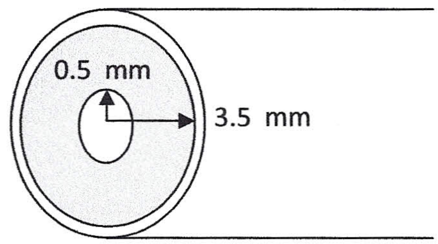
ECE 107 – Electromagnetism   Spring 2010   Midterm Exam #1

Name ______________________________   PID______________________________

#1 Transmission Lines: (do not forget units!)



A coaxial cable has the following characteristics:

- **Radius of center conductor: 0.5 mm**
- **Radius of outer conductor (inside surface): 3.5 mm**
- **Relative dielectric constant: 2.4 (lossless)**
- **Conductivity of conductors: infinite**
- **The cable contains no magnetic materials**
- **Assume the frequency is 5 GHz**

(a) Find the inductance per unit length (1 point)

$$L' = \frac{\mu_0}{2\pi} \ln(b/a) = \frac{4\pi \times 10^{-7}}{2\pi} \ln\left(\frac{3.5}{0.5}\right) = 3.9 \times 10^{-7}\ H/m$$

(b) Find the capacitance per unit length (1 point)

$$C' = \frac{2\pi \epsilon_r \epsilon_0}{\ln(b/a)} = \frac{2\pi \times 2.4 \times 8.854 \times 10^{-12}}{\ln(3.5/0.5)} = 6.9 \times 10^{-11}\ F/m$$

(c) Find the characteristic impedance (1 point)

$$Z_0 = \sqrt{\frac{L'}{C'}} = 75\Omega$$

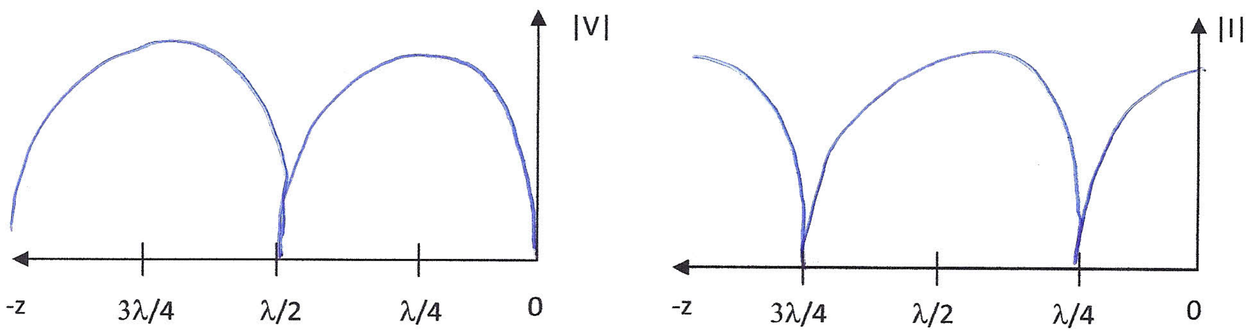
(d) Find the phase velocity of a wave traveling on this cable (1 point)

$$u_p = \frac{c}{\sqrt{\epsilon_r}} = \frac{3 \times 10^8}{\sqrt{2.4}} = 1.9 \times 10^8\ m/s$$

(e) Find the wavelength of a 5 GHz wave traveling on this cable (1 point)

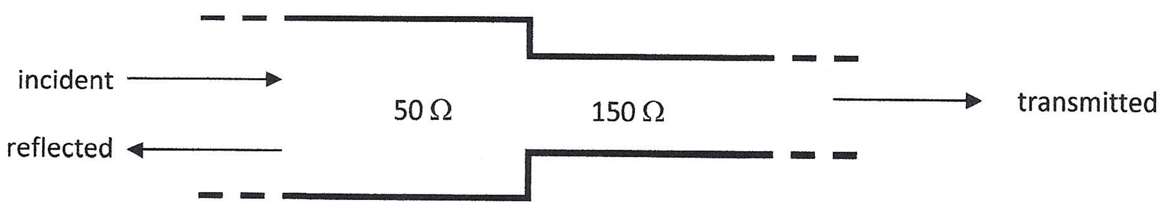
$$\lambda = \frac{c}{f\sqrt{\epsilon_r}} = 3.9\,cm$$

(f) Now assume the cable is terminated in a short circuit. A wave is traveling in the +z direction toward the short circuit at z=0, and forms a standing wave. Sketch the magnitude of the voltage and current in the standing wave as a function of z. You do not need to calculate the value at the voltage or current – just sketch the shape of each function. (3 points)



#2 Power flow:

A 50 Ohm transmission line is extends from z=-∞ to z=0, where it is attached to a 150 Ohm transmission line that extends from z=0 to z=∞. A wave with a magnitude of 5 Volts is traveling along the 50 Ohm line in the +z direction.



(a) How much average power is incident on the 150 Ohm line? (1 point)

$$\frac{|V_0^+|^2}{2Z_0} = \frac{5^2}{2\times 50} = 0.25\,W$$

(b) What is the voltage reflection coefficient at the interface between the two lines? (1 point)

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{150-50}{150+50} = 0.5$$

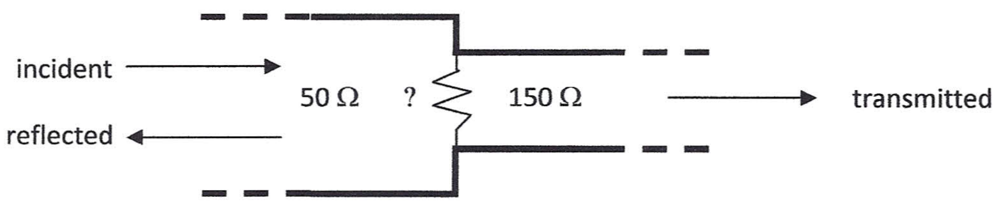
(c) How much average power is reflected from the interface? (1 point)

$$\frac{|\Gamma \cdot V_0^+|^2}{2Z_0} = \frac{(0.5 \times 5)^2}{2 \times 50} = 0.0625\ W$$

(d) How much average power is transmitted into the 150 Ohm line? (1 point)

$$0.25 - 0.0625 = 0.1875\ W$$

Now assume a resistor is placed at the interface.



(e) What resistance value would eliminate the reflected power? (2 points)

$$\frac{1}{50} = \frac{1}{R} + \frac{1}{150} \rightarrow R = 75\,\Omega$$

(f) With the resistor, how much average power is transmitted down the 150 Ohm line? (2 points)

$$\frac{5^2}{2 \times 150} = 0.0833\ W$$

(g) How much average power is absorbed into the resistor? (2 points)

$$\frac{5^2}{2 \times 75} = 0.1667\ W$$

#3 Impedance matching:

A 100 Ohm transmission line is terminated with a load of 30+j140 Ohms.

(a) Determine the magnitude of the reflection coefficient. (1 point)

$$|\Gamma| = \left|\frac{z_L - 1}{z_L + 1}\right| = \left|\frac{0.3 + j1.4 - 1}{0.3 + j1.4 + 1}\right| = 0.82$$

(b) Determine the voltage standing wave ratio. (1 point)

$$S = \frac{1 + |\Gamma|}{1 - |\Gamma|} = 10$$

(c) How far (in wavelengths) from the load is the first voltage maximum? (1 point)

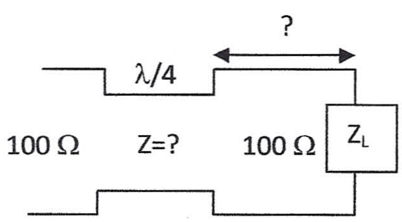
See Smith Chart

$0.25 - 0.154 = 0.096\lambda$

(d) How far (in wavelengths) from the load is the first voltage minimum? (1 point)

$0.096 + 0.25 = 0.346\lambda$

You are now designing a lossless quarter-wavelength transformer to match this load.



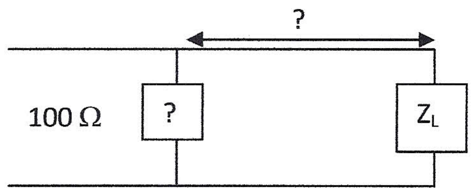
(e) How far from the load (in wavelengths) do you start the transformer? (2 points)

0.096λ from the load, at the voltage maximum

(f) What is the impedance of the quarter-wavelength section? (2 points)

$Z = \sqrt{Z_o \times 10 Z_o} = 316\ \Omega$

You are now designing a reactive lumped element matching circuit. You would like to place it as close as possible to the load, in shunt with the transmission line.



(g) What is the normalized admittance of the load? (2 points)

$\frac{1}{0.3 + j1.4} = 0.15 - j0.68$

(h) How far from the load (in wavelengths) do you place the shunt element? (2 points)

See Smith Chart

$(0.5 - 0.404) + 0.2 = 0.296\lambda$

(i) What is the normalized susceptance, b, of the shunt element, and is it capacitive or inductive? (2 points)

See Smith Chart

$b = -2.8 \rightarrow$ Inductive

(j) For a design frequency of 1 GHz, what is its value in Farads or Henrys? (2 points)

$b = \frac{B}{Y_o} = -2.8$

$B = -2.8 Y_o = \frac{-2.8}{Z_o} = \frac{-1}{\omega L}$

$L = \frac{Z_o}{2.8\omega} = \frac{100}{2.8 \times 2\pi \times 10^9} = 5.68\ nH$

The Complete Smith Chart

Black Magic Design