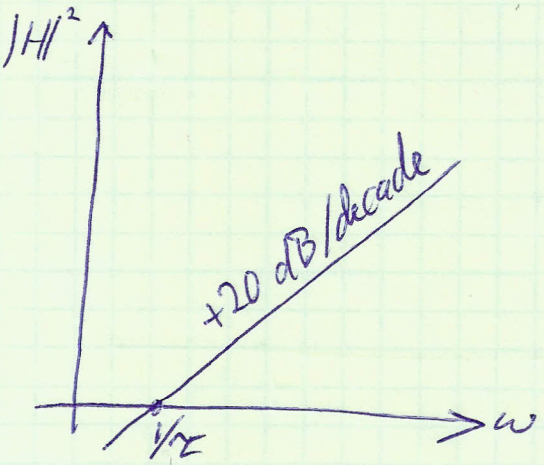
Lab #4: Design of a Differentiator

Laplace transform: $f'(t) \leftrightarrow sF(s) - f(0)$ ← initial value

We need a system where $H(s) = s\tau$ ← scale factor

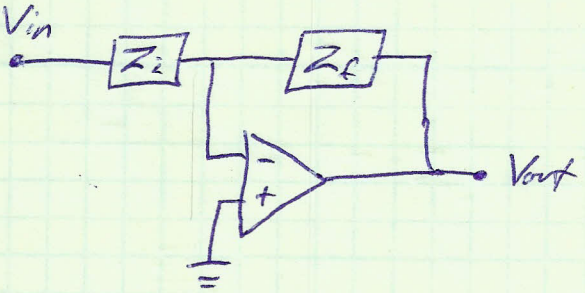


phase = +90° at all ω.

Problems with this system:

- Non causal: can't anticipate phase of incoming signal at all frequencies
- Can't have Gain → ∞ for $\omega \to \infty$ (can't have more zeros than poles)

Consider this circuit:

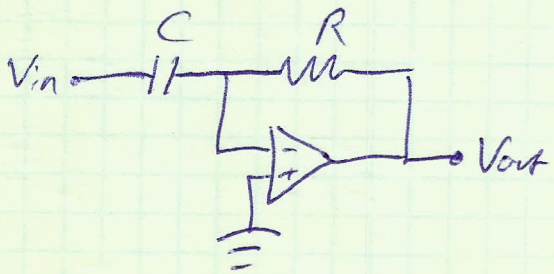


Simple model: $V_- = 0$

$$\frac{0 - V_{in}}{Z_i} + \frac{0 - V_{out}}{Z_f} = 0$$

$$\frac{V_{out}}{V_{in}} = -\frac{Z_f}{Z_i}$$

$$\frac{V_{out}}{V_{in}} = -sCR$$



or $V_{out}(s) = -s\tau V_{in}(s)$

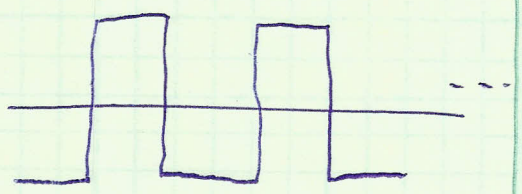
$$v_{out}(t) = -\tau \frac{dv_{in}(t)}{dt}$$

$\tau = RC$

note RC has units of time

$v_{in}(t)$:

$v_{out}(t)$:

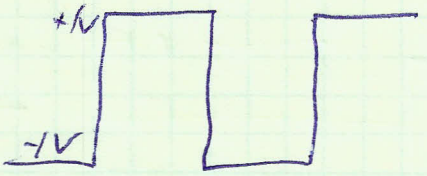


what about $\tau = RC$ factor?

example: $RC = \underline{10^{-3}}$

$\frac{dV}{dt} = \frac{2V}{2ms} = \underline{10^3 V/s}$

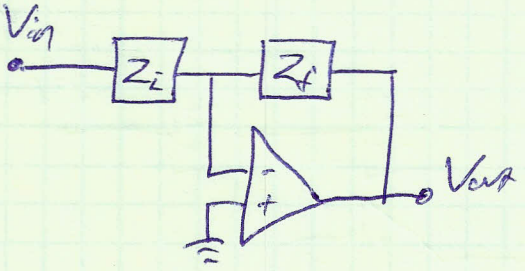
$-RC\frac{dV}{dt} = -1V$



however, what you actually see in the lab is:

This must be a second-order system.

More detailed analysis:



$V_{out} = -A V_-$

$$\frac{V_- - V_{out}}{Z_f} + \frac{V_- - V_{in}}{Z_i} = 0$$

$$V_-\left(\frac{1}{Z_f} + \frac{1}{Z_i}\right) = \frac{V_{out}}{Z_f} + \frac{V_{in}}{Z_i}$$

$$V_-(Z_i + Z_f) = V_{out}Z_i + V_{in}Z_f$$

$$V_- = V_{out}\frac{Z_i}{Z_i+Z_f} + V_{in}\frac{Z_f}{Z_i+Z_f}$$

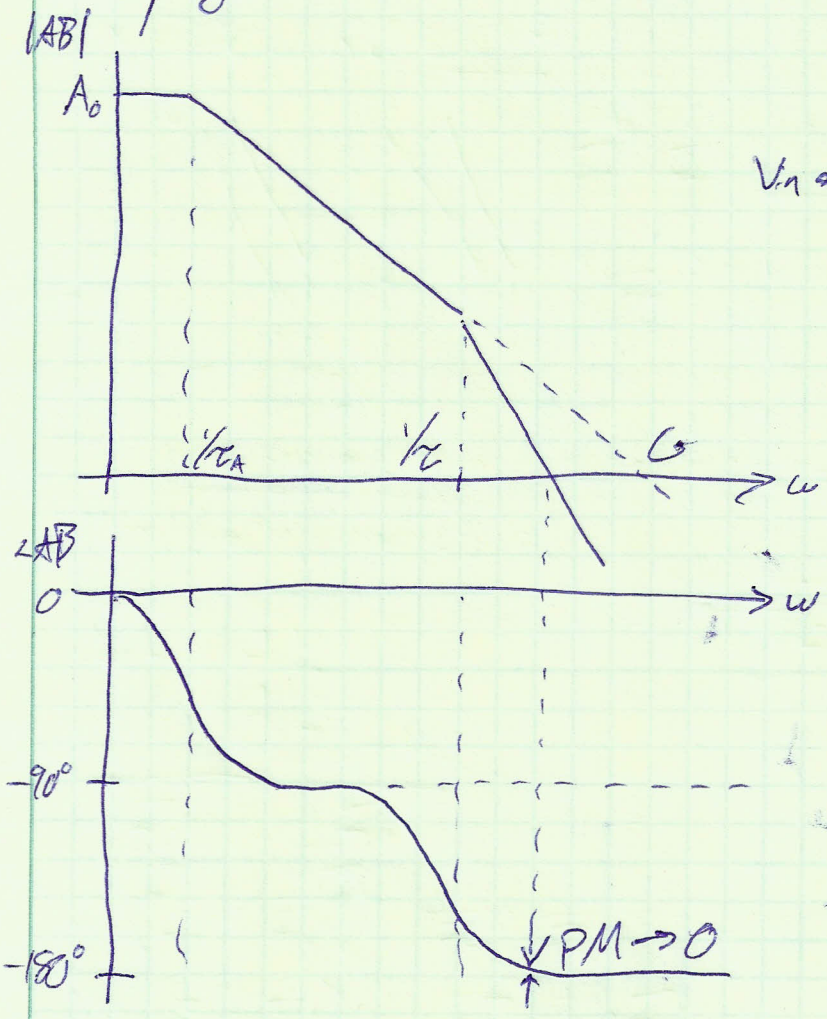
$$V_{out} = -AV_- = -A\left(V_{out}\frac{Z_i}{Z_i+Z_f} + V_{in}\frac{Z_f}{Z_i+Z_f}\right)$$

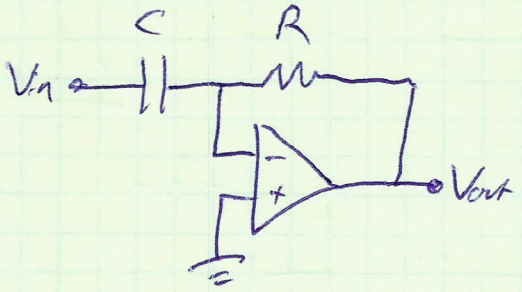
$$V_{out}\left(1 + A\frac{Z_i}{Z_i+Z_f}\right) = -AV_{in}\frac{Z_f}{Z_i+Z_f}$$

$$\frac{V_{out}}{V_{in}} = \frac{-Z_f}{Z_i}\cdot\frac{AB}{1+AB} \quad \text{where } B = \frac{Z_i}{Z_i+Z_f}$$

$$H_{ideal} = \frac{-Z_f}{Z_i} \quad \text{and} \quad H_{real} = H_{ideal}\cdot\frac{AB}{1+AB}$$

Loop gain AB for Differentiator





Phase margin approaches zero

- response will be ∫ with overshoot and oscillations
- may even be unstable (considering higher op-amp poles)

How do we eliminate this? - Compensation (next lecture)

Remainder of this lecture - review for Midterm #1