Is the vector magnetic potential a "real" field?

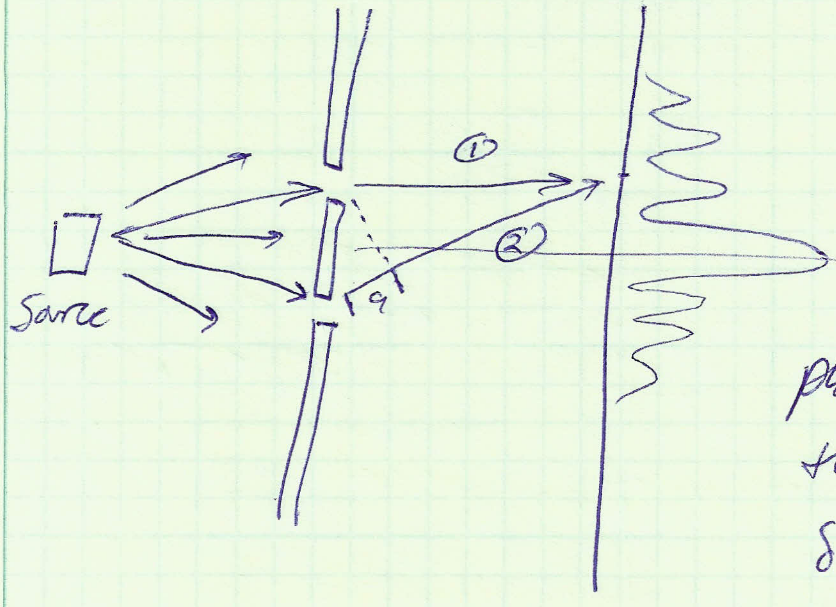
- The magnetic field $\vec{B}$ produces the forces on moving charges.
- $\vec{A}$ is somewhat arbitrary

  $$\vec{B} = \nabla \times \vec{A} \quad \text{and} \quad \nabla \times \nabla \Psi = 0$$

  so we can always add $\vec{A}'$, the gradient of a scalar field

  $$\nabla \times (\vec{A} + \vec{A}') = \nabla \times \vec{A} + \nabla \times \vec{A}' = \nabla \times \vec{A} + \nabla \times \nabla \Psi = \nabla \times \vec{A}$$

  without affecting the magnetic field $\vec{B}$
- Is $\vec{A}$ just a fabricated field to make solving problems easier?
- Define a "real" field as a mathematical function to avoid the idea of action at a distance
  - charges don't affect other charges that are far away
  - charges create fields, those fields affect other charges
  - behavior of charges only determined by functions (fields) <u>at that point</u>
- For decades (until 1956) $\vec{A}$ was considered to have no direct physical significance - just a tool for solving certain problems
- Test whether $\vec{A}$ is "real" by designing an experiment where $\vec{B} = 0$ and $\vec{A} \neq 0$, see if there are measurable physical effects



Classic double-slit experiment

phase difference between the two paths

$$\delta = \frac{2\pi a}{\lambda}$$

In quantum mechanics:

Magnetic change in phase $= \frac{q}{\hbar}\int_l \vec{A}\cdot d\vec{l}$ ← vector potential

Electric change in phase $= \frac{q}{\hbar}\int_t \Phi \, dt$ ← scalar potential

These are the quantum equivalent of $\vec{F} = q(\vec{E} + \vec{u}\times\vec{B})$

phase of path 1 $\quad \Phi_1 = \Phi_1(B=0) + \frac{q}{\hbar}\int_{①} \vec{A}\cdot d\vec{l}$

phase of path 2 $\quad \Phi_2 = \Phi_2(B=0) + \frac{q}{\hbar}\int_{②} \vec{A}\cdot d\vec{l}$

$$\delta = \Phi_1(B=0) - \Phi_2(B=0) + \frac{q}{\hbar}\int_{①}\vec{A}\cdot d\vec{l} - \frac{q}{\hbar}\int_{②}\vec{A}\cdot d\vec{l}$$

$$\delta = \delta(B=0) + \frac{q}{\hbar}\oint_{1-2} \vec{A}\cdot d\vec{l}$$

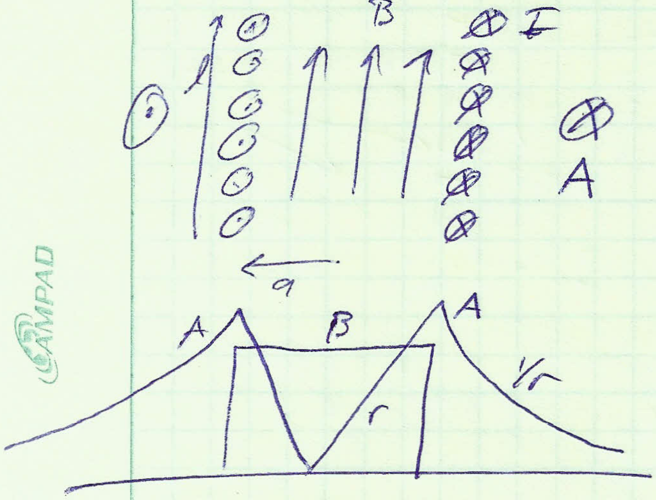
Although we can add an arbitrary $\nabla\Psi$ to A,

$$\oint \nabla\Psi \cdot d\vec{l} = 0 \text{ for a closed path}$$

so that arbitrary $\nabla\Psi$ component has no effect on the physics

$$\delta = \delta(B=0) + \frac{q}{\hbar}\left[\text{flux of } B \text{ between ① and ②}\right]$$

Vector potential of a solenoid



$$B = \frac{\mu_0 N I}{l} = \mu_0 n I \quad \text{where } n = \frac{N}{l}$$
turns/unit length

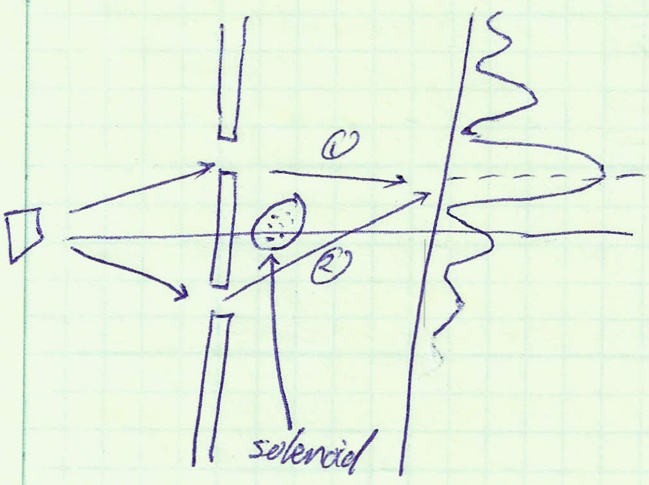
$$\vec{B} = \nabla \times \vec{A}$$

$$\int \vec{B} \cdot d\vec{s} = \oint \vec{A} \cdot d\vec{l}$$

$$\pi a^2 B = 2\pi r A$$

$$A = \frac{a^2 \mu_0 n I}{2r} \quad \text{outside the coil}$$

B is 0 outside the solenoid, but $A \neq 0$



A magnetic field can affect the motion of electrons even if the electrons do not pass through the field (but they do pass through the vector potential)

The experiment was done with microscopic iron whiskers. When magnetized, they appear as a tiny solenoid. The experiment validated the expected effect of $\vec{A}$.