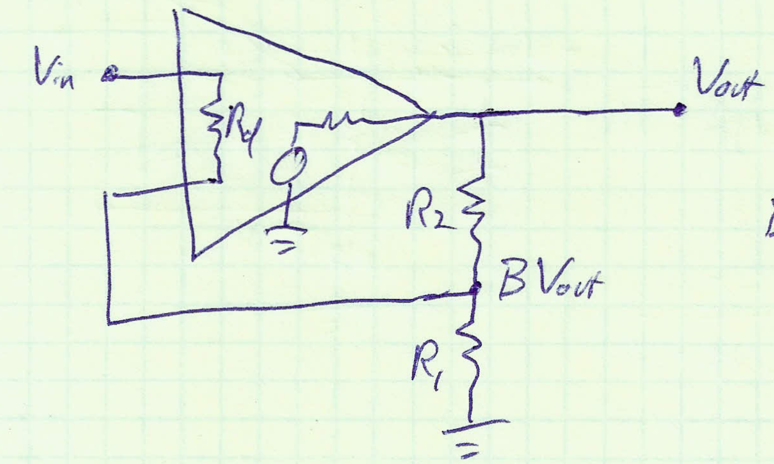
Input Impedance



$$B = \frac{R_1}{R_1 + R_2}$$

$$I_{in} = \frac{V_{in} - BV_{out}}{R_d} = \frac{V_{in}}{R_d}(1 - BH) = \frac{V_{in}}{R_d}\left(1 - \frac{AB}{1+AB}\right) = \frac{V_{in}}{R_d} \cdot \frac{1}{1+AB}$$

$$A = G/s$$

$$I_{in} = \frac{V_{in}}{R_d} \cdot \frac{1}{1 + \frac{GB}{s}}$$

$$Z_{in} = \frac{V_{in}}{I_{in}} = R_d\left(1 + \frac{GB}{s}\right) = R_d + \frac{1}{sC_{eff}} \leftarrow C_{eff} = \frac{1}{GBR_d}$$

Input impedance is R_d in series with small capacitor.

e.g. $G = 2\pi \times 10^6$, $B = 0.1$, $R_d = 10^7 \rightarrow$ 0.16 pF

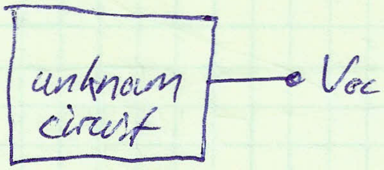
$Z = R + jX$

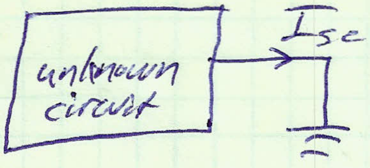
at $\omega = GB$, $X_{eff} = \frac{GBR_d}{GB} = R_d$

for $\omega < GB$, $X_{eff} > R_d \rightarrow$ input is capacitive

for $\omega > GB$, $X_{eff} < R_d \rightarrow$ input is resistive

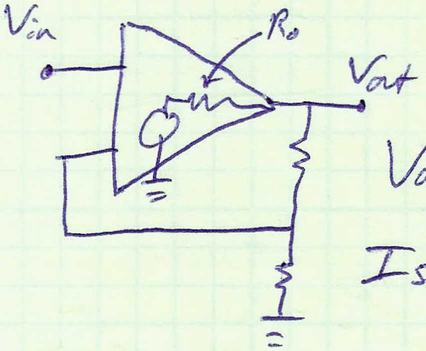
Output Impedance





$$Z_{out} = \frac{V_{oc}}{I_{sc}}$$

note: $I_{sc} = V_{out}/R_{out}$
$V_{oc} = V_{out}$



$$V_{out} = H V_{in} = V_{in} \frac{A}{1+AB}$$

$$I_{sc} = A V_{in} / R_o$$

$$Z_{out} = \frac{V_{in} \frac{A}{1+AB}}{A V_{in}/R_o} = \frac{R_o}{1+AB} \quad A = G/s$$

$$Z_{out} = \frac{R_o}{1+GB/s} = \frac{1}{\frac{1}{R_o} + \frac{GB}{R_o s}}$$

Output impedance is R_o in <u>parallel</u> with inductor $L_{eff} = \frac{R_o}{GB}$

for $\omega = GB$, $X_{eff} = R_o$

for $\omega < GB$, $X_{eff} < R_o \rightarrow$ output is inductive

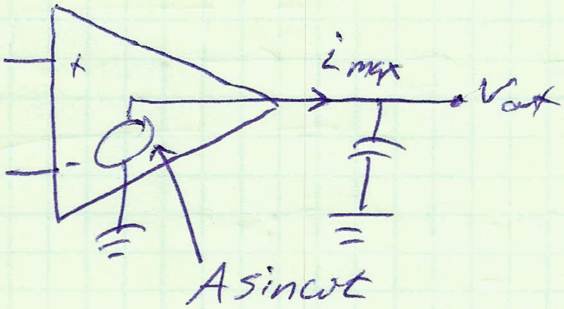
for $\omega > GB$, $X_{eff} > R_o \rightarrow$ output is resistive

3 effects of feedback

1. Reduces and stabilizes <u>gain</u>, increasing <u>bandwidth</u>
2. <u>Increases Zin</u>, but makes it <u>capacitive</u>
3. <u>Reduces Zout</u>, but makes it <u>inductive</u>

Slew Rate Limiting

$$i = C\frac{dV_{out}}{dt}$$

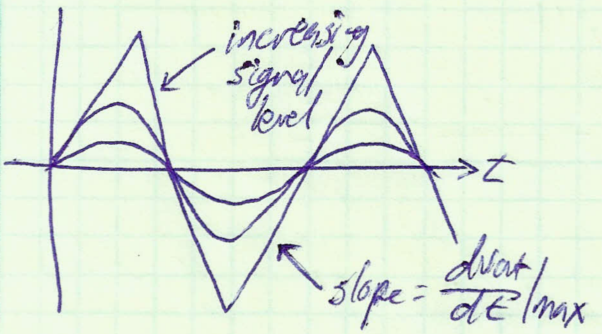


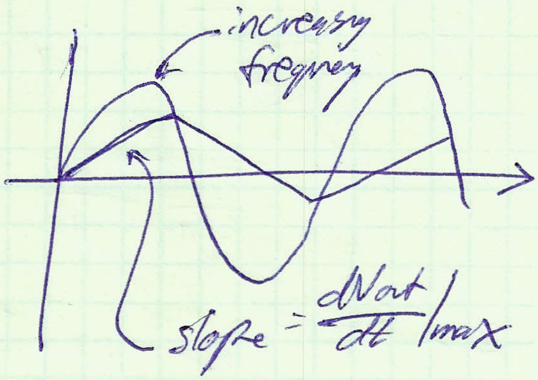
$$V_{out} = A\sin\omega t$$

$$i = AC\omega\cos\omega t$$

$|i| = AC\omega \rightarrow$ current scales with:
- Signal amplitude
- Capacitance
- Frequency

$$\left.\frac{dV_{out}}{dt}\right|_{max} = \frac{i_{max}}{C}$$





Overshoot and Rise Time

$$\%\ \text{overshoot} = \left(\frac{y}{x} - 1\right) \times 100$$

10% to 90% rise time

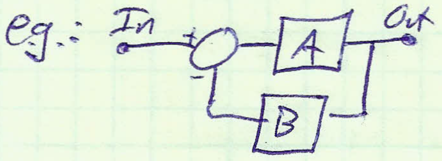
Stability Theory

Transfer function always has form of ~ $H(s) = \frac{1}{B}\left(\frac{AB}{1+AB}\right) = \frac{A}{1+AB}$

Always has $1+AB$ in denominator

Numerator depends on how the input enters the circuit

eg.:



$A(In - B\,Out) = Out$

$A\,In - AB\,Out = Out$

$A\,In = Out(1+AB)$

$$\frac{Out}{In} = \frac{A}{1+AB}$$

another common form:

$AB(In - Out) = Out$

$AB\,In - AB\,Out = Out$

$AB\,In = Out(1+AB)$

$$\frac{Out}{In} = \frac{AB}{1+AB}$$

If we cut the feedback loop here

then we have $Out = \underline{AB}\,In$ where AB is the open-loop transfer func.

We can analyze AB to determine stability of the closed loop system

Circuit cannot be stable if any subcircuit (A, B) is unstable

How to determine stability:

Does $1+AB$ have any zeros in right half plane?

Is $AB = -1$ for any s?

Remember $-1 = 1 \angle -180°$

Point of neutral stability (just barely stable/unstable):

$$|AB| = 1 \text{ and } \angle AB = \pm 180°$$

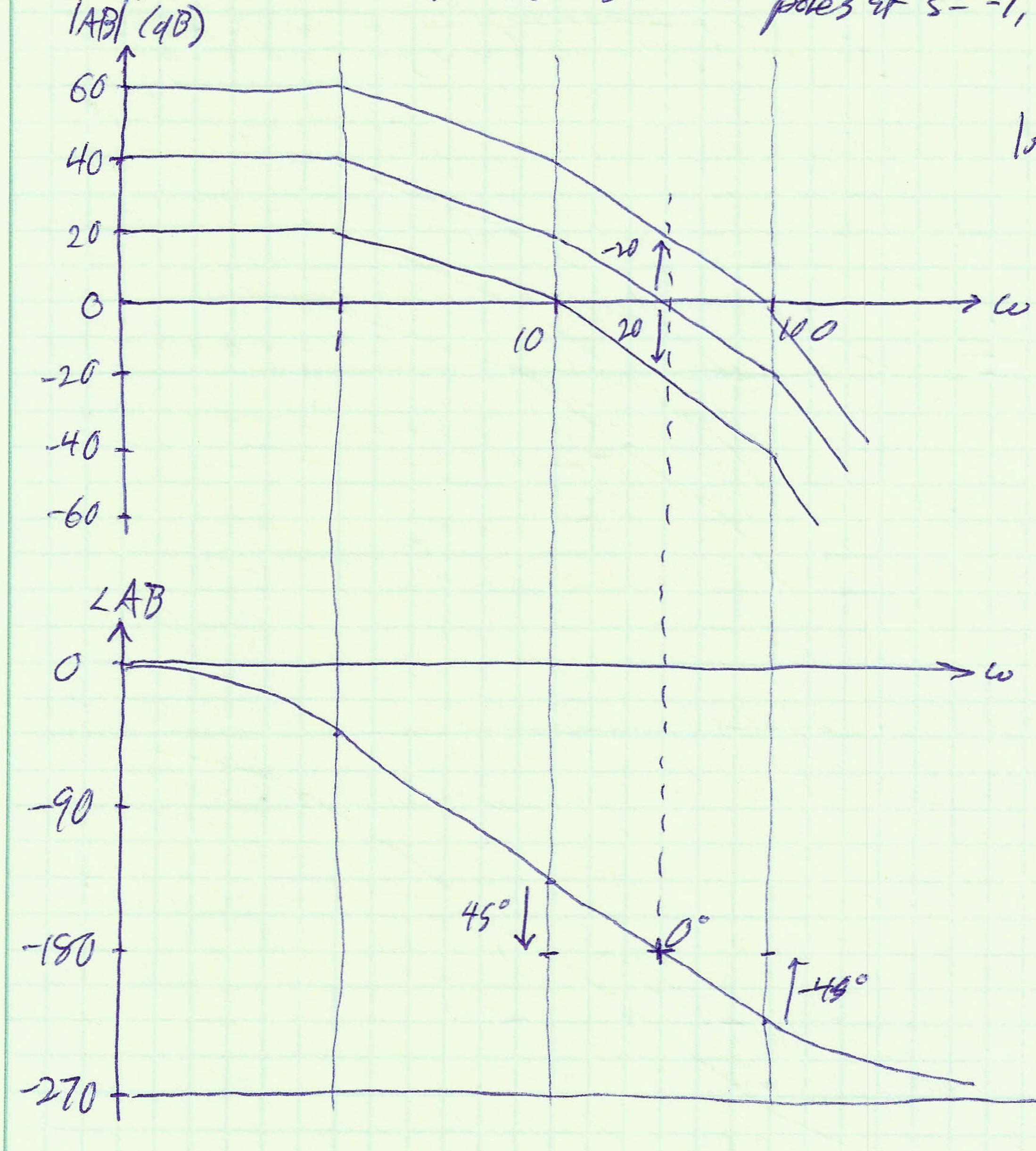
Gain and Phase Margin

Gain Margin $= -20\log_{10}|AB|$ at ω where $\angle AB = -180^\circ$

Phase Margin: $\angle AB + 180^\circ$ at ω where $|AB| = 1$ (0dB)

Example: $AB = \dfrac{K}{(s+1)(s+10)(s+100)}$ for $K = 10^4, 10^5, 10^6$

poles at $s = -1, -10, -100$



$|AB| = 1$ at

K	ω
10^4	10
10^5	32
10^6	100

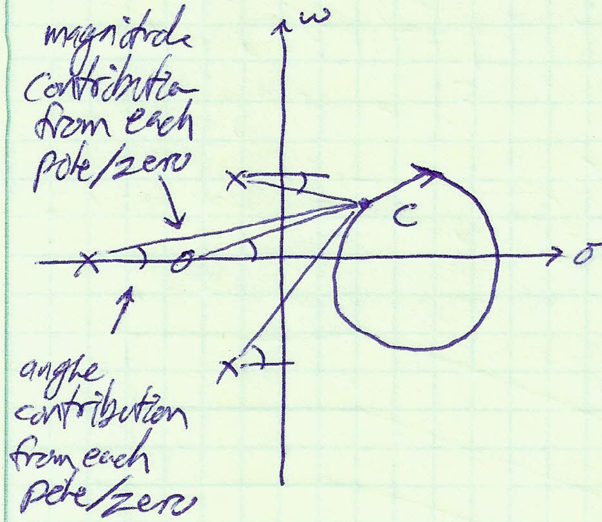
$\angle AB = -180^\circ$ at $\omega = 32$

K	GM	PM	stability
10^4	20dB	45°	stable
10^5	0dB	0°	marginally stable
10^6	-20dB	-45°	unstable

Roots of $(s+1)(s+10)(s+100)+K=0$?
$-101,\ -5 \pm 9j$
$-109,\ -0.8 \pm 30j$
$-148,\ 18 \pm 80j$

in RHP!

Nyquist Stability Criterion



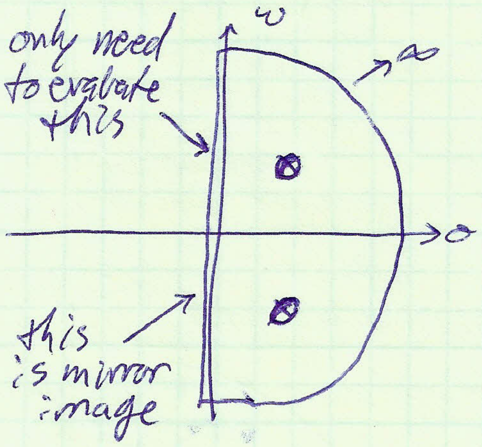
Consider a contour C in the s plane

If C does not enclose any poles or zeros, it will not go through a full 360° of phase

$$1 + AB = \frac{\prod_i (s - z_i)}{\prod_j (s - p_j)} = Mag\, e^{j Ph}$$

$$Mag = \frac{\prod \text{magnitude from zeros}}{\prod \text{magnitude from poles}}$$

$$Ph = \Sigma \text{ phase from zeros} - \Sigma \text{ phase from poles}$$



we take a contour along imaginary axis and extending to ∞ on positive real side

total number of times contour of $1+AB$ encircles origin: $N = Z - P$ (clockwise)

Z = # Zeros of $1+AB$ in RHP

P = # Poles of $1+AB$ in RHP

but poles of $1+AB$ = poles of AB

we know these have no poles in RHP if A and B are stable

If contour of $1+AB$ encircles origin clockwise, system is unstable

We actually look for encirclements of -1 in plot of AB

If $AB = -1$, there is a pole right on the ω axis.