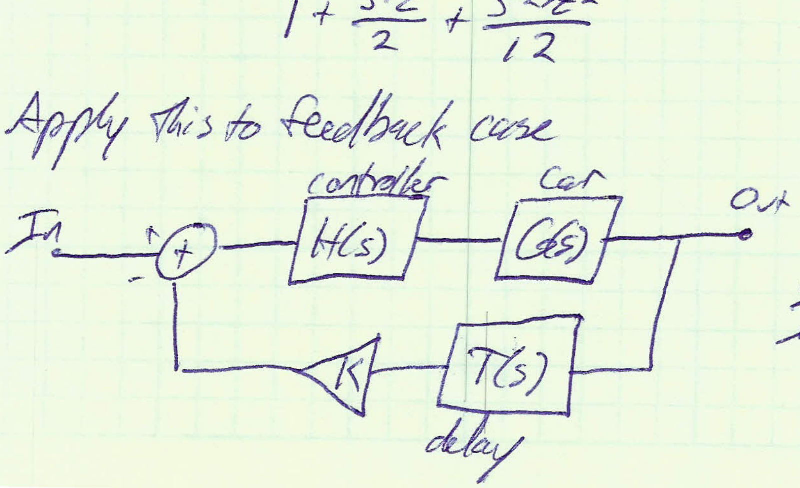
s^3 terms: $\dfrac{k\tau^2}{2} = 0 \rightarrow k = 0$ (don't use this result)

so $\underline{k = \tau/2}$ and $e^{-s\tau} \simeq \dfrac{1 - \frac{\tau}{2}s}{1 + \frac{\tau}{2}s}$ this is called a Pade approximation (first order)

2nd order Pade approximation:

$$e^{-s\tau} \simeq \frac{1 - \frac{s\tau}{2} + \frac{s^2\tau^2}{12}}{1 + \frac{s\tau}{2} + \frac{s^2\tau^2}{12}}$$

Apply this to feedback case



$$\frac{Out}{In} = \frac{G(s)\,H(s)}{1 + G(s)H(s)T(s)\,K}$$

Is the system still stable?

poles of $\frac{Out}{In}$ must be in LHP. (look for zeros of $1+GHTK$)

$1+G(s)H(s)T(s)K=0$ does the delay make the system unstable?

using the car example from last lecture:

$G(s)=\frac{1/m}{s+b/m}$ $m=1000$ kg $b=50$ Ns/m $G=\frac{10^{-3}}{s+5\times10^{-2}}$

$H(s)=h$ (to be determined)

$K=50$ (chosen to make the final velocity the same as the car alone)

$$1+\frac{10^{-3}}{s+5\times10^{-2}}h\cdot50\cdot\frac{2/\tau-s}{2/\tau+s}=0$$

$$(s+5\times10^{-2})\left(\frac{2}{\tau}+s\right)+5\times10^{-2}h\left(\frac{2}{\tau}-s\right)=0$$

$$s^2+s\left(5\times10^{-2}+\frac{2}{\tau}-5\times10^{-2}h\right)+\left(5\times10^{-2}\cdot\frac{2}{\tau}+5\times10^{-2}h\frac{2}{\tau}\right)=0$$

$$s^2+\left(\frac{2}{\tau}-(h-1)5\times10^{-2}\right)s+\frac{(h+1)10^{-1}}{\tau}=0$$

This is a 2nd order system

$$s^2+2\zeta\omega_0 s+\omega_0^2=0$$

if $\zeta<0$, the system is <u>unstable</u>

$\frac{2}{\tau}-(h-1)\times5\times10^{-2}>0$ for system to be stable

This puts limits on the gain of the system

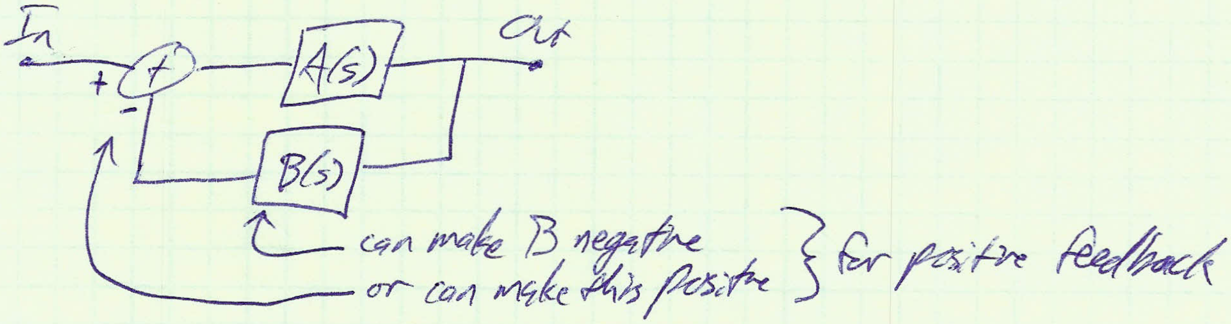
If h is too big, the system becomes unstable

In this case, $h<1+\frac{2}{5\times10^{-2}\tau}$ for stability

if $\tau=1$ sec, $h<41$

So far in this class we have covered:

- Filters, 2nd order behavior
- Behavior of 2nd order systems
- Feedback in circuits, and stability criterion
- General applications of feedback
  - make things more stable
  - make things more responsive
  - make things less stable (oscillators)

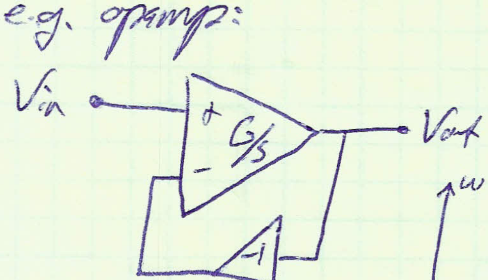


Negative feedback → usually to make something stable

Positive feedback → usually to make something unstable

(but not all positive feedback results in an oscillator)

e.g. opamp:



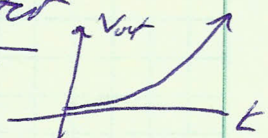
$$V_{out} = G/s\,(V_{in} + V_{out})$$

$$V_{out}(1 - G/s) = V_{in}\,G/s$$

$$\frac{V_{out}}{V_{in}} = \frac{G}{s-G} \leftarrow \text{pole at } s = G$$

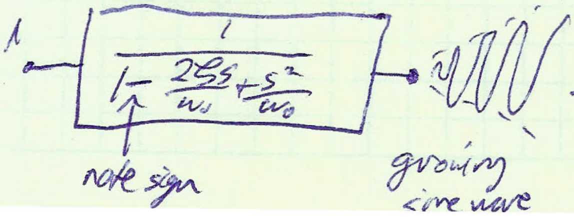
unstable, but <u>not an oscillator</u>

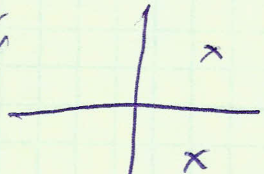
Laplace transform $\frac{1}{s-a} \leftrightarrow e^{at}$



output grows indefinitely with infinitesimal input (e.g. noise)

<u>This</u> is an oscillator (almost)

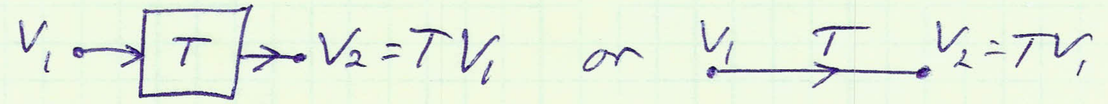




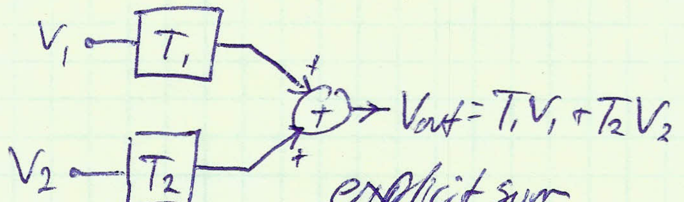
A good oscillator needs the poles to be right on the imaginary axis!

Signal Flow Graphs

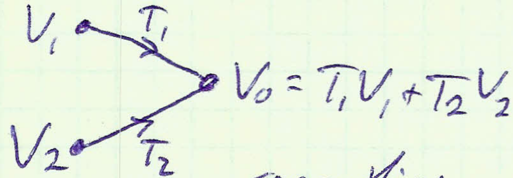
- Elements are <u>nodes</u> and <u>branches</u>
- Nodes carry values (V_{in}, V_{out}, V_x, ...)
- Branches have transmittances (e.g. $V_{out} = H V_{in}$)
  they are directed → there is an <u>input</u> and an <u>output</u>



- If two or more branches terminate in a node, the values <u>sum</u>

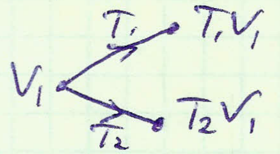


<u>explicit sum</u>



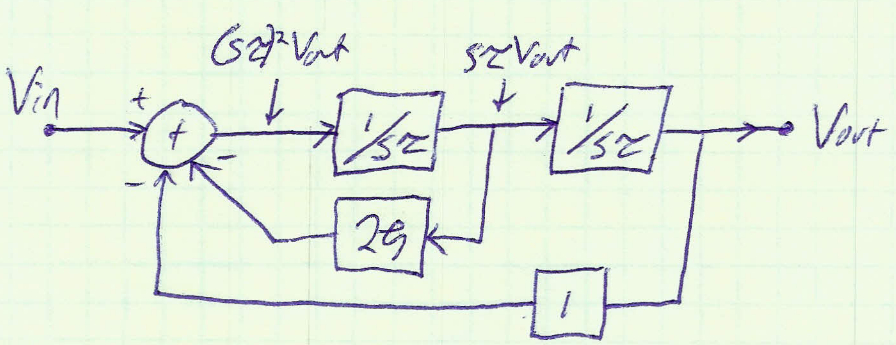
<u>same thing</u>

- Any number of branches leaving a node <u>distribute</u> the same value



These are particularly useful for op-amp circuits because op-amps are good buffers (one-way circuits)

Consider a circuit like this:

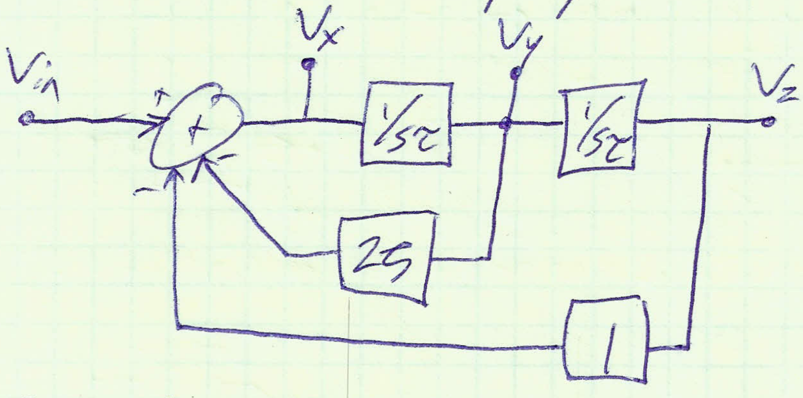


$$V_{in} - V_{out} - 2\zeta s\tau V_{out} = s^2\tau^2 V_{out}$$

$$V_{in} = V_{out}\left(1 + 2\zeta s\tau + s^2\tau^2\right)$$

$$\frac{V_{out}}{V_{in}} = \frac{1}{1+2\zeta s\tau + s^2\tau^2} \rightarrow \text{low-pass filter}$$

we can also take other output points:

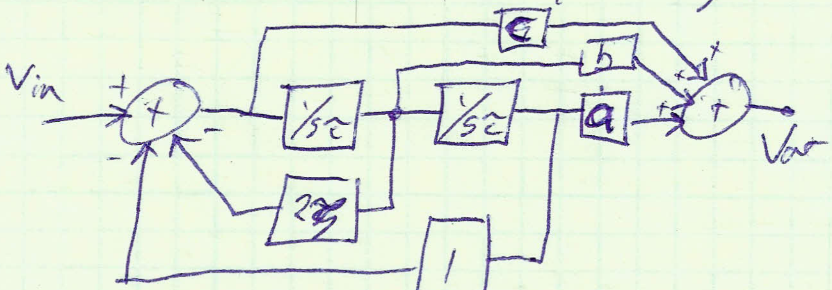


$$\frac{V_x}{V_{in}} = \frac{s^2\tau^2}{1+2\zeta s\tau+s^2\tau^2} : \text{HPF}$$

$$\frac{V_y}{V_{in}} = \frac{s\tau}{1+2\zeta s\tau+s^2\tau^2} : \text{BPF}$$

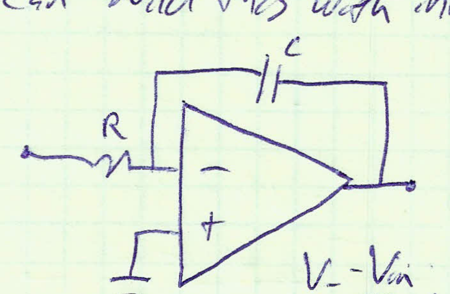
$$\frac{V_z}{V_{in}} = \frac{1}{1+2\zeta s\tau+s^2\tau^2} : \text{LPF}$$

General case: Biquad ("biquadratic")

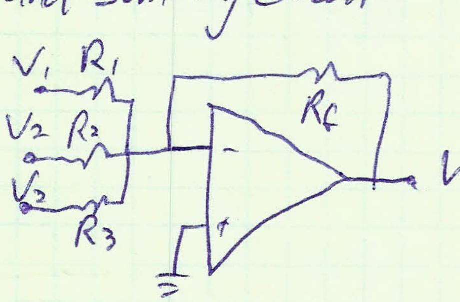


$$\frac{V_{out}}{V_{in}} = \frac{a + bs\tau + cs^2\tau^2}{1+2\zeta s\tau+s^2\tau^2}$$

can build this with integrators and summing circuit



$V_- = V_+ = 0$



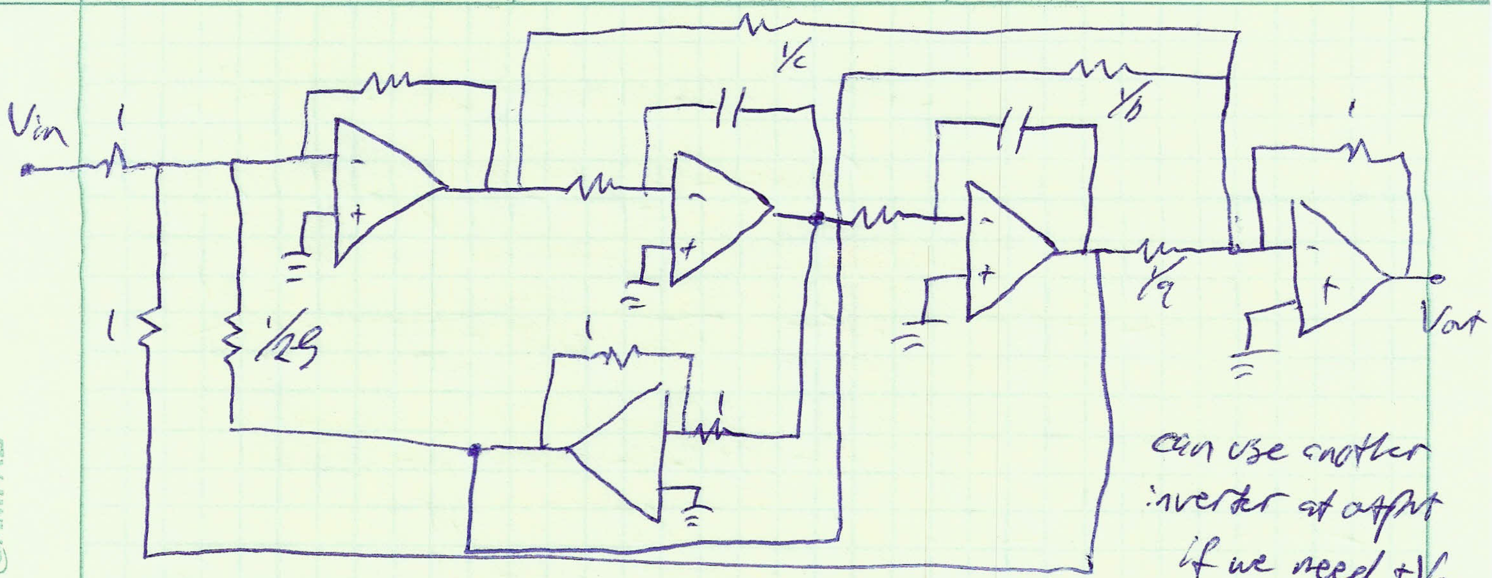
$$\frac{V_- - V_{in}}{R} + \frac{V_- - V_{out}}{1/sC} = 0$$

$$\frac{-V_{in}}{R} = V_{out}sC$$

$$\frac{V_{out}}{V_{in}} = \frac{-1}{sRC}$$

$$\frac{V_- - V_1}{R_1} + \frac{V_- - V_2}{R_2} + \frac{V_- - V_3}{R_3} + \frac{V_- - V_{out}}{R_f} = 0$$

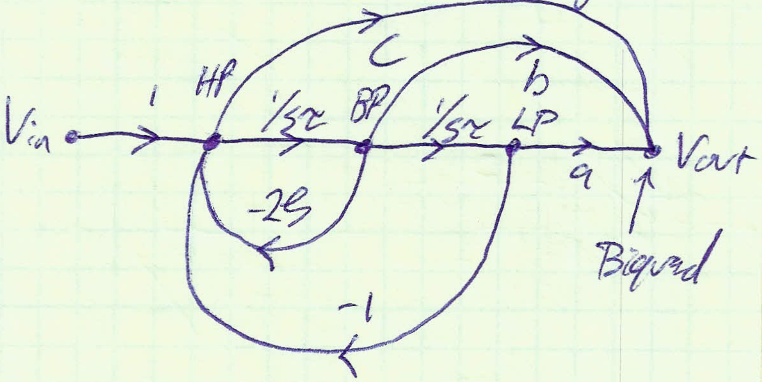
$$V_{out} = -R_f\left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3}\right)$$



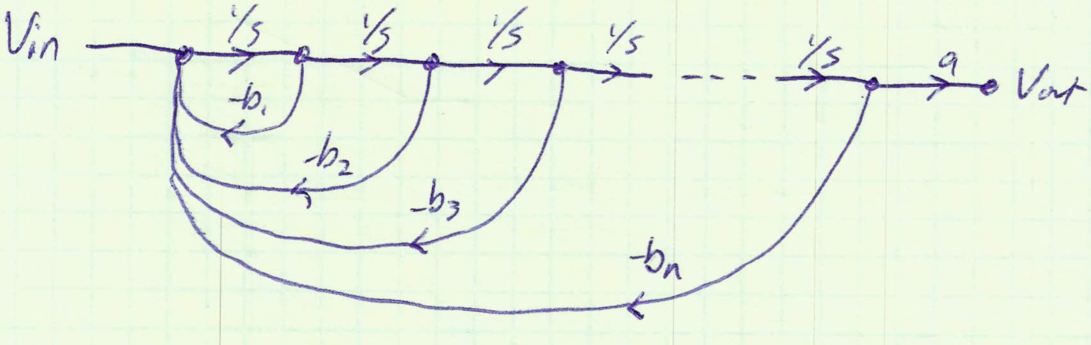
can use another inverter at output if we need $+V_o$

this would give $\dfrac{V_{out}}{V_{in}} = \dfrac{a + bs\tau + cs^2\tau^2}{1 + 2\zeta s\tau + s^2\tau^2}$

We can also draw this as a signal flow graph:



Can also extend the biquad concept:



$$\frac{V_{out}}{V_{in}} = \frac{1}{s^n + b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n}$$