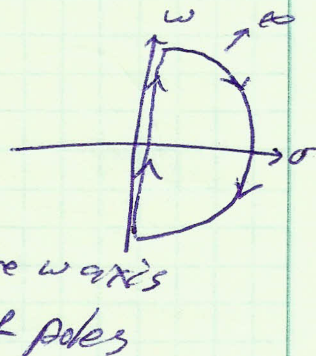
Nyquist Stability Criterion (continued)

- A contour evaluation of a complex function will only encircle the origin if the contour contains a singularity of the function.
- Number of clockwise encirclements $N = Z - P$
  - Z = number of zeros within contour
  - P = number of poles within contour



- Contour is taken to be the entire RHP

  Negative ω axis is just mirror image of positive ω axis

  Contour at $s \to \infty$ contributes nothing if number of poles in AB is greater than number of zeros (which must be true!)

  $\rightarrow$ transfer function cannot go to ∞ as s (or ω) go to ∞
- If A and B are stable individually, they have no poles in RHP

  Subcircuits must all be stable for whole circuit to be stable

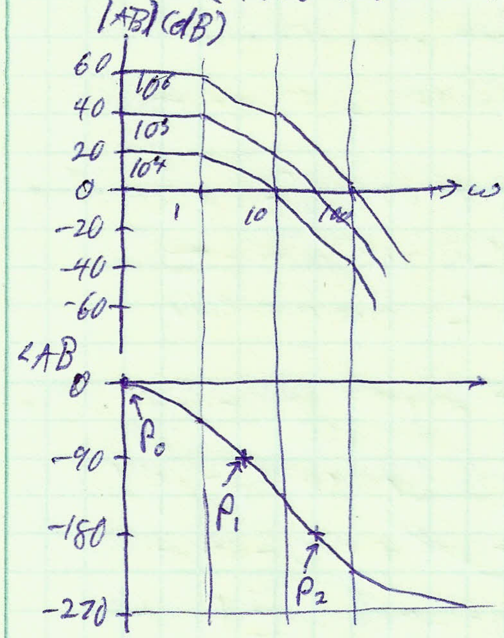
  Poles of $1+AB$ are the same as poles of AB

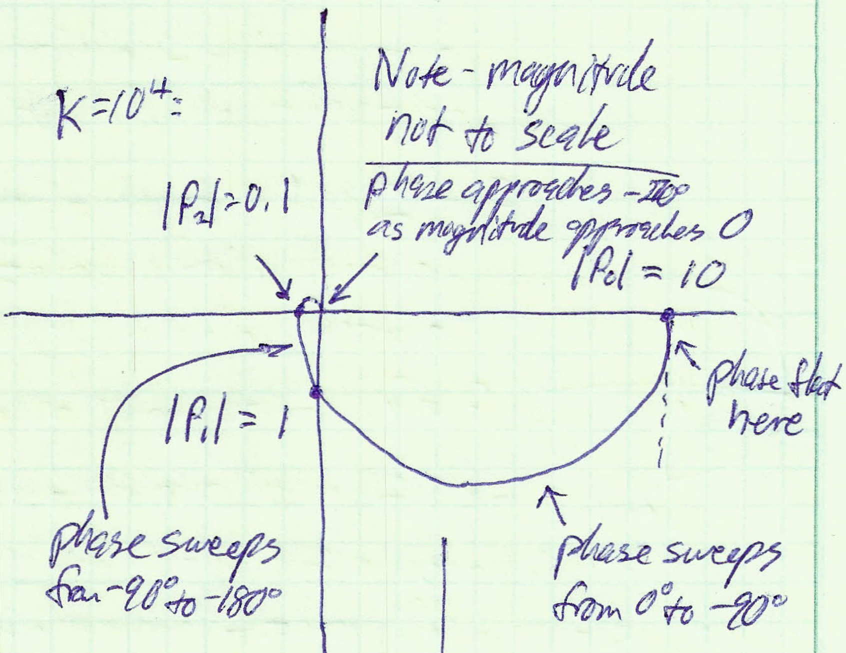
  We are now only concerned with Zeros of $1+AB$ in RHP
- We typically plot AB, look for clockwise encirclements of -1
- Procedure for drawing simple Nyquist plots:
  1. Draw a Bode plot of open-loop transfer function AB
  2. Look for points where phase crosses $0, -90, -180, \ldots$
  3. Plot these intercept points based on magnitude plot
  4. Connect the dots, based on shape of magnitude + phase functions

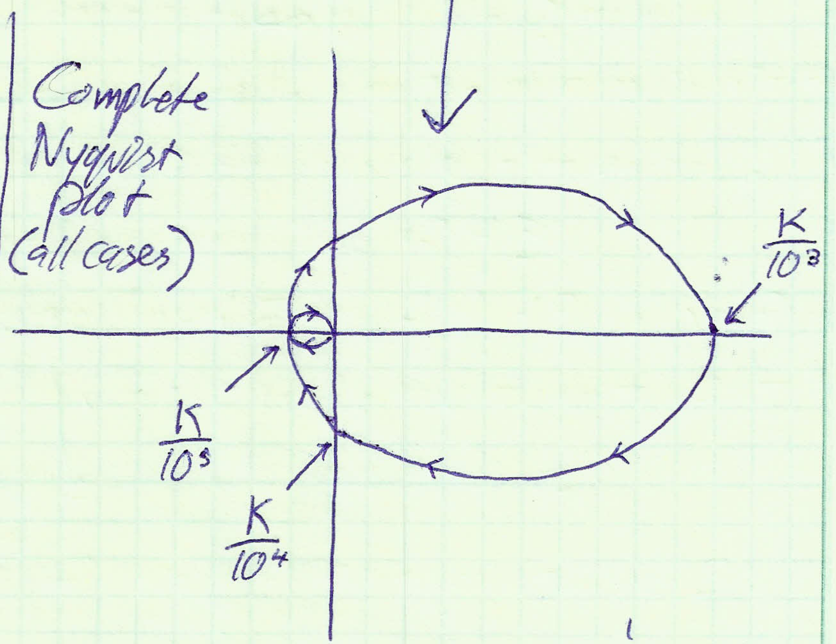
Be sure to include where magnitude may approach 0 or ∞

Revisit our previous example

$$AB = \frac{K}{(s+1)(s+10)(s+100)} \quad \text{for } K = 10^4, 10^5, 10^6$$







Does the plot encircle -1?

Depends on the value of K.

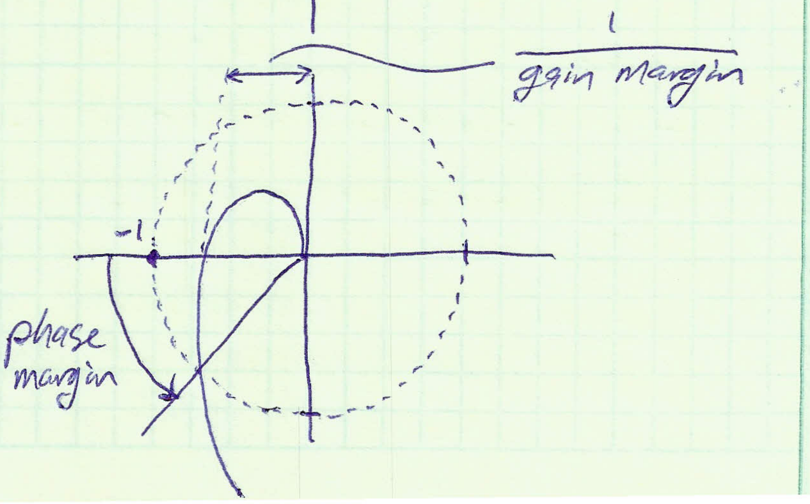
$K = 10^4 \rightarrow$ no

$K = 10^5 \rightarrow$ just barely

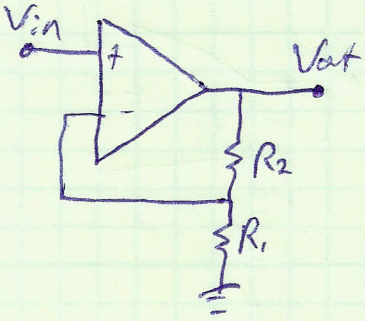
$K = 10^6 \rightarrow$ yes

Same result as our previous mag/phase margin analysis!

We can also get mag/phase margin from Nyquist plots:



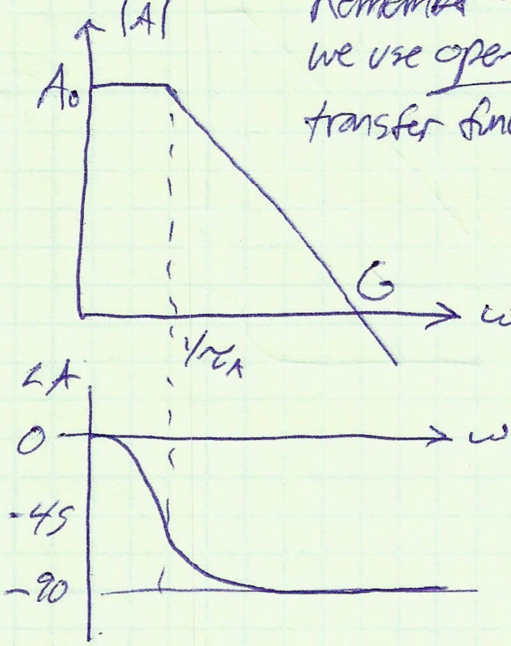
Example: Voltage follower circuit



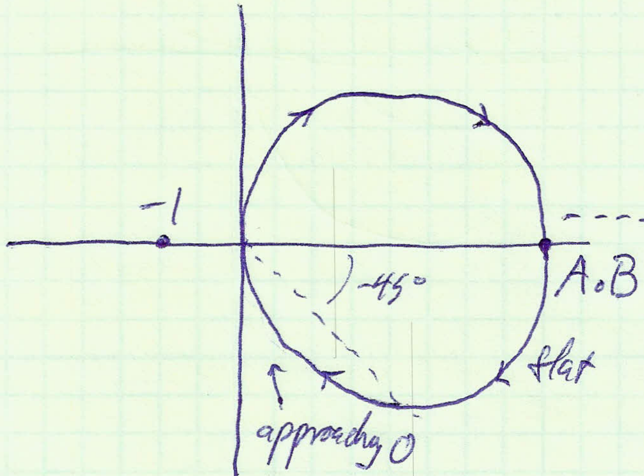
$$A = \frac{A_0}{1+s\tau_A}$$

$$B = \frac{R_1}{R_1+R_2}$$

(constant)



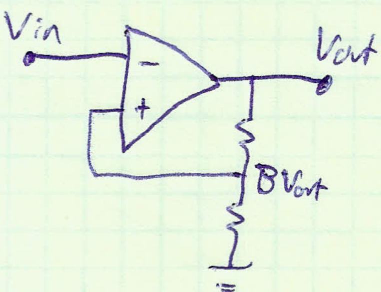
Remember - we use open loop transfer function: AB

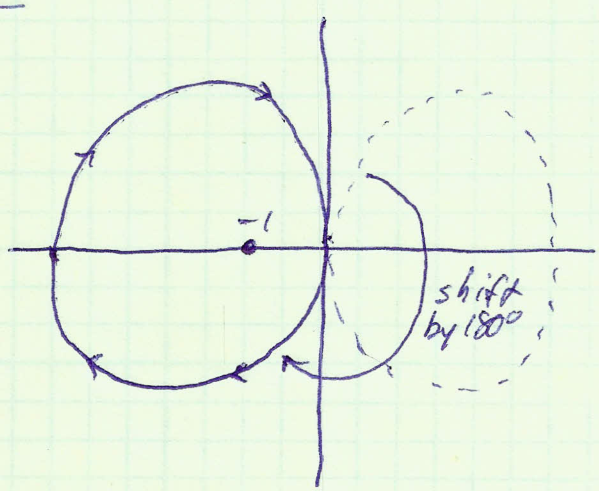


→ ∞ if we use $A = G/s$ model

Does not encircle −1
Unconditionally stable

Example: Schmitt trigger





$$V_{out} = A(BV_{out} - V_{in})$$

$$A\,V_{in} = AB\,V_{out} - V_{out}$$

$$\frac{V_{out}}{V_{in}} = \frac{-A}{1-AB} = \frac{-A}{1+T}$$

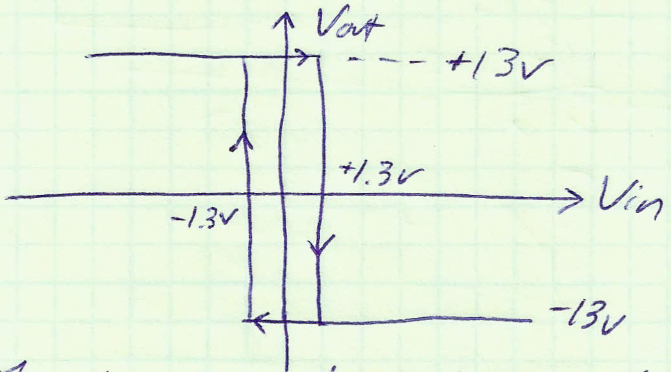
$T = -AB \rightarrow$ shifted by 180°

Encircles −1 one time, clockwise
closed loop transfer function has one pole in RHP

Schmitt trigger (continued)

Assume ±15v bias, and amplifier saturates at ±13v

Assume B=0.1, let $V_{in} = -15v \rightarrow +15v$



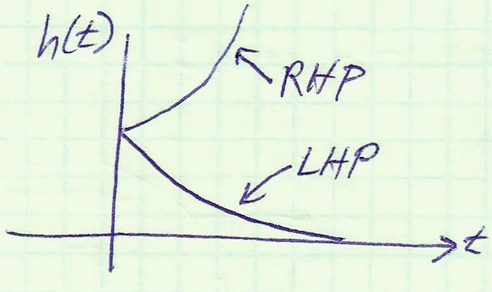
when $V_{in} > 1.3v$
output switches to $-13v$

The circuit is bistable, with hysteresis
Used in many digital circuits

$$H(s) = \frac{-A}{1-AB} = \frac{1}{B}\cdot\frac{1}{1-\frac{1}{AB}} = \frac{1}{B}\cdot\frac{1}{1-\frac{s}{GB}} = -G\cdot\frac{1}{s-GB}$$

pole in RHP

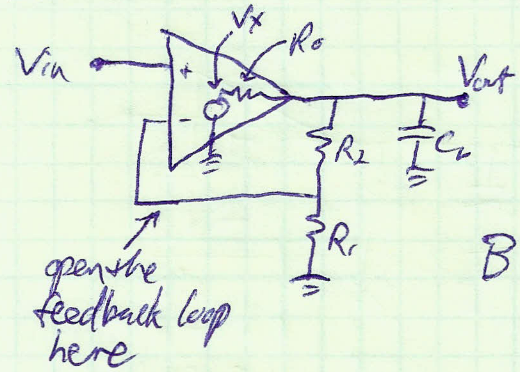
recall Laplace transform $\frac{1}{1+\alpha} \leftrightarrow e^{-\alpha t}u(t)$



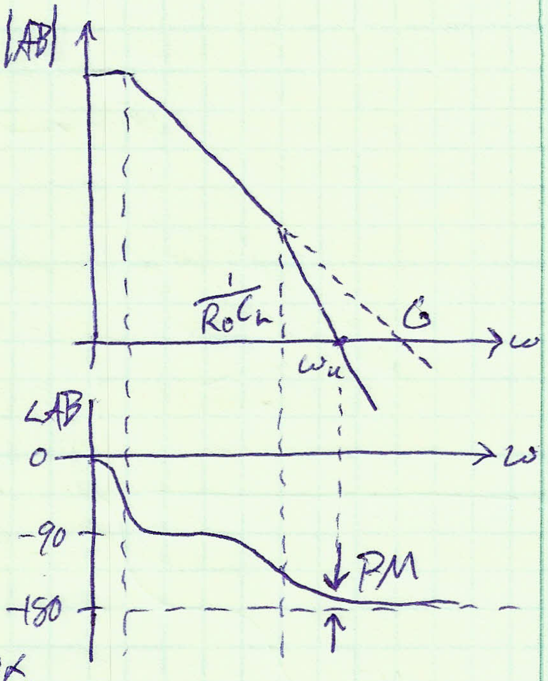
pole in LHP: exponential decay: e^{-tGB}
pole in RHP: exponential growth: e^{+tGB}

The circuit switches very quickly because
the feedback increases the drive to change state.

Example: Voltage follower with Capacitive Load (lab #3)



$$B = \frac{R_1}{R_1+R_2}$$



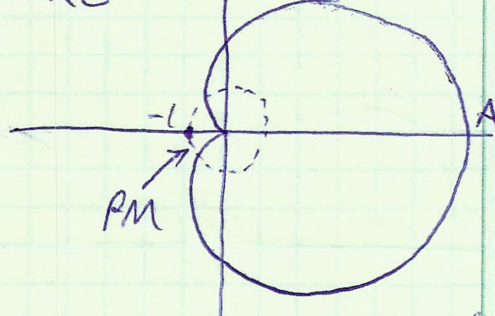
$$\frac{V_x}{V_{in}} = \frac{A_o}{1+s\tau_A}, \quad \frac{V_{out}}{V_x} = \frac{1/sC_L}{R_o + 1/sC_L} = \frac{1}{1+sR_oC_L}$$

$$AB = \frac{A_o}{1+s\tau_A} \cdot \frac{1}{1+sR_oC_L} \cdot \frac{R_1}{R_1+R_2}$$

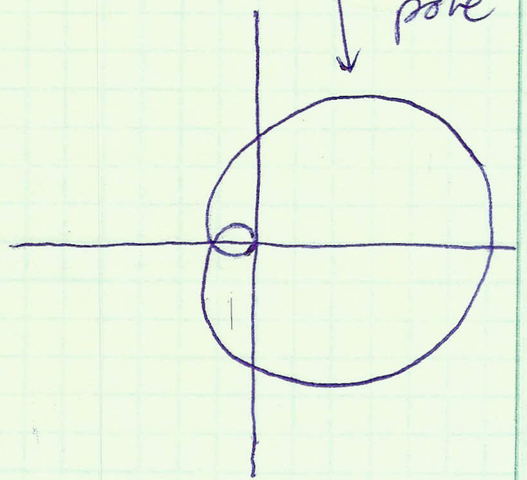
Phase gets close to -180°, but never exceeds it.

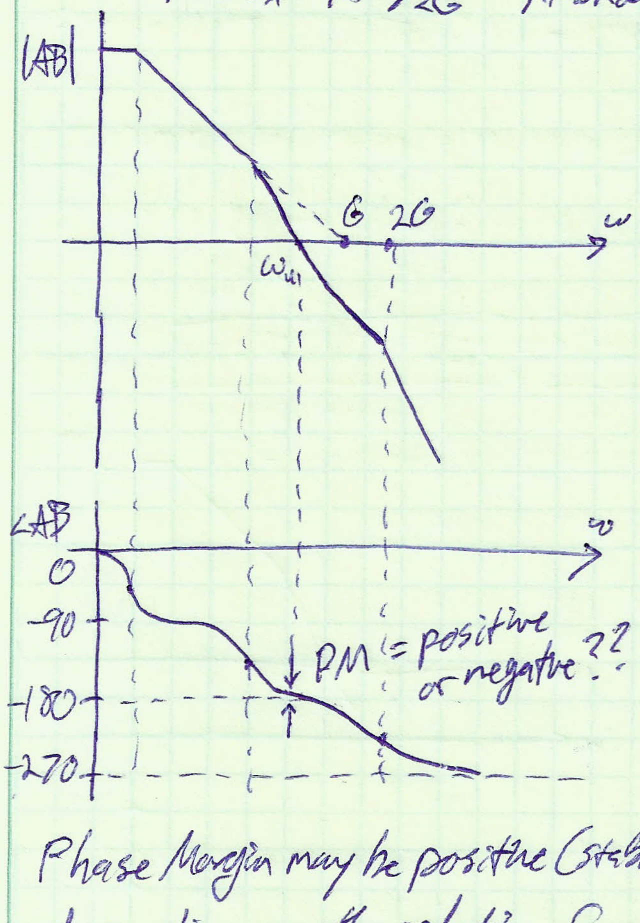
However, real op-amps have a second pole near ~2G

$$AB = \frac{A_o}{1+s\tau_A} \cdot \frac{1}{1+s/2G} \cdot \frac{1}{1+sR_oC_L} \cdot \frac{R_1}{R_1+R_2}$$



with extra pole

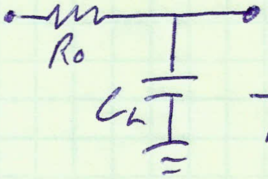




Phase Margin may be positive (stable) or negative (unstable) depending on the relative frequencies of the various poles

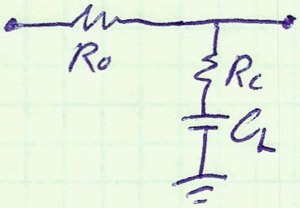
Effect of Compensation

Change this:



$$\frac{1}{1+sR_oC_L}$$

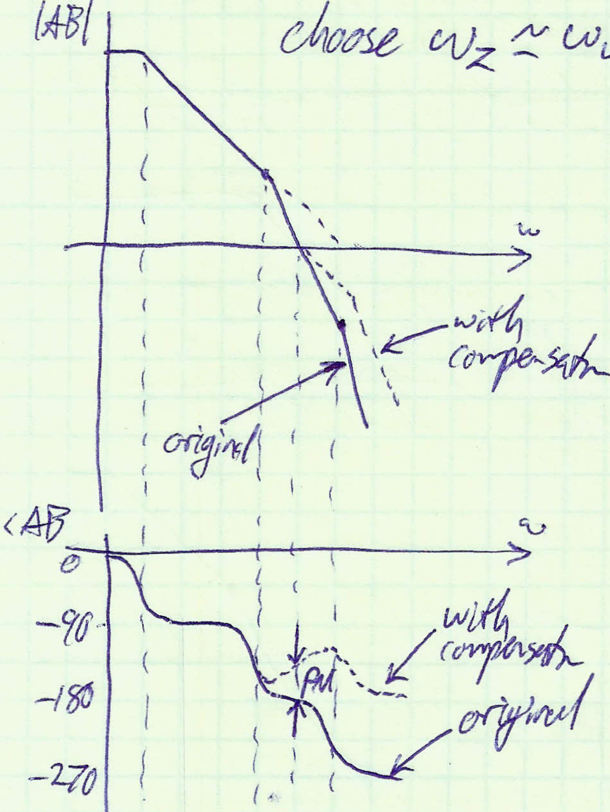
to this:

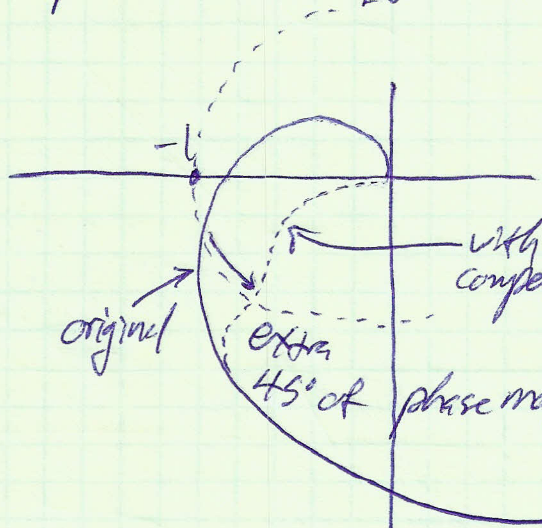


$$\frac{1+sR_cC_L}{1+s(R_o+R_c)C_L}$$

Compensation resistor adds a zero to the transfer function

choose $\omega_z \simeq \omega_u$ → put zero near unity gain frequency





General approach to compensation:

Look for a way to add a zero to the feedback path or transfer function

Effect of Delay

$$f(t-a)u(t-a) \longleftrightarrow e^{-as}F(s)$$

Causes a phase lag, but does not reduce gain
(compare this to a pole, which does both)
tends to cause instability

Examples:

- loudspeaker to microphone feedback
- attempt to control water temperature in a shower
- psycho-acoustic effect of delay between speaking and hearing