Classical electron radius

Assume that the charge in an electron is assembled from a shell of charge at infinity which is brought together to a radius r_e

From Gauss's law, $4\pi R^2 E = \frac{Q}{\epsilon_0}$ for $R > r_e$

$$E = \frac{Q}{4\pi R^2 \epsilon_0}$$

Total energy in the electric field =

$$\int_0^{\pi}\int_0^{2\pi}\int_{r_e}^{\infty} \frac{\epsilon_0}{2} E^2 R^2 \cos\theta \, dR \, d\phi \, d\theta$$

$$= \int_0^{\pi}\int_0^{2\pi}\int_{r_e}^{\infty} \frac{\epsilon_0}{2} \left(\frac{Q}{4\pi R^2 \epsilon_0}\right)^2 R^2 \cos\theta \, dR \, d\phi \, d\theta$$

$$= \frac{4\pi \epsilon_0}{2} \left(\frac{Q}{4\pi \epsilon_0}\right)^2 \int_{r_e}^{\infty} \frac{dR}{R^2}$$

$$= \frac{1}{2} \frac{Q^2}{4\pi \epsilon_0} \left[\frac{1}{R}\right]\Big|_{r_e}^{\infty}$$ ← Q = electron charge, e

$$= \frac{1}{2} \frac{e^2}{4\pi \epsilon_0 r_e} = m_e c^2$$ ← set $E = mc^2$

energy, not electric field

this factor depends on what distribution we assume

$1/2$ for a shell, $3/5$ for a uniform sphere of charge, usually ignored

$$r_e = \frac{e^2}{4\pi \epsilon_0 m_e c^2}$$ classical electron radius

$$r_e = \alpha \frac{\hbar}{m_e c} = \alpha \frac{\lambda_e}{2\pi}$$ ← compton wavelength

$$\alpha = \frac{e^2}{4\pi \epsilon_0 \hbar c} \approx \frac{1}{137}$$ fine structure constant

$$= \alpha^2 a_0 \qquad a_0 = \frac{4\pi \epsilon_0 \hbar^2}{m_e e^2}$$ Bohr radius