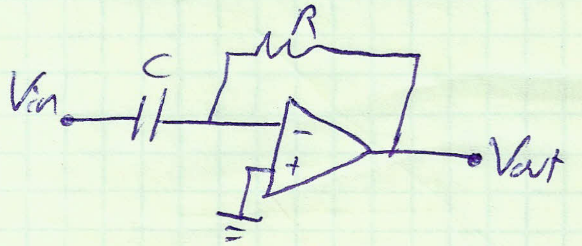
More on Lab #4 – Design of a Differentiator



$$\frac{V_{out}}{V_{in}} = \frac{-Z_f}{Z_i}\left[\frac{AB}{1+AB}\right] = H_{ideal}\left[\frac{AB}{1+AB}\right]$$

$Z_i = \frac{1}{sC}$, $Z_f = R$, $B = \frac{Z_i}{Z_i + Z_f}$ $\quad B = \frac{1}{1+sCR} \quad A = G/s$

Check last lecture for derivation and other details

$$\frac{AB}{1+AB} = \frac{1}{1+1/AB} = \frac{1}{1+\frac{s}{G}(1+sCR)} = \frac{1}{1+s/G + \frac{s^2CR}{G}}$$

This is a 2nd order Low pass filter. Define $\tau \equiv RC$

$$\frac{1}{1+\frac{s}{G}+\frac{s^2\tau}{G}} = \frac{1}{1+\frac{2\zeta s}{\omega_0}+\frac{s^2}{\omega_0^2}}$$

$$\frac{\tau}{G} = \frac{1}{\omega_0^2} \rightarrow \omega_0 = \sqrt{\frac{G}{\tau}}$$

$$\frac{2\zeta}{\omega_0} = \frac{1}{G} \rightarrow \zeta = \frac{\omega_0}{2G} = \sqrt{\frac{G}{\tau}}\cdot\frac{1}{2G} = \frac{1}{\sqrt{4G\tau}}$$

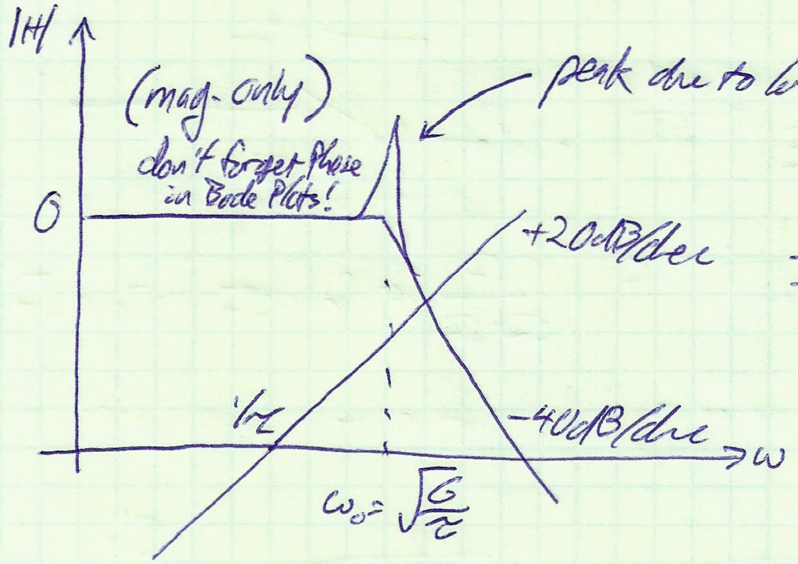
for $RC = 2.5\times10^{-4}$

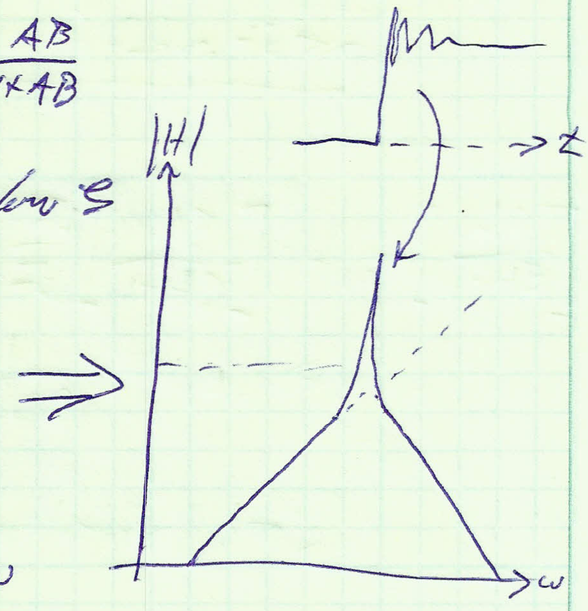
and $G = 2\pi\times10^6$ (1 MHz unity gain frequency)

$$\zeta = \frac{1}{\sqrt{4\times2\pi\times10^6\times2.5\times10^{-4}}} \simeq \frac{1}{80} \rightarrow \text{very small damping}$$

Compare to Butterworth, $\zeta = \frac{1}{\sqrt{2}} = 0.707$

Bode plot is composite of $\frac{Z_f}{Z_i}$ and $\frac{AB}{1+AB}$

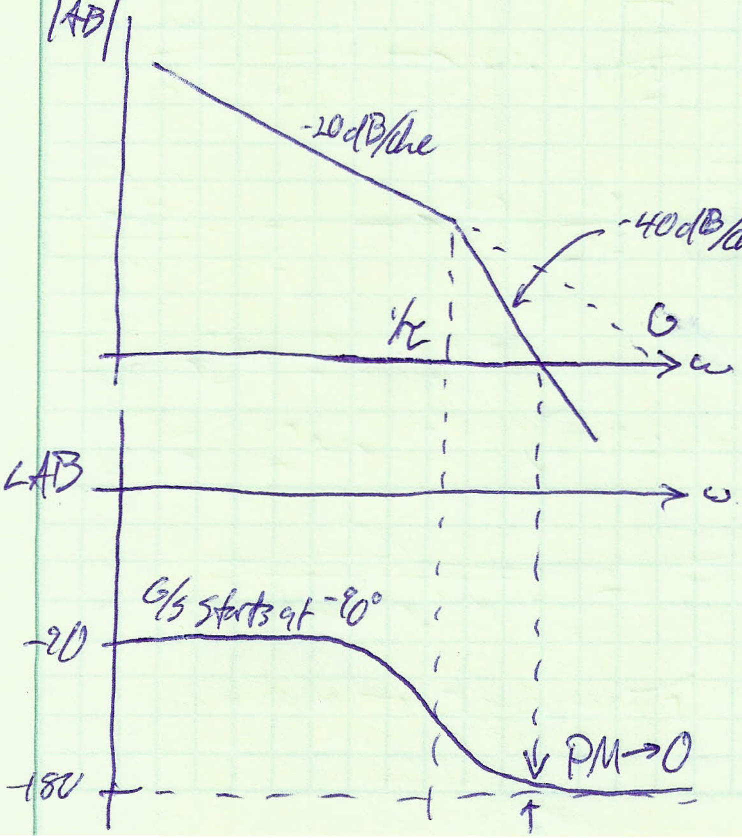




Low Q causes: (1) Ringing and overshoot in time domain
(2) Peaking in frequency domain

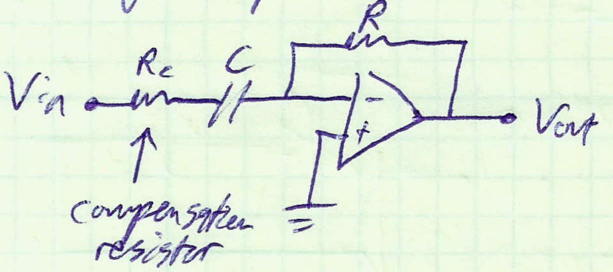
We can also see the problem in the loop gain:

$$AB = \frac{G}{s} \cdot \frac{1}{1+s\tau}$$



Phase Margin approaches 0

Fixing the problem with Compensation



(add a zero to the transfer function)

$$\frac{AB}{1+AB} = \frac{1}{1+\frac{1}{AB}}, \quad B = \frac{Z_i}{Z_f+Z_i} = \frac{R_c + \frac{1}{sC}}{R+R_c+\frac{1}{sC}}$$

$$B = \frac{1+sCR_c}{1+sC(R+R_c)}$$ ← added a zero $\left(\text{old } B = \frac{1}{1+sCR}\right)$

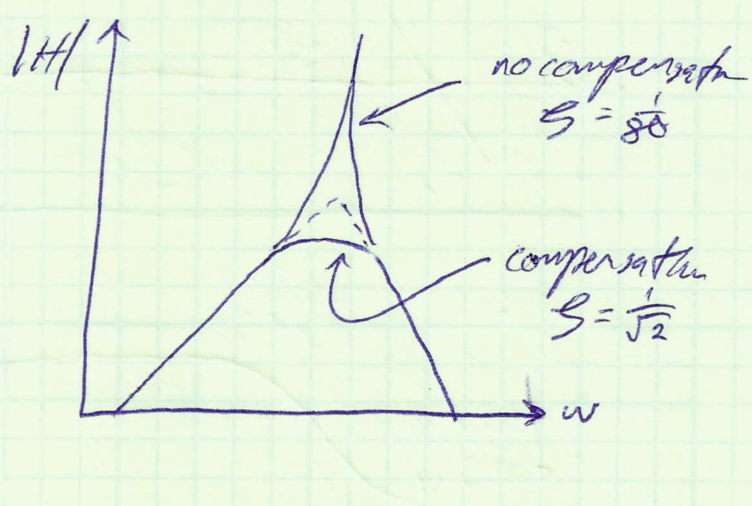
for $A = \frac{G}{s}$, $$\frac{1}{1+\frac{1}{AB}} = \frac{1}{1+\frac{s}{G}\frac{1+s(\tau+\tau_c)}{1+s\tau_c}}$$ $\tau = RC$, $\tau_c = R_cC$

$$\frac{1}{1+\frac{1}{AB}} = \frac{1+s\tau_c}{1+s\tau_c+\frac{s}{G}+s^2\frac{(\tau+\tau_c)}{G}}$$

$$\omega_o = \sqrt{\frac{G}{\tau+\tau_c}} \qquad \zeta = \frac{1}{2}\sqrt{\frac{G}{\tau+\tau_c}}\left(\tau_c + \frac{1}{G}\right)$$

allows good control over ζ

making $\zeta = \frac{1}{\sqrt{2}}$ gives Butterworth response



Relationship between ζ and phase margin
(only valid for 2nd order systems)

Using example of differentiator

$$A = \frac{G}{s} \qquad B = \frac{1}{1+s\tau}$$

$$|AB| = \left|\frac{G}{s}\cdot\frac{1}{1+s\tau}\right| = 1 \longrightarrow \left|\frac{G}{s}\right|^2 = |1+s\tau|^2$$

$$\frac{G^2}{\omega^2} = 1+\omega^2\tau^2 \qquad \text{usually } \omega^2\tau^2 \gg 1$$

$$\frac{G^2}{\omega^2} \simeq \omega^2\tau^2 \quad \text{or} \quad \omega_u \simeq \sqrt{\frac{G}{\tau}} \leftarrow \text{frequency where } |AB| = 1$$

calculate the phase at this frequency

$$PM = 180 + \angle AB \text{ at } \omega_u$$

$$= 180 + \angle A + \angle B \qquad \text{but } \angle A \text{ is } -90 \text{ always} \quad (A = G/s)$$

$$PM = 90 + \angle B$$

$$B = \frac{1}{1+s\tau} \rightarrow \angle B = -\tan^{-1}\omega_u\tau$$

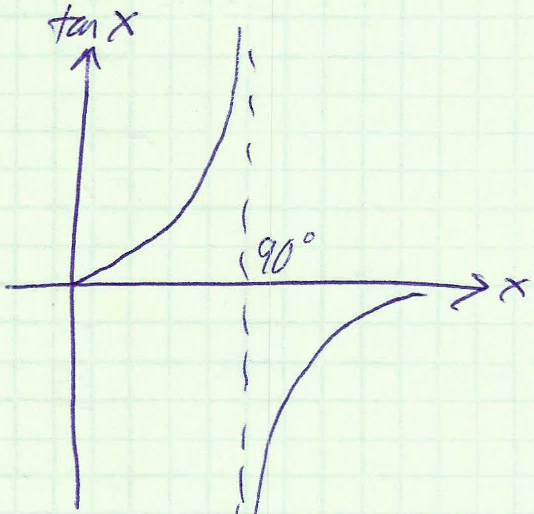
$$B = -\tan^{-1}\sqrt{\frac{G}{\tau}}\,\tau = -\tan^{-1}\sqrt{G\tau}$$

$$PM = 90 - \tan^{-1}\sqrt{G\tau}$$

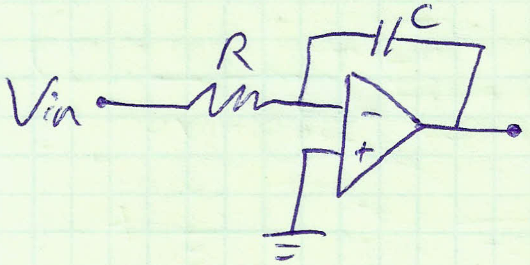
remember $\zeta = \frac{1}{2\sqrt{G\tau}}$

$$\boxed{PM = 90 - \tan^{-1}\left(\frac{1}{2\zeta}\right)}$$

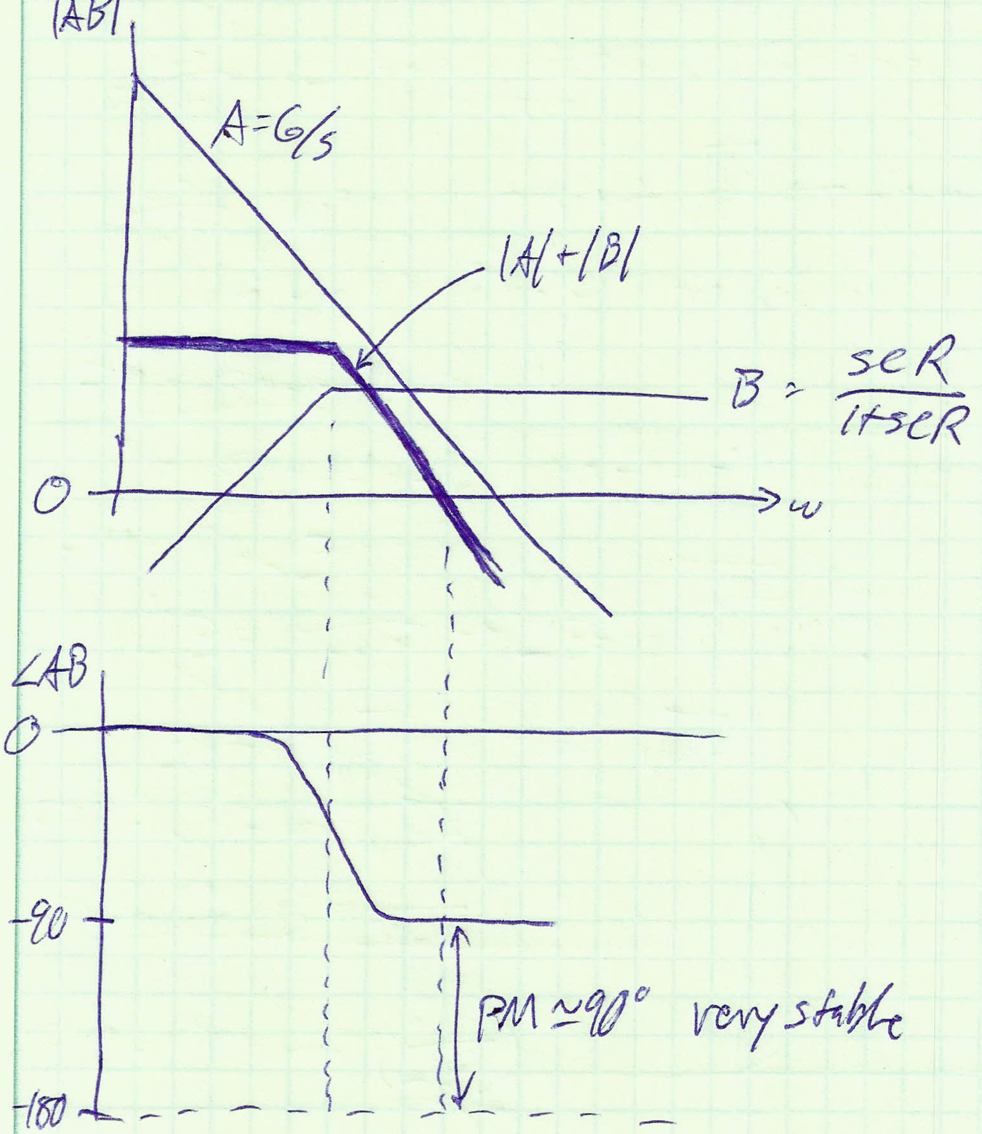
general equation for 2nd order systems



Another similar example - Integrator



$$A = G/s \,, \quad B = \frac{R}{R + 1/sC} = \frac{sCR}{1+sCR}$$



Rest of lecture: review midterm