Mason's Gain Formula

$$G = \frac{Out}{In} = \frac{\sum_{k=1}^{N} G_k \Delta_k}{\Delta}$$

$$\Delta = 1 - \Sigma L_i + \Sigma L_i L_j - \Sigma L_i L_j L_k + \cdots$$

Δ = determinant of the graph

N = number of forward paths between In and Out

G_k = gain of kth forward path

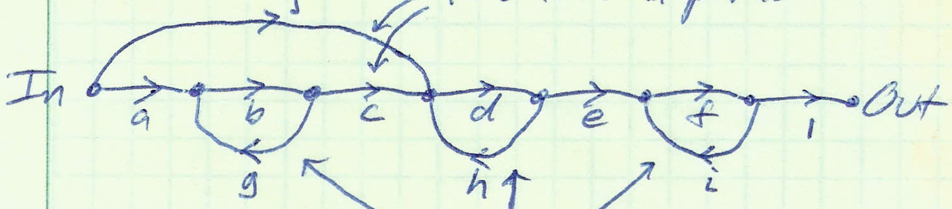
L_i = loop gain of each closed loop

$L_i L_j$ = product of loop gains of any non-touching loops (no common nodes)

Δ_k = cofactor of Δ for the kth forward path with loops touching the kth forward path removed

Example:

two forward paths



three loops

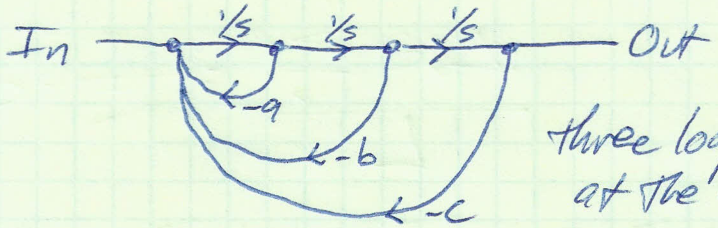
$\Delta = 1 - bg - dh - fi + bgdh + bgfi + dhfi - bgdhfi$

path 1: $G_1 = abcdef$, $\Delta_1 = 1$ (touches all loops)

path 2: $G_2 = jdef$, $\Delta_2 = 1 - bg$ (this path does not touch loop bg)

$$G = \frac{abcdef + jdef(1-bg)}{\Delta}$$

Another example:



three loops, all touching at the first node

$L_1 = -a/s$

$L_2 = -b/s^2$

$L_3 = -c/s^3$

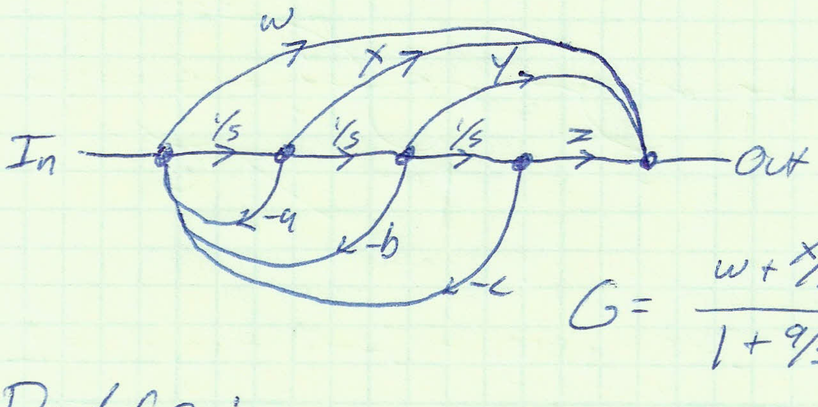
$\Delta = 1 + a/s + b/s^2 + c/s^3$

One forward path, $G_1 = 1/s^3$, that touches all loops

$$G = \frac{1/s^3}{1 + a/s + b/s^2 + c/s^3} = \frac{1}{s^3 + as^2 + bs + c}$$

We can add paths without adding complexity if the paths touch all loops $\rightarrow \Delta_i = 1$ for all i

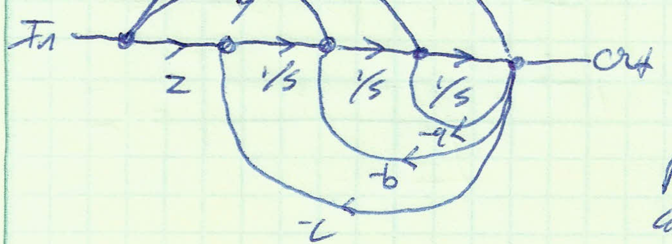
Here, all loops touch the first summing node so all forward paths should go through that node.



$$G = \frac{w + x/s + y/s^2 + z/s^3}{1 + a/s + b/s^2 + c/s^3}$$

Dual form:

$$G = \frac{ws^3 + xs^2 + ys + z}{s^3 + as^2 + bs + c}$$

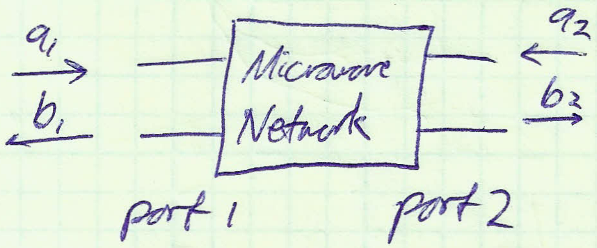


<u>Flow reversal theorem</u>:

Reverse direction of all branches and exchange In and Out, the transfer function is unchanged

Example: Microwave Circuits

Traveling waves instead of simple voltage and current



$$\begin{bmatrix} b_1 \\ b_2 \end{bmatrix} = \begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \end{bmatrix}$$

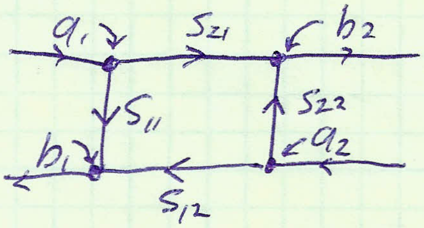
$$b_1 = S_{11} a_1 + S_{12} a_2$$

$$b_2 = S_{21} a_1 + S_{22} a_2$$

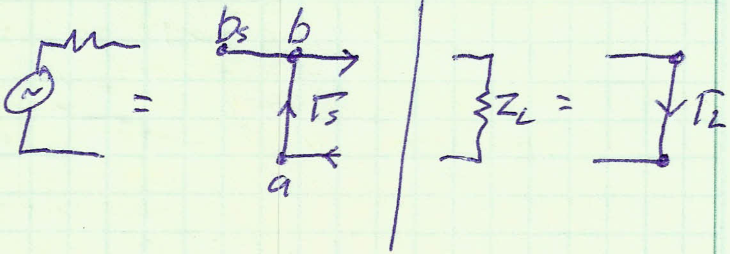
Ideal amplifier:

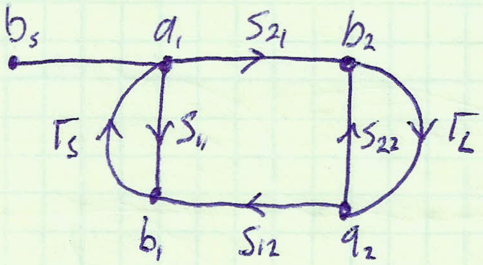
$S_{11} = S_{22} = S_{12} = 0, \quad S_{21} = G$

General case:



Sources and loads have reflection coefficients:





3 loops

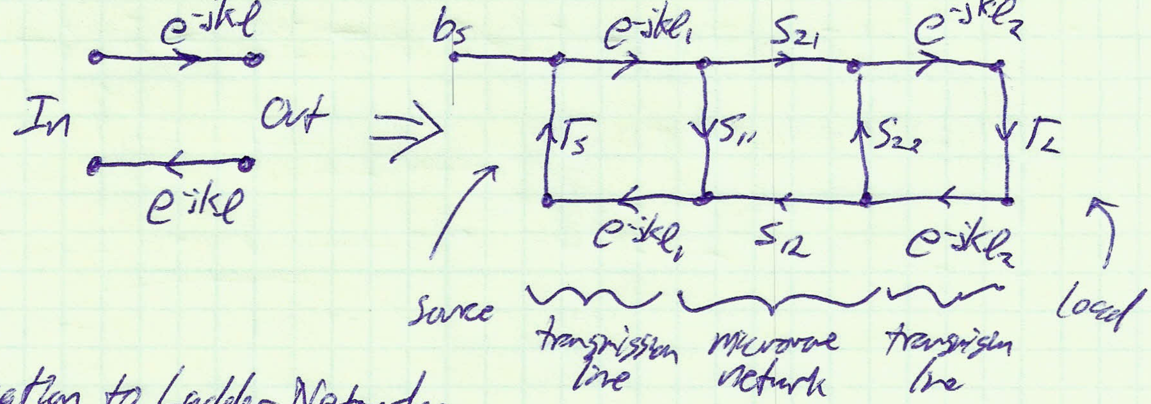
$$\Delta = 1 - S_{11}\Gamma_s - S_{22}\Gamma_L - S_{21}\Gamma_L S_{12}\Gamma_s \underbrace{+ S_{11}\Gamma_s S_{22}\Gamma_L}_{\text{two non-touching loops}}$$

$$\frac{b_2}{b_s} = \frac{S_{21}}{1 - S_{11}\Gamma_s - S_{22}\Gamma_L - (S_{21}S_{12} - S_{11}S_{22})\Gamma_L\Gamma_s}$$

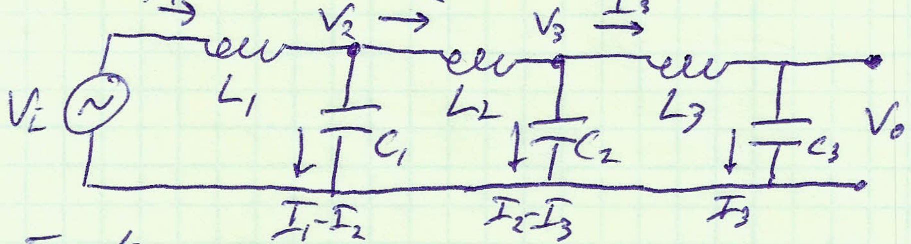
one forward path

$$\frac{b_1}{b_s} = \frac{S_{11}(1 - S_{22}\Gamma_L) + S_{21}\Gamma_L S_{12}(1)}{1 - S_{11}\Gamma_s - S_{22}\Gamma_L - (S_{21}S_{12} - S_{11}S_{22})\Gamma_L\Gamma_s}$$

We can also easily include a length of transmission line



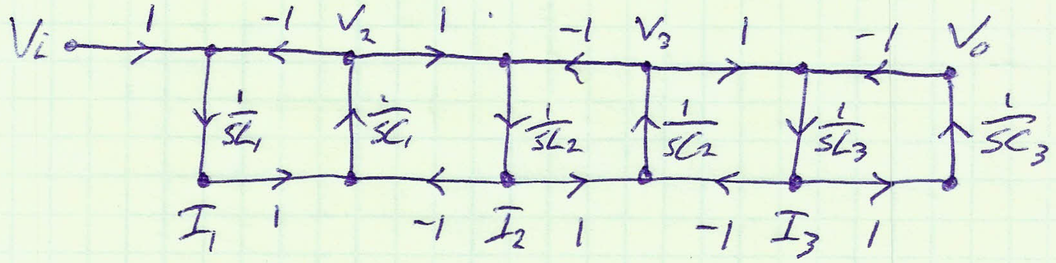
Application to Ladder Networks



Equations:

$$I_1 = \frac{V_i - V_2}{sL_1}, \quad I_2 = \frac{V_2 - V_3}{sL_2}, \quad I_3 = \frac{V_3 - V_o}{sL_3}$$

$$V_2 = \frac{I_1 - I_2}{sC_1}, \quad V_3 = \frac{I_2 - I_3}{sC_2}$$



loops: L_1 L_2 L_3 L_4 L_5

$$L_1 = \frac{-1}{s^2 L_1 C_1}, \quad L_2 = \frac{-1}{s^2 L_2 C_1}, \quad \ldots$$

Only one forward path touching all loops

$$G_1 = \frac{1}{sL_1}\,\frac{1}{sC_1}\,\frac{1}{sL_2}\,\frac{1}{sC_2}\,\frac{1}{sL_3}\,\frac{1}{sC_3} = -L_1 L_3 L_5$$

↑ loop gains, not inductors

$$\Delta = 1 - L_1 - L_2 - L_3 - L_4 - L_5 + L_1L_3 + L_1L_4 + L_1L_5 + L_2L_4 + L_2L_5 + L_3L_5 - L_1L_3L_5$$

$$H = \frac{-L_1L_3L_5}{\Delta}$$

Lab #6 System Analysis and Design

1. Dual-Integrator Analysis

   find overshoot based on ζ (analytical)

   find step response (numerical), blocks for $\boxed{1/s}$, $g\triangleright$, Σ in Simulink

2. Dual Integrator Realization

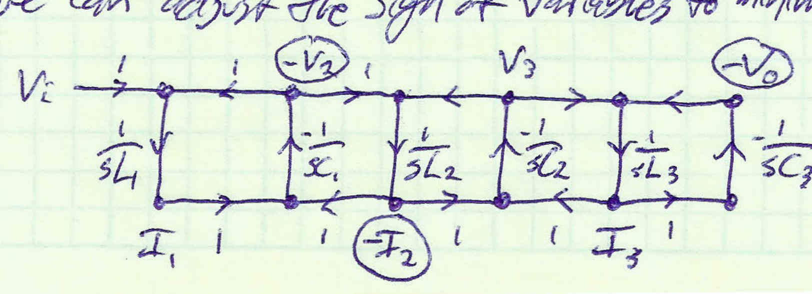
   try minimizing the number of op-amps
   by absorbing the sum function into the integrator where possible

3. Simulink is very good for nonlinear systems and
   mixed analog/digital systems, so it's good for <u>oscillators</u>

4. We discussed the effect of <u>time delay</u> but didn't try it.
   Now is your chance

5. Signal flow graph of ladder network

   We can adjust the sign of variables to minimize # of inverters



Op-amp implementation of LC ladder network
Called "Leapfrog ladder" circuit, used in IC filters
Realize the equations of a passive circuit so that the active circuit has the stability and sensitivity of passive circuit

Sensitivity : how accurately we must specify component values

example:

$$H(s) = \frac{1}{1 + sCR + s^2LC}$$

$$\omega_0 = \frac{1}{\sqrt{LC}}, \quad \frac{2\zeta}{\omega_0} = CR$$

$$\zeta = \frac{1}{2} R \sqrt{\frac{C}{L}}$$

$$\underset{\substack{\uparrow \\ \text{fractional} \\ \text{error in } \omega_0}}{\frac{\delta \omega_0}{\omega_0}} = S_L^{\omega_0} \underset{\substack{\uparrow \\ \text{fractional} \\ \text{error in } L}}{\frac{\delta L}{L}} + S_C^{\omega_0} \underset{\substack{\uparrow \\ \text{fractional} \\ \text{error in } C}}{\frac{\delta C}{C}}$$

We want $S_L^{\omega_0}$ and all others $\lesssim 1$

Calculate S using logs,

$$\omega_0^2 = \frac{1}{LC} \longrightarrow 2 \log \omega_0 = -\log L - \log C$$

take $\frac{\partial}{\partial L}$ of whole equation

$$\frac{2}{\omega_0} \frac{\partial \omega_0}{\partial L} = \frac{-1}{L}$$

$$\frac{\partial \omega_0}{\omega_0} = \frac{-1}{2} \frac{\partial L}{L} \quad \text{so } S_L^{\omega_0} = -\frac{1}{2}$$