ECE 107, HW2 Solutions

(1) 2.36

We need $Z_{in} = Z_0 \dfrac{Z_L + jZ_0 \tan(\beta l)}{Z_0 + jZ_L \tan(\beta l)} = 40j\ \Omega$

$Z_0 = 50\Omega$

$Z_L = 0$ (short circuit)

$$\beta = \frac{2\pi}{\lambda} = \frac{2\pi f}{.75c} = 2\pi \frac{300\times10^6 \text{ Hz}}{.75\times(3\times10^8 \frac{m}{s})} = 8.34\ m^{-1}$$

so $Z_{in} = jZ_0 \tan(\beta l) = 40j$

$$l = \frac{1}{\beta}\tan^{-1}\left(\frac{40}{Z_0}\right)$$

The smallest positive solution corresponds to $0 < \tan^{-1}\left(\frac{40}{50}\right) < \frac{\pi}{2}$

$$= 8.05 \text{ cm}$$

(2) 2.40

(a) Electrical length $= \beta l = \frac{2\pi}{\lambda}\cdot\frac{\lambda}{4} = \pi/2$

For matching, we need $Z_{in}(-l) = \dfrac{Z_{02}^2}{Z_L} = Z_{01}$

$$Z_{02} = \sqrt{Z_L Z_{01}}$$

$$= \sqrt{73\Omega \cdot 300\Omega}$$

$$= 148\ \Omega$$

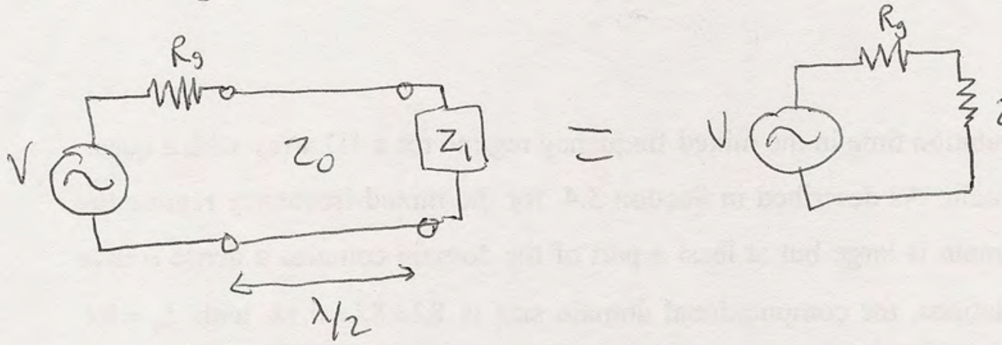
(b) $$l = \frac{\lambda}{4} = \frac{c}{4f\sqrt{\varepsilon_r}} = \frac{3\times10^8 \text{ m/s}}{4\cdot(100\times10^6 \text{ Hz})\sqrt{2.6}} = 46.5 \text{ cm}$$

$$Z_{02} = \frac{120\Omega}{\sqrt{\varepsilon_r}} \ln\left(D/d + \sqrt{(D/d)^2 - 1}\right)$$

$$\left(\frac{D}{d} - e^{\frac{Z_{02}\sqrt{\varepsilon_r}}{120}}\right)^2 = \left(\frac{D}{d}\right)^2 - 1 \implies \frac{D}{d} = \frac{e^{2\frac{Z_{02}\sqrt{\varepsilon_r}}{120}} + 1}{2e^{Z_{02}\sqrt{\varepsilon_r}/120}} \implies d = 0.67 \text{ cm}$$

3 2.44

Taking into account input impedance, the setup becomes



because $\frac{\lambda}{2}$ sections contribute nothing to input impedance

$$\Gamma = \frac{Z_1 - Z_0}{Z_1 + Z_0} = \frac{100\,\Omega - 50\,\Omega}{100\,\Omega + 50\,\Omega} = \frac{1}{3}$$

In the circuit representation, the voltage across the load is

$$V_L = V\frac{Z_1}{R_g + Z_1}, \qquad I_L = \frac{V}{R_g + Z_1}$$

The power dissipated by the load in the circuit representation equals the power transmitted into the infinite transmission line

$$\text{Transmitted power} \quad P_{av}^{t} = \frac{V_L I_L}{2} = \frac{V^2}{2(R_g + Z_1)^2} Z_1 = \frac{2V^2 \cdot 100\,\Omega}{2(50\,\Omega + 150\,\Omega)^2} = \boxed{8.89\,\text{mA}}$$

$$= \frac{|V_0^+|^2}{2Z_0}\left(1 - |\Gamma|^2\right)$$

$$P_{av}^{i} = \frac{|V_0^+|^2}{2Z_0} = \frac{P_{av}}{1 - |\Gamma|^2} = \frac{8.89\,\text{mA}}{1 - \left(\frac{1}{3}\right)^2} = \boxed{10\,\text{mA}}$$

$$P_{av}^{r} = -|\Gamma|^2 P_{av}^{i} = -\left(\frac{1}{3}\right)^2 \cdot 10\,\text{mA} = \boxed{-1.1\,\text{mA}}$$

Note that the input impedance of an infinite line is <u>always</u> equal to its input impedance, even if there is no loss. (The book is wrong)

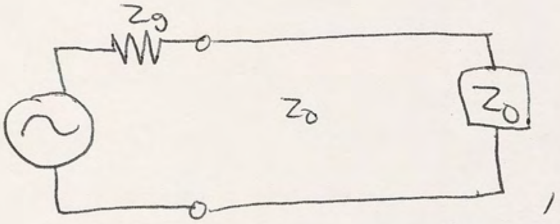
$$\text{Remember,} \quad Z_{in}(z) = \frac{\tilde{V}(z)}{\tilde{I}(z)} = \frac{V_0^+ e^{-\gamma z}}{\frac{V_0^+}{Z_0} e^{-\gamma z}}$$

(because there can be no reflected wave)

$$= Z_0$$

4) 2.46

Under matched conditions, meaning



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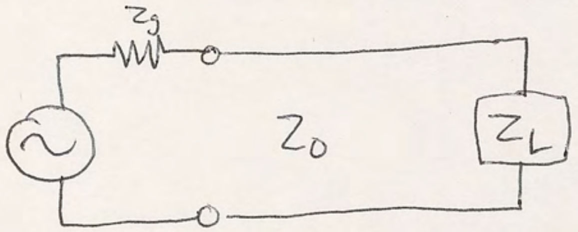
20 W is delivered to the load. Because there is no reflected power, $P^i_{av} = 20\,W$.

When the transmitter is connected to the antenna,



the incident power is still 20W, but some power is reflected. The power delivered to the load is

$$P_{av} = P^i_{av} - |\Gamma|^2 P^i_{av}$$

$$= P^i_{av}(1-|\Gamma|^2)$$

Note that $Z_g = Z_0$ in this case, so there are no multiple reflections from the generator

where $\Gamma = \dfrac{Z_L - Z_0}{Z_L + Z_0} = \dfrac{25 + 25j\,\Omega}{125 + 25j\,\Omega} = \dfrac{1+j}{5+j}$

$$|\Gamma| = \frac{|1+j|}{|5+j|}$$

$$|\Gamma| = \sqrt{\frac{2}{26}}$$

so $P_{av} = \left(1 - \frac{1}{13}\right) 20\,W = 18.5\,W$

(5.)

$$r_L + j x_L = \frac{1+\Gamma_r + j\Gamma_i}{1-\Gamma_r + j\Gamma_i} \cdot \frac{1-\Gamma_r + j\Gamma_i}{1-\Gamma_r + j\Gamma_i}$$

$$= \frac{1-\Gamma_r^2 - \Gamma_i^2 + 2j\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

so $$x_L = \frac{2\Gamma_i}{(1-\Gamma_r)^2 + \Gamma_i^2}$$

$$(1-\Gamma_r)^2 x_L + \Gamma_i^2 x_L = 2\Gamma_i$$

$$(1-\Gamma_r)^2 + \Gamma_i^2 - \frac{2}{x_L}\Gamma_i = 0$$

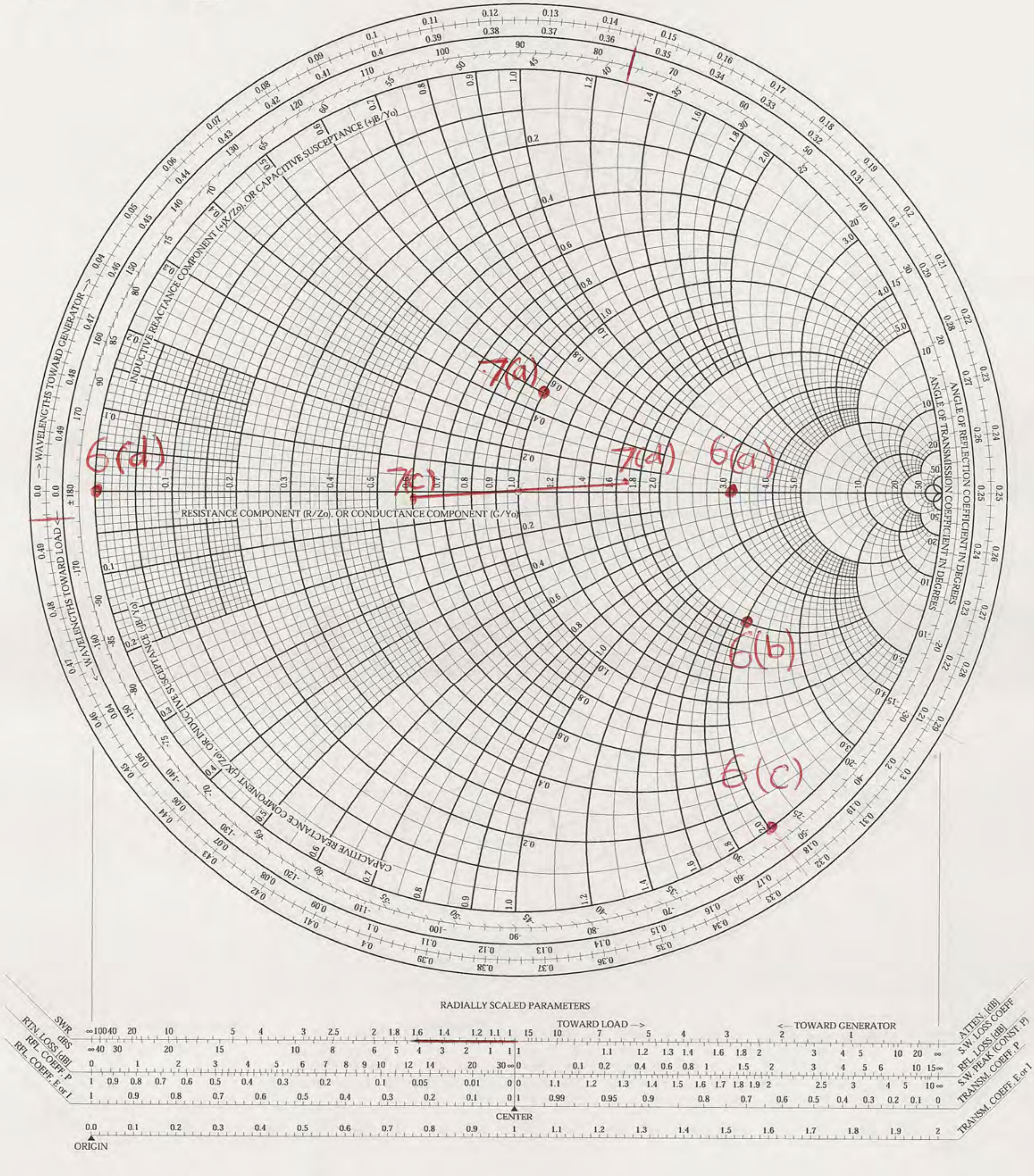
but $$\Gamma_i^2 - \frac{2}{x_L}\Gamma_i = \left(\Gamma_i - \frac{1}{x_L}\right)^2 - \left(\frac{1}{x_L}\right)^2$$

So $$(1-\Gamma_r)^2 + \left(\Gamma_i - \frac{1}{x_L}\right)^2 = \frac{1}{x_L^2}$$

6 and 7

The Complete Smith Chart

Black Magic Design



(6) 2.47

For the smith chart given in class, there is a distance of 8cm from the origin ($\Gamma = 0$) to the outer circle ($|\Gamma| = 1$). Therefore, $|\Gamma|$ is measured from the formula $|\Gamma| = \dfrac{\text{distance from origin (cm)}}{8\text{cm}}$

and the angle of Γ is read off from the outer scale.

(a) $|\Gamma| = \dfrac{4\text{cm}}{8\text{cm}} = \dfrac{1}{2}$, $\theta_r = 0$

$\Gamma = \frac{1}{2}$

(b) $|\Gamma| = \dfrac{5\text{cm}}{8\text{cm}} = 0.625$, $\theta_r \approx -30°$

$\Gamma = .625e^{-j\pi/6} = .54 - .31j$

(c) $|\Gamma| = \dfrac{8\text{cm}}{8\text{cm}} = 1$, $\theta_r = -53°$

$\Gamma = e^{-j \cdot 0.93} = .60 - .80j$

(d) $|\Gamma| = \dfrac{8\text{cm}}{8\text{cm}} = 1$, $\theta_r = 180°$

$\Gamma = 1e^{+j\pi} = -1$

(7) (a) $|\Gamma| = \dfrac{2\text{cm}}{8\text{cm}} = \dfrac{1}{4}$, $\theta_r \approx 75.5°$

$\Gamma = \frac{1}{4}e^{j\theta_r} = 0.063 + 0.24j$

(b) Measuring 2cm on the SWR scale gives

SWR ≈ 1.65

(7) (c) Using the outer scale, $.145 + .35 =$

This gives a normalized input impedance of

$$\frac{Z_{in}(-.35\lambda)}{Z_0} = .60 - .02j$$

$$Z_{in}(-.35\lambda) = 30 - j\ \Omega$$

(d) Inverting the input impedance gives the normalized input admittance:

$$\frac{Y_{in}(-.35\lambda)}{Y_0} = 1.7 + .05j \qquad Y_0 = \frac{1}{Z_0}$$

$$Y_{in} = .034 + .001j\ \Omega^{-1}$$

(e) Using the outer scale, it requires a length

$$\ell = .25\lambda - .145\lambda$$
$$= .105\lambda$$

to get to the real axis, where Z_{in} is purely real

(f) The first voltage maximum coincides with the first point where Z_{in} is purely real, so the answer is

$$\ell = .105\lambda$$

8 2.60

$$Z_L = 50 - 25j\ \Omega$$

$$Y_L = \frac{1}{Z_L} = .016 + .008\ \Omega^{-1}$$

$$\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{50 - 25j - 200}{50 - 25j + 200} = -.584 - .158j$$

$$\Gamma = .605\, e^{-2.88j}$$

need to solve for d:

$$\cos(\theta_r - 2\beta d) = -|\Gamma|$$

$$\theta_r - 2\beta d = \pm \cos^{-1}(-|\Gamma|)$$

$|\Gamma| = .605$

$$d = \frac{\theta_r \mp \cos^{-1}(-|\Gamma|)}{2\beta}$$

$\theta_r = -2.88$

$\beta = \frac{2\pi f}{c/\sqrt{\varepsilon_r}} = \frac{2\pi\, 8\cdot 10^8\,\text{Hz}}{3\times 10^8\,\text{m/s} \cdot \frac{1}{2}} = 33.5$

Two solutions exist:

$d_1 = 1.77\text{cm}$ where $\cos^{-1}(-|\Gamma|)$ is chosen so that d

$d_2 = 8.40\text{cm}$ is the smallest possible positive value

These are the two positions at which $\mathrm{Re}\{Y_{in}\} = Y_0$

a) Now, for each case, the imaginary part must be matched

for $d = d_1$ the shunt reactive component has impedance

$$Z_{s1} = \frac{Z_0}{j b_{s1}} = \frac{200\Omega}{j} \frac{1 + |\Gamma|^2 + 2|\Gamma|\cos(\theta_r - 2\beta d_1)}{2|\Gamma|\sin(\theta_r - 2\beta d_1)}$$

$$= -131.5j\ \Omega$$

In this case use a capacitor with capacitance satisfying

$$\frac{1}{j\omega C} = -131.5j\ \Omega$$

$$C = \frac{1}{2\pi \cdot 8\times10^{8}\,\text{Hz} \cdot 131.5} = \boxed{1.51\,\text{pF}}$$

b) for $d = d_2$ the matching element has impedance

$$Z_{S2} = 131.5j\,\Omega$$

Use an inductor with inductance

$$j\omega L = 131.5j$$

$$L = \frac{131.5}{2\pi\ 8\cdot10^{8}\,\text{Hz}} = \boxed{26.2\,\text{nH}}$$

9. $z_L = \dfrac{Z_L}{Z_0} = .25 - .125j$

Using the Smith chart, the normalized admittance is

$y_L = 3.2 + 1.7j$

Tracing the arc to the point where $\text{Re}\{y_{in}\} = y_0$ yields two possible solutions

(a) at a distance from the load (given by the outer scale)

$$d_1 = .322\lambda - .228\lambda \qquad \lambda = \frac{2\pi}{\beta} = 18.75\,\text{cm}$$

$$= 1.76\,\text{cm}$$

(9)

The Complete Smith Chart

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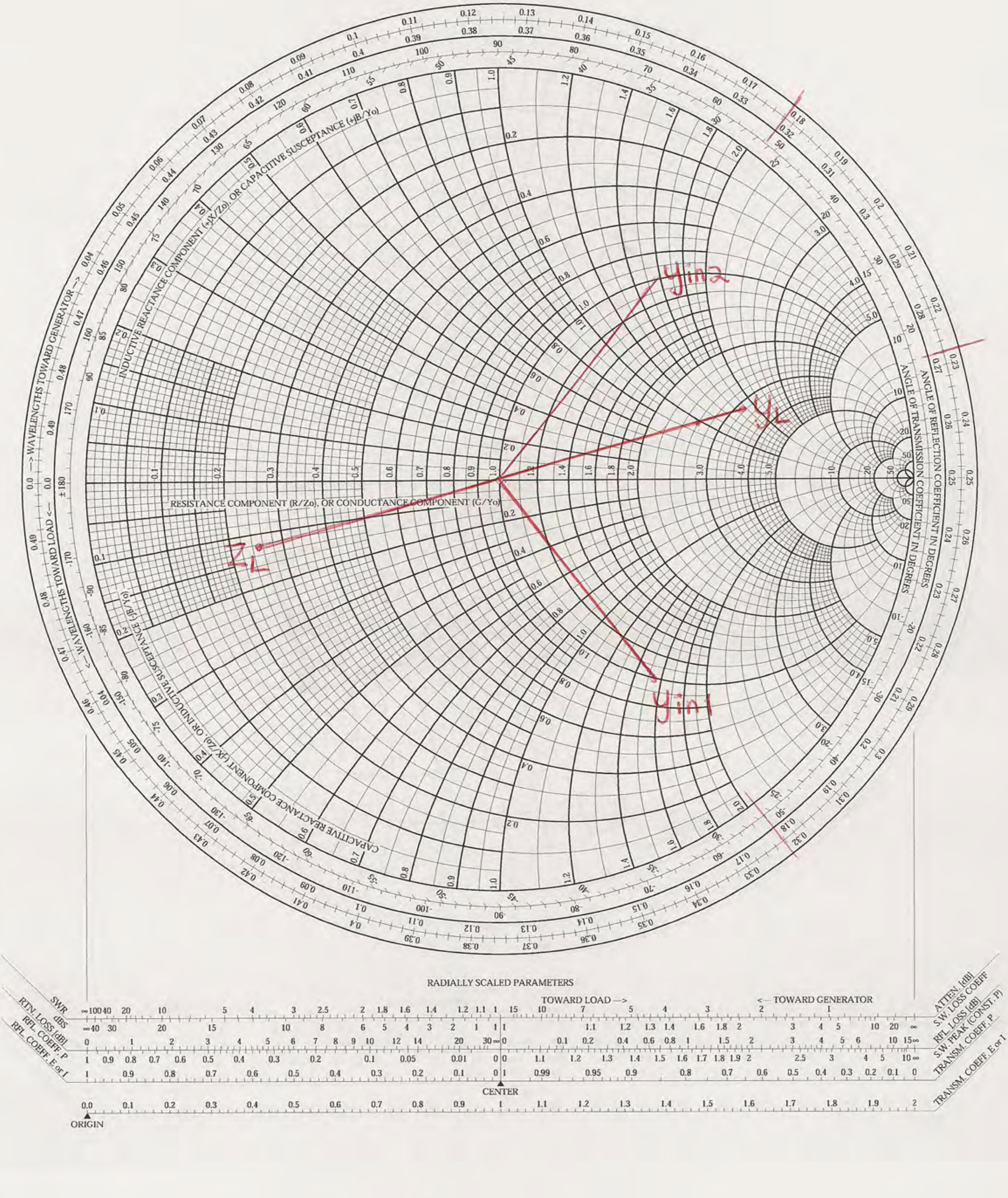

(9) at this position the normalized input admittance is

$$y_{in1} = 1 - 1.55j$$

so the shunt element must have normalized susceptance

$$b_{s1} = 1.55$$

and impedance

$$Z_{s1} = \frac{Z_0}{j\,b_{s1}} = -129j\ \Omega$$

using a capacitor,

$$C = \frac{1}{j\omega Z_s} = \boxed{1.54\,pF}$$

The other solution is found a distance

$$d_2 = \underbrace{\frac{\lambda}{2}}_{\text{full circle}} - \underbrace{(.228 - .178)\lambda}_{\text{"wedge"}} = 8.44\,cm$$

at this point $y_{s2} = 1 + 1.55j$

$$b_{s2} = -1.55j$$

$$Z_{s2} = \frac{Z_0}{j\,b_{s2}} = +129j\ \Omega$$

use an inductor

$$L = \frac{Z_{s2}}{j\omega} = \boxed{25.7\,nH}$$

10 2-74

I will use a reactive shunt impedance, and solve using the Smith chart

$$z_L = \frac{Z_L}{Z_0} = \frac{25\Omega}{75\Omega} = \frac{1}{3}$$

so $y_L = \frac{1}{z_L} = 3$

tracing the arc (with radius 4cm), the closest matching admittance is

$y_{in} = 1 - 1.15j \implies b_s = +1.15$ = susceptance of shunt element

It is found a distance

$$\ell = (.334 - .25)\lambda$$

$= 0.084\lambda$ from the load

so $Z = \frac{Z_0}{j b_s} = \frac{75\Omega}{j\,1.15} = \boxed{-65.2j}$

This would require a capacitor

Another option is to place the shunt element a distance

$\ell = \frac{\lambda}{2} - .084\lambda = .416\lambda$ from the load

In this case

$\boxed{Z = +65.2j}$ and an inductor is required

(10)

The Complete Smith Chart

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