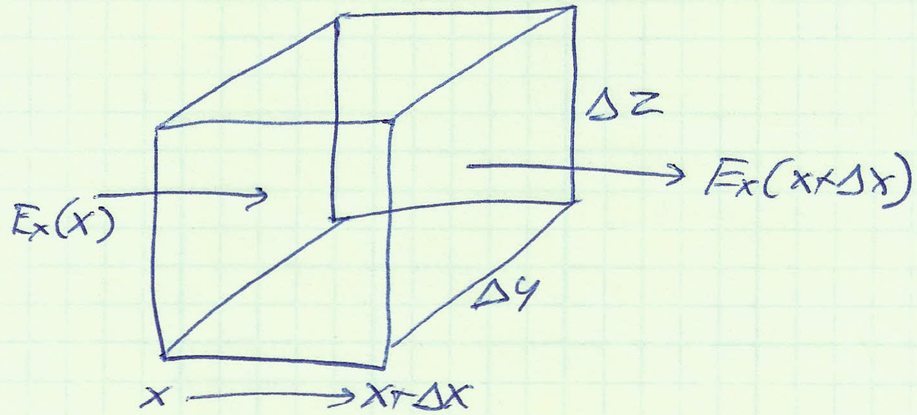
More on Divergence and Curl

$$\nabla \cdot E = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

- net outward change in flux per unit volume over closed incremental surface



total change in E_x through area $\Delta y \Delta z$:

$$\frac{dE_x}{\cancel{dx}} \Delta y \Delta z = \underbrace{\frac{E(x+\Delta x) - E(x)}{\Delta x}}_{\frac{dE_x}{dx}} \cdot \underbrace{\Delta x \Delta y \Delta z}_{\Delta V}$$

$$\boxed{\int_V \nabla \cdot \vec{E}\, dV = \oint_S \vec{E} \cdot d\vec{s} \quad \text{Divergence theorem}}$$

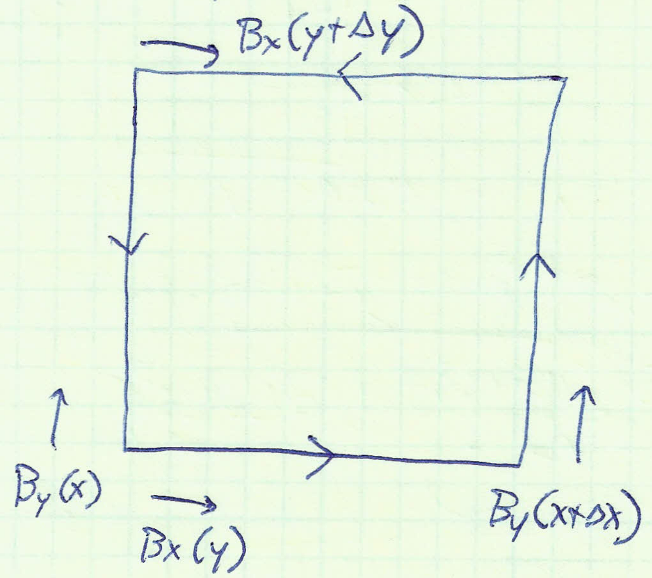
$\nabla \cdot \vec{E}$ integrated over small volume ΔV is equal to total flux of E passing through surface S surrounding that volume

Curl

$$\nabla \times B = \hat{x}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) + \hat{y}\left(\frac{\partial B_x}{\partial z} - \frac{\partial B_z}{\partial x}\right) + \hat{z}\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)$$

$$= \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ B_x & B_y & B_z \end{vmatrix}$$

For z component:



total B along closed path C:

$$B_x(y)\Delta x + B_y(x+\Delta x)\Delta y - B_x(y+\Delta y)\Delta x - B_y(x)\Delta y$$

$$\left[\frac{B_y(x+\Delta x) - B_y(x)}{\Delta x}\right]\Delta y \Delta x - \left[\frac{B_x(y+\Delta y) - B_x(y)}{\Delta y}\right]\Delta x \Delta y$$

$$\left(\frac{\partial B_y}{\partial x} - \frac{\partial B_x}{\partial y}\right)\Delta x \Delta y$$

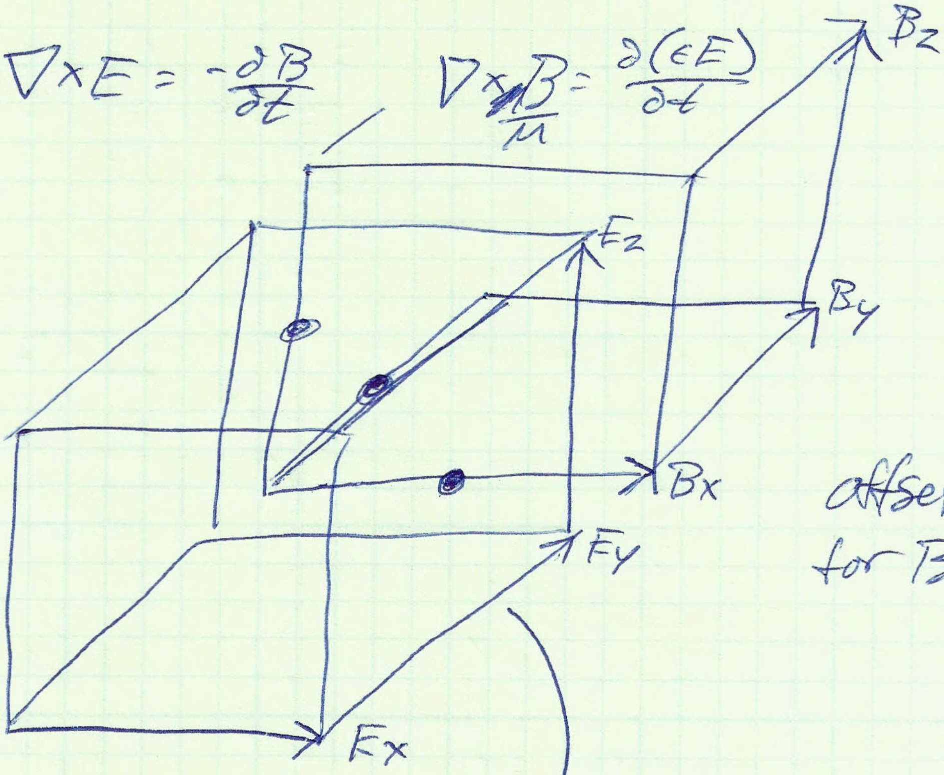
$$\boxed{\int_S (\nabla \times \vec{B}) \cdot d\vec{s} = \oint_C \vec{B} \cdot d\vec{l} \quad \text{Stokes Theorem}}$$

$\nabla \times \vec{B}$ integrated over surface S is equal to circulation around perimeter C

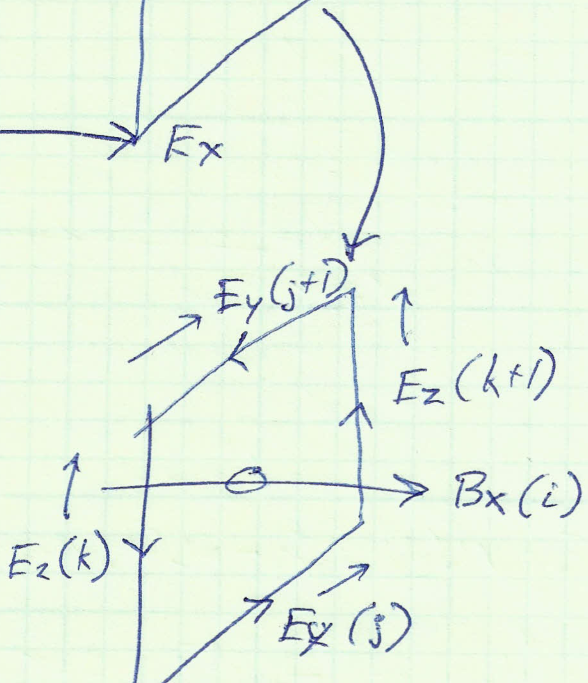
Application of curl in numerical simulations

example: FDTD (finite difference time domain)

$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times \frac{B}{\mu} = \frac{\partial (\epsilon E)}{\partial t}$$



offset lattice for B and E



$$B(t+1) = B(t) - \left[E_y(j) + E_z(k+1) - E_y(j+1) - E_z(k) \right]$$

E and B are incremented each timestep