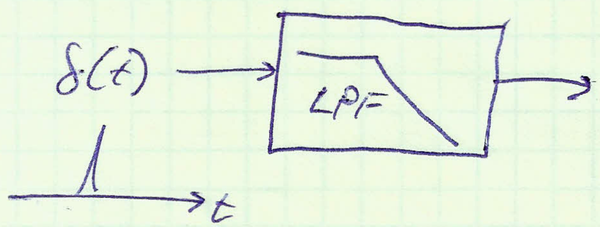
Impulse Response of Some Systems We Have Studied:

$\delta(t) \rightarrow V_{in}(s) = 1 \longrightarrow V_{out}(s) = H(s)$

arbitrary $V_{in}(s) \longrightarrow V_{out}(s) = H(s) \cdot V_{in}(s)$

Perform the inverse transform $V_{out}(s) \rightarrow V_{out}(t)$ using <u>tables</u>

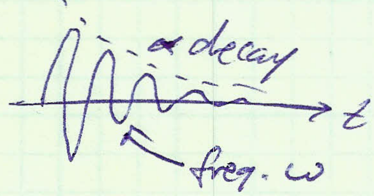
<u>Low Pass Filter</u>



$V_{out}(s) = H(s) = \dfrac{1}{1 + \frac{2\zeta s}{\omega_0} + \frac{s^2}{\omega_0^2}}$

fits form of $H(s) = \dfrac{\omega}{(s+\alpha)^2 + \omega^2}$

$V_{out}(t) = e^{-\alpha t} \sin \omega t$



$$H(s) = \frac{\omega_0^2}{s^2 + 2\zeta s \omega_0 + \omega_0^2} \quad \times \omega_0^2$$

$$= \frac{\omega_0^2}{(s^2 + 2\zeta s\omega_0 + \zeta^2\omega_0^2) + \omega_0^2 - \zeta^2\omega_0^2}$$

complete square

$$= \frac{\omega_0^2}{(s+\zeta\omega_0)^2 + \omega_0^2(1-\zeta^2)} \times \left[\frac{\omega_0^2(1-\zeta^2)}{\omega_0^2(1-\zeta^2)}\right]^{1/2}$$

$$= \frac{\omega_0^2}{[\omega_0^2(1-\zeta^2)]^{1/2}} \cdot \frac{[\omega_0^2(1-\zeta^2)]^{1/2}}{(s+\zeta\omega_0)^2 + \omega_0^2(1-\zeta^2)} = K \frac{\omega}{(s+\alpha)^2 + \omega^2}$$

constant (scale factor)

$$V_{out}(t) = \frac{\omega_0}{(1-\zeta^2)^{1/2}} e^{-\zeta\omega_0 t} \sin\left[\omega_0(1-\zeta^2)^{1/2} t\right]$$

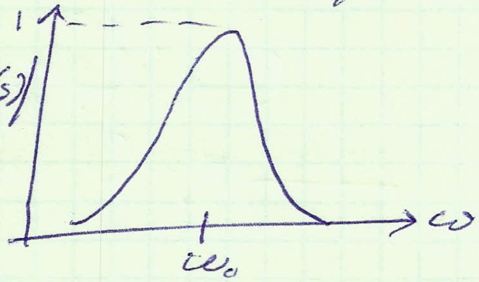
Band Pass Filter

$$H(s) = \frac{2\zeta s/\omega_0}{1 + \frac{2\zeta s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

at $s=0$, $|H(s)| = \frac{0}{1} = 0$

at $s=\infty$, $|H(s)| = \frac{2\zeta s/\omega_0}{s^2/\omega_0^2} = 0$

at $s=j\omega_0$, $|H(s)| = \frac{2\zeta s/\omega_0}{2\zeta s/\omega_0} = 1$



we can solve this with:

$$\frac{(s+\alpha)}{(s+\alpha)^2 + \omega^2} \rightarrow e^{-\alpha t}\cos\omega t$$

and

$$\frac{\omega}{(s+\alpha)^2 + \omega^2} \rightarrow e^{-\alpha t}\sin\omega t$$

Why? Force $H(s)$ into this form

$$\frac{K_1(s+\alpha) + K_2\omega}{(s+\alpha)^2 + \omega^2}$$

This term is whatever is left after finding $(s+\alpha)$ term.

$$H(s) = \frac{2\zeta s/\omega_0}{1 + \frac{2\zeta s}{\omega_0} + \frac{s^2}{\omega_0^2}} \cdot \frac{\omega_0^2}{\omega_0^2} = \frac{2\zeta\omega_0 s}{(s+\zeta\omega_0)^2 + \omega_0^2(1-\zeta^2)}$$

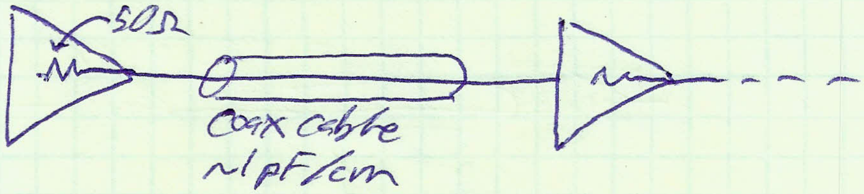
see LPF solution

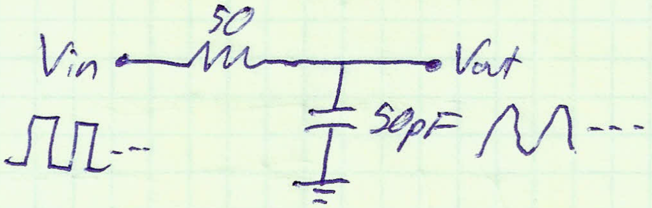
$$= \frac{2\zeta\omega_0(s+\zeta\omega_0)}{(s+\zeta\omega_0)^2 + \omega_0^2(1-\zeta^2)} - \frac{2\zeta^2\omega_0^2}{(s+\zeta\omega_0)^2 + \omega_0^2(1-\zeta^2)}$$

$$h(t) = 2\zeta\omega_0 e^{-\zeta\omega_0 t}\cos\left[\omega_0(1-\zeta^2)^{1/2}t\right]$$

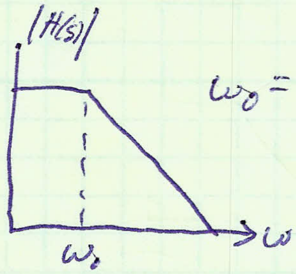
$$- \frac{2\zeta^2\omega_0^2}{\omega_0(1-\zeta^2)^{1/2}} e^{-\zeta\omega_0 t}\sin\left[\omega_0(1-\zeta^2)^{1/2}t\right]$$

Lab #3 - Voltage Follower Circuit





$$H(s) = \frac{1/sC}{R + 1/sC} = \frac{1}{1+sRC} = \frac{1}{1+s/\omega_0}$$



$$\omega_0 = \frac{1}{RC} = \frac{1}{50 \times 50 \times 10^{-12}}$$

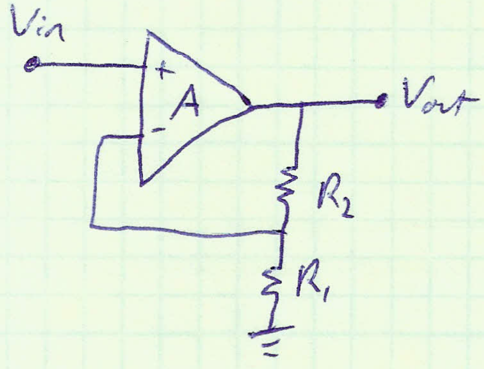
$$f_0 = 64\,MHz$$

Data Rate limited to < 64Mb/s

How do we send 10Gb/s?

Lower Capacitance? - use optical fiber → expensive!

Lower Rout? 50Ω → 0.5Ω Use Feedback!

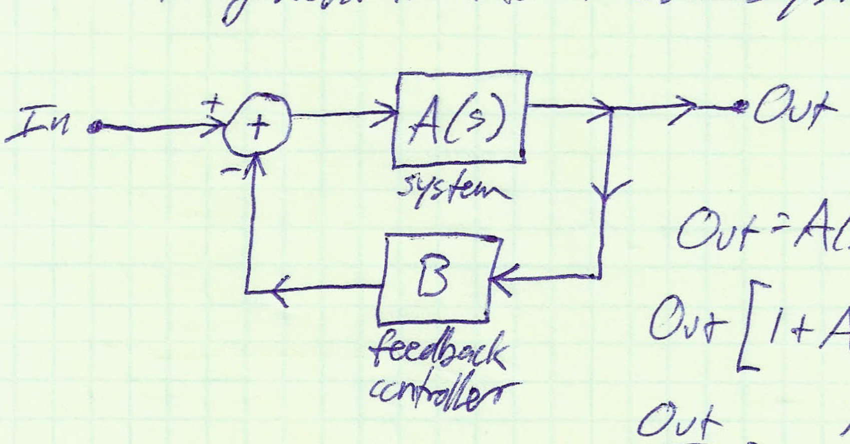


$$V_- = V_{out}\frac{R_1}{R_1+R_2} = \underline{\underline{B}}\,V_{out}$$

$$V_{out} = A(V_+ - V_-) = A(V_{in} - BV_{out})$$

$$\underline{\frac{V_{out}}{V_{in}} = \frac{A}{1+AB}}$$

This is the general form for feedback systems:

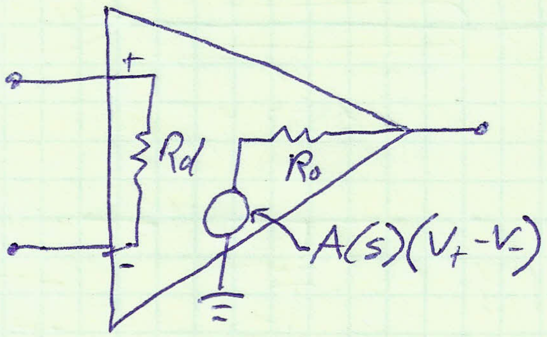


$$Out = A(s)\left[In - B \cdot Out\right]$$

$$Out\left[1 + A(s)\cdot B\right] = In \cdot A(s)$$

$$\frac{Out}{In} = \frac{A(s)}{1+A(s)\cdot B}$$

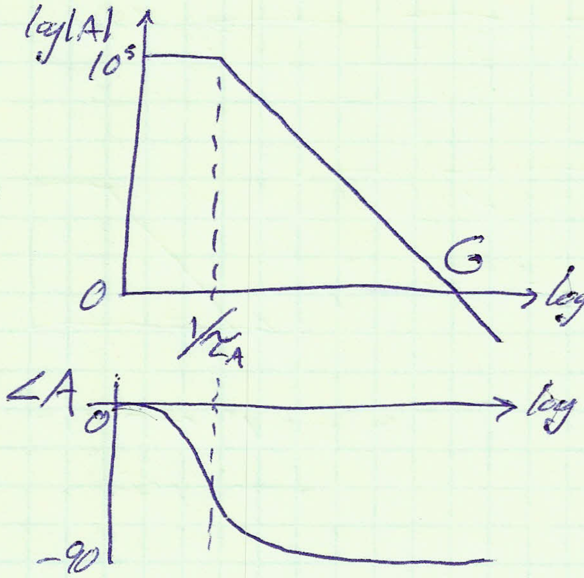
Real Op-Amp Behavior



$R_d \approx 10^7$ (ideally ∞)

$R_o \approx 100$ (ideally 0)

A = ideally ∞



A(s) actually has a pole near zero on the negative real axis

Consider case of $B=1$

$$H(s) = \frac{A(s)}{1+A(s)} = \frac{1}{1+\frac{1}{A(s)}}$$

$$= \frac{1}{1+\frac{1+s\tau}{A_o}} = \frac{1}{1+\frac{1}{A_o}+\frac{s\tau}{A_o}} \approx \frac{1}{1+\frac{s\tau}{A_o}} = \frac{1}{1+s/G}$$

we can usually ignore this term

Usually $A(s) = G/s$ is a good approximation for op-amp gain

For general B,

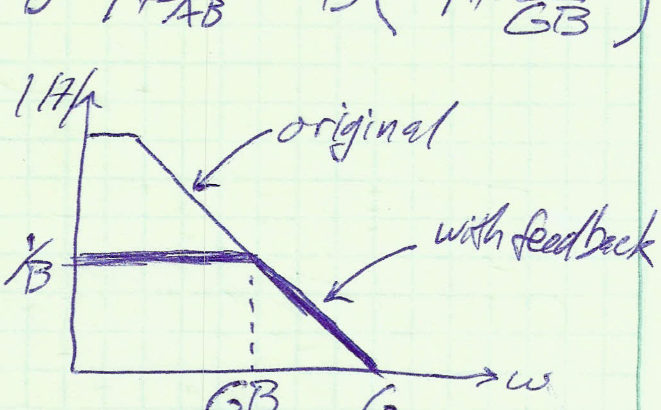
$$H(s) = \frac{A}{1+AB} = \frac{1}{B}\cdot\frac{AB}{1+AB} = \frac{1}{B}\cdot\frac{1}{1+\frac{1}{AB}} = \frac{1}{B}\left(\frac{1}{1+\frac{s}{GB}}\right)$$

DC gain $= 1/B$

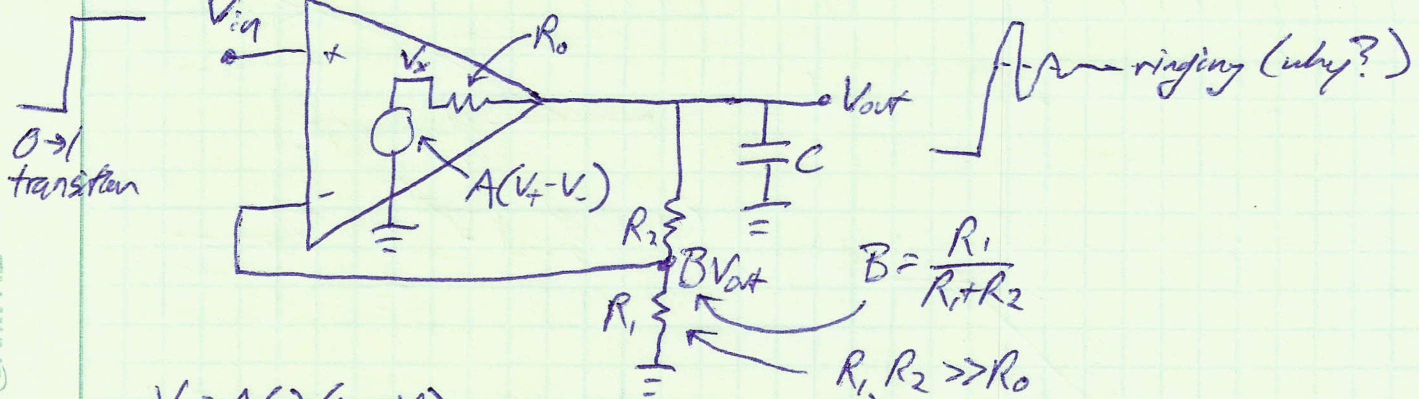
Bandwidth (-3dB) $= GB$

Gain-Bandwidth product

$\frac{1}{B}\cdot GB = G \rightarrow$ constant



Apply voltage follower to capacitive load



$$V_x = A(s)(V_+ - V_-)$$

$$V_x = A(s)(V_{in} - BV_{out}) \qquad V_{out} = \frac{V_x}{1+sR_oC} \quad \text{low pass filter}$$

$$V_x = A(s)\left(V_{in} - \frac{BV_x}{1+sR_oC}\right)$$

$$V_x\left(1 + \frac{A(s)B}{1+sR_oC}\right) = A(s)V_{in}$$

$$\frac{V_x}{V_{in}} = \frac{A(s)}{1+\frac{A(s)B}{1+sR_oC}} = \frac{A(s)(1+sR_oC)}{1+sR_oC+A(s)B} \qquad \tau = R_oC$$

$$\frac{V_{out}}{V_{in}} = \frac{A(s)}{1+s\tau+A(s)B} = \frac{1}{B}\left[\frac{1}{1+\frac{(1+s\tau)}{A(s)B}}\right] = \frac{1}{B}\left[\frac{1}{1+\frac{s(1+s\tau)}{GB}}\right]$$

$A(s) = G/s$

$$\frac{V_{out}}{V_{in}} = \frac{1}{B}\left[\frac{1}{1+\frac{s}{GB}+\frac{s^2\tau}{GB}}\right] \longrightarrow H(s)_{LPF} = \frac{1}{1+\frac{2\zeta s}{\omega_0}+\frac{s^2}{\omega_0^2}}$$

$$\omega_0 = \sqrt{\frac{GB}{\tau}}, \quad \zeta = \frac{1}{2\sqrt{GB\tau}}$$

make $\zeta \geq 1$ to stop ringing $\longrightarrow \tau \geq \frac{1}{4GB}$

<u>Example:</u> $|A| = 1$ at 1 MHz $\longrightarrow G = 2\pi \times 10^6$

$B = 0.1 \rightarrow$ DC gain of 10

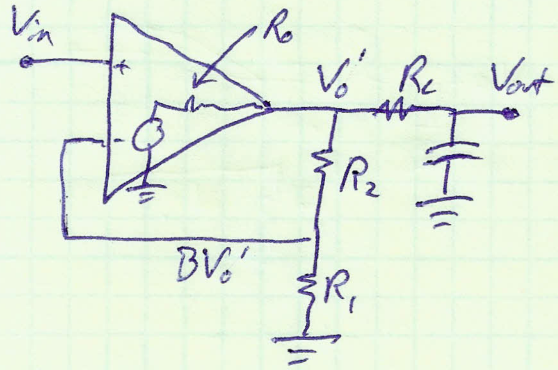
$R_o = 100$

$$\tau = \frac{1}{4GB} = \frac{1}{8\pi \times 10^5} \simeq 4\times10^{-7} = R_oC$$

Circuit will ring if C is larger than this

$C = 4\times10^{-9}$ or 4nF

Add compensation Resistor to reduce ringing



$$V_o' = A(V_{in} - BV_o')\left[\frac{R_c + 1/sC}{R_o + R_c + 1/sC}\right]$$

$$\frac{1 + sCR_c}{1 + sC(R_o + R_c)} = \frac{1 + s\tau_c}{1 + s(\underset{\text{load}}{\tau_L} + \underset{\text{compensation}}{\tau_c})}$$

$$\frac{V_o'}{V_{in}} = \frac{1}{B}\cdot\frac{1}{1 + \frac{s}{GB}\cdot\frac{1+s(\tau_L+\tau_c)}{1+s\tau_c}} = \frac{1}{B}\cdot\frac{1+s\tau_c}{1+s\left(\tau_c+\frac{1}{GB}\right)+s^2\left(\frac{\tau_L+\tau_c}{GB}\right)}$$

$$\frac{V_{out}}{V_o'} = \frac{1}{1+s\tau_c} \quad \text{so} \quad \frac{V_{out}}{V_{in}} = \frac{1}{B}\cdot\frac{1}{1+s\left(\tau_c+\frac{1}{GB}\right)+s^2\left(\frac{\tau_L+\tau_c}{GB}\right)}$$

$$\omega_o = \sqrt{\frac{GB}{\tau_L+\tau_c}} \quad \text{slightly lower}$$

$$\zeta_c = \frac{1}{2}\left(\tau_c + \frac{1}{GB}\right)\sqrt{\frac{GB}{\tau_L+\tau_c}}$$

Using R_c we can adjust ζ_c to ≈ 1 to remove ringing.