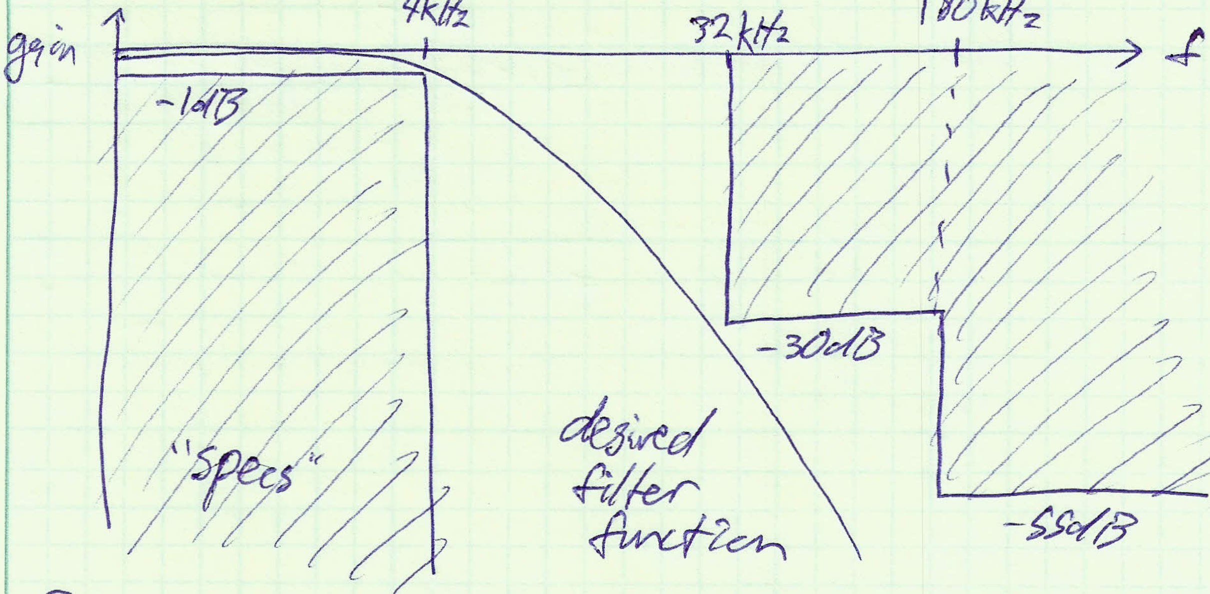
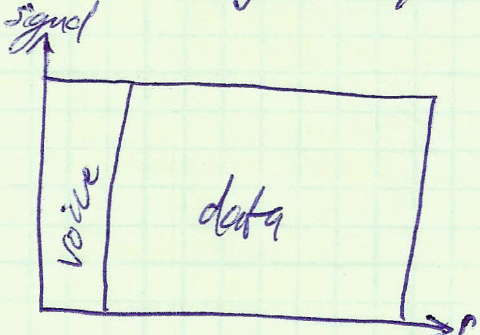
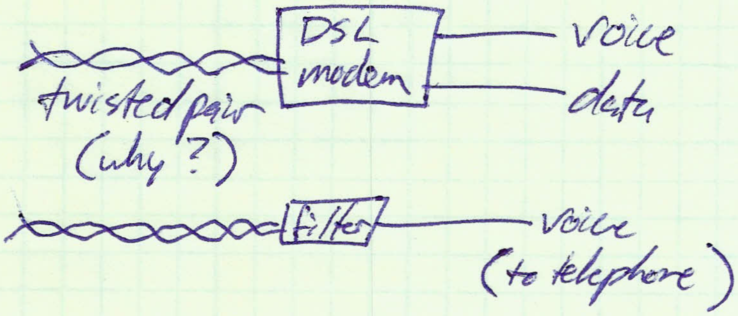
Lab #2 Discussion



Purpose: Pass voice signals (<4kHz) but reject higher frequency DSL signals (~4kHz - 4MHz)





First - try simple Butterworth filter (2nd order)

Note - rule of -20dB/decade per pole

is same as -6dB/octave per pole

4kHz → 32kHz is 3 octaves

$3 \times 6 \times 2$ poles ⇒ 36 dB ... should be fine. (?)

However, $\omega = \omega_0$ is the -3dB point $|H(s)|^2 = \frac{1}{1+(\omega/\omega_0)^{2n}}$

We need -1dB at 4kHz, so ω_0 must be >4kHz

We do not have 3 full octaves available (

2nd order Butterworth:

$$|H(\omega)|^2 = \frac{1}{1+(f/f_0)^4}$$

$|H(\omega)|^2 = -1\text{dB}$ at 4 kHz

$$10^{-0.1} = 0.7943 = \frac{1}{1+\left(\frac{4\times10^3}{f_0}\right)^4} \quad \text{solve for } f_0$$

$f_0 = 5.61$ kHz

Next check -30dB spec at 32 kHz

$$|H(\omega)|^2 = \frac{1}{1+(f/f_0)^4} = \frac{1}{1+\left(\frac{32}{5.61}\right)^4} = 9.4\times10^{-4}$$

(note dB is $20\log_{10}$ Gain or $10\log_{10}$ Gain^2)

$$10\log_{10}(9.4\times10^{-4}) = -30.26 \text{ dB}$$

Just barely meets both specs, by a fraction of a dB.
This is not enough for typical device tolerances
Electronic devices have values based on "preferred numbers"

Divide 1 to 10 into 6, 12, 24, 48, 96 steps
these become 20%, 10%, 5%, 2%, 1% tolerances

Equal ratio between adjacent steps: $X^n = 10 \longrightarrow X = 10^{1/n}$

X: ratio; n: number of steps; 10: 1-10

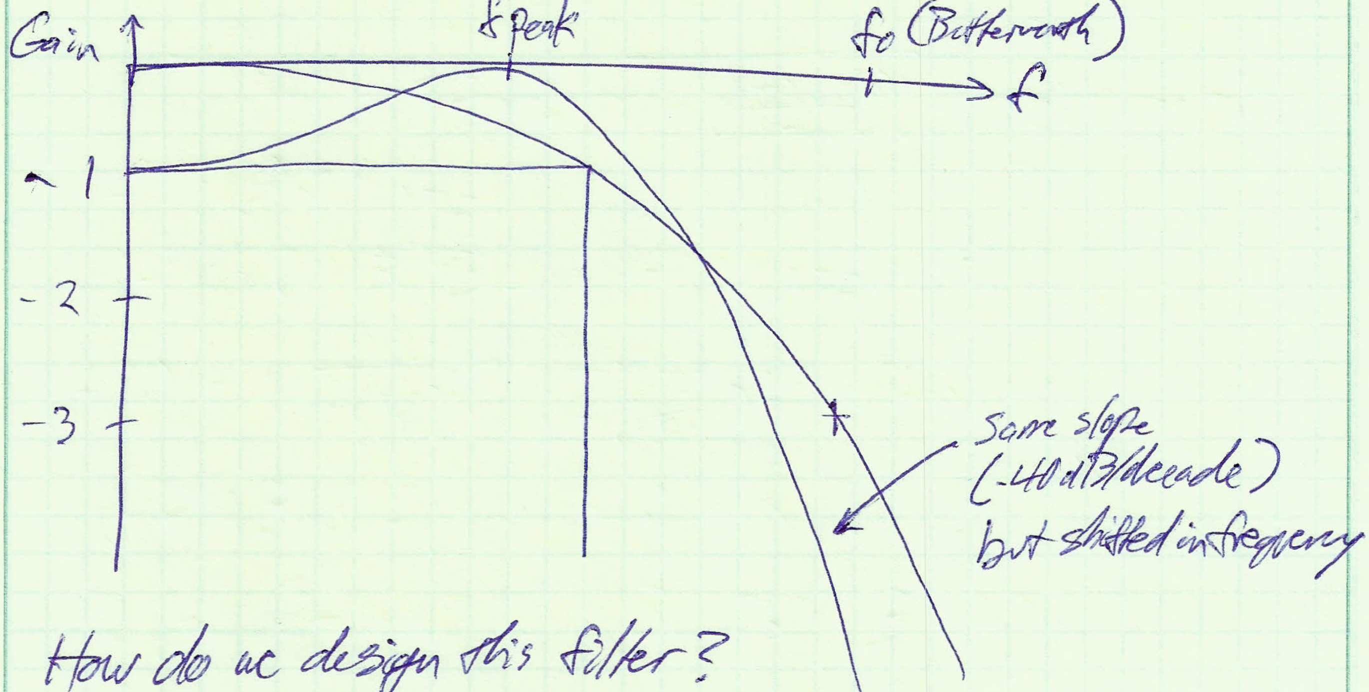
Ex: $n = 6$
$X = 1.4678$

$$X^m\Big|_{m=0}^{n} = 1, 1.47, 2.15, 3.16, 4.64, 6.81, 10$$

Standard 20% resistors: 10, 15, 22, 33, 47, 68, 100 . . .

How to have better performance margin:

1. Use 3rd order filter → 50% more expensive
2. Use something other than Butterworth.



How do we design this filter?

$$H(s) = \frac{(0.7943)^{1/2}}{1 + \frac{2\zeta s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$ ← DC gain = -1dB

find ζ and ω_0

remember $\frac{\partial}{\partial\omega}\left(\frac{1}{|H(\omega)|^2}\right) = 0$

1dB

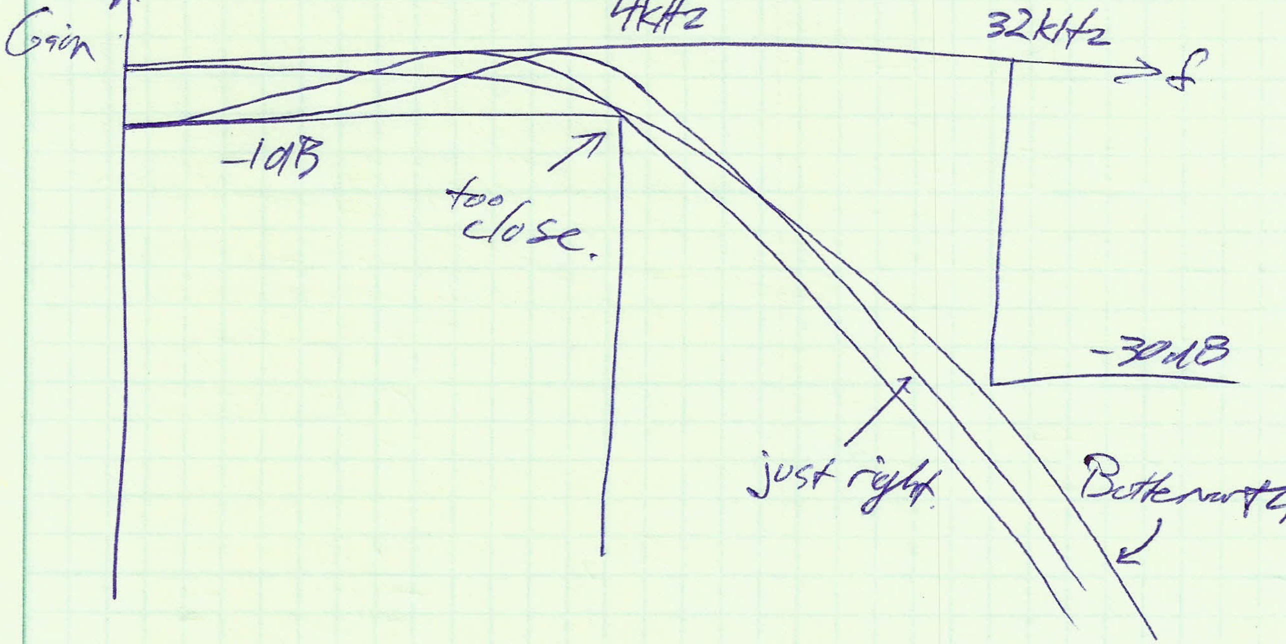
$$\frac{|H(\omega_{peak})|^2}{|H(\omega=0)|^2} = \frac{1}{4\zeta^2(1-\zeta^2)} = 10^{0.1} = 1.2589$$

Solve for ζ. Optimum value in range of 0.1-0.7

Next solve for ω_0

$$|H(\omega)|^2 = \frac{0.7943}{(1-\omega^2/\omega_0)^2 + (2\zeta\omega/\omega_0)^2}$$

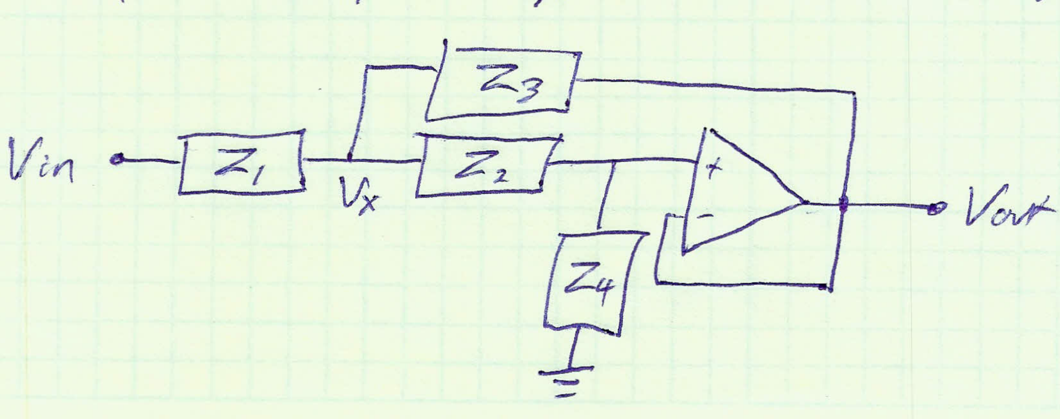
Optimize values of ζ and ω_0 to have wide margin on all specs



Why are we using an active filter instead of RLC?

Inductors are
- Lossy (low Q)
- Large (especially on-chip)
- Have low resonance frequency
- Less accurate than R, C
- More expensive (often wire wound)

Instead we use Sallen-Key active filter
(R.P. Sallen, E.L. Key, MIT Lincoln Lab, 1955)



Op-amp with negative feedback: $V_+ = V_- = V_{out}$

KCL at node x:

$$\frac{V_x - V_{in}}{Z_1} + \frac{V_x - V_{out}}{Z_3} + \frac{V_x - V_+}{Z_2} = 0$$

KCL at node +: (equal to V_{out})

$$\frac{V_{out} - V_x}{Z_2} + \frac{V_{out}}{Z_4} = 0$$

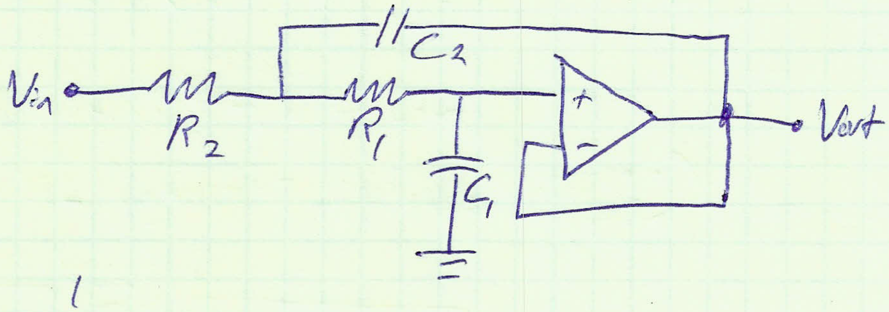
$$V_x = V_{out}\left(\frac{Z_2}{Z_4} + 1\right)$$

substitute into this equation and rearrange

$$\frac{V_{out}}{V_{in}} = \frac{Z_3 Z_4}{Z_1 Z_2 + Z_3(Z_1 + Z_2) + Z_3 Z_4}$$

or $$\frac{V_{out}}{V_{in}} = \frac{1}{1 + \frac{Z_1 + Z_2}{Z_4} + \frac{Z_1 Z_2}{Z_3 Z_4}}$$

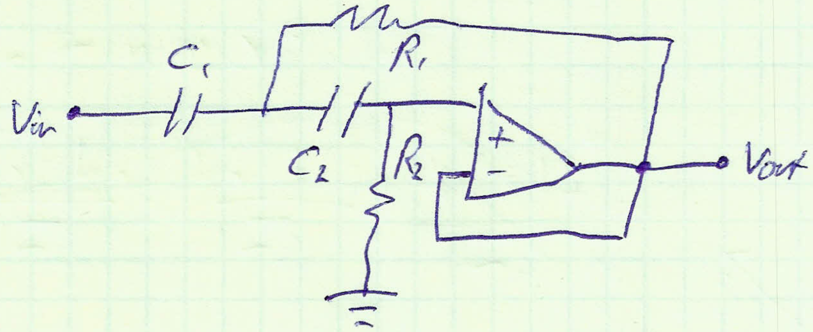
For our case:



$$\frac{V_{out}}{V_{in}} = \frac{1}{1 + sC_1(R_1 + R_2) + s^2 C_1 C_2 R_1 R_2}$$

$$= \frac{1}{1 + \frac{2\zeta s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

Note that we can also make a high-pass filter:

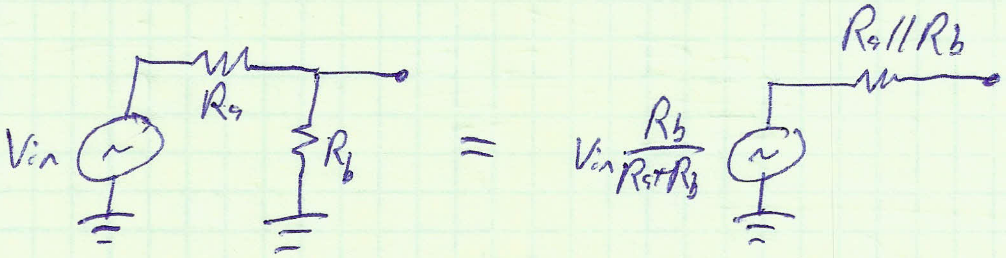


$$\frac{V_{out}}{V_{in}} = \frac{R_1 R_2}{\frac{1}{s^2 C_1 C_2} + R_1\left(\frac{1}{sC_1} + \frac{1}{sC_2}\right) + R_1 R_2}$$

$$= \frac{s^2 C_1 C_2 R_1 R_2}{1 + R_1(sC_1 + sC_2) + s^2 C_1 C_2 R_1 R_2}$$

Other notes on lab #2:

Reducing DC gain:



Output resistance: