HW 6 solutions ECE 107

① 6.2

$$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} = -\frac{d}{dt}\left(B_0 A \sin\omega t\right) = -\omega B_0 A\cos\omega t$$

(a) at $t=0$, $V_{emf} = -B_0 A\omega < 0$, so I flows in $-\hat{\phi}$ direction

(b) at $\omega t = \pi/4$, $V_{emf} = -B_0 A \frac{\sqrt{2}}{2}\omega$ I flows in $-\hat{\phi}$ direction

(c) at $\omega t = \pi/2$ $V_{emf} = 0 \Rightarrow$ no current flows

② 6.6

(a) $$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s}$$

$$= -\frac{d}{dt}\int_0^{10cm}\int_{5cm}^{10cm} \frac{\mu_0 I(t)}{2\pi y}(-\hat{x})\cdot(dy\,dz\,\hat{x})$$

$$= \frac{\mu_0}{2\pi}\left(\frac{d}{dt}I(t)\right)\cdot 10cm \cdot \ln\frac{15cm}{5cm}$$

$$= -\frac{\mu_0 \cdot 2\pi\times10^4}{2\pi}\cdot 0.1m \cdot \ln(3)\cdot 5\sin(2\pi\times10^4 t)$$

$$= 6.90\times10^{-3}\sin(2\pi\times10^4 s^{-1} t)\ V$$

(b) $$\text{Magnitude of current} = \frac{|V_{emf}|}{5\Omega} = 1.38\times10^{-3}\ A$$

Direction is counterclockwise when $\sin(2\pi\times10^4 t) < 0$,
clockwise when $\sin(2\pi\times10^4 t) > 0$

③ 6.8

(a) $$\vec{H} = \frac{I}{2\pi r}\hat{\phi} = \frac{I_0\cos(\omega t)}{2\pi r}$$

$$\vec{B} = \mu\vec{H} = \mu_0\mu_r\vec{H}$$

(3) $$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} = -\frac{d}{dt}\frac{\mu_0\mu_r N I_0 \cos(\omega t)}{2\pi}\int_a^b\int_0^c \frac{1}{r}\,dr\,dz$$

$$\frac{\mu_0\mu_r N I_0 \omega \sin(\omega t)}{2\pi}\cdot c\ln\left(\frac{b}{a}\right)$$

(b) $$V_{emf} = \frac{4000\cdot 4\pi\times10^{-7}(50A)(2\pi\cdot 60Hz)}{2\pi}\cdot(.02\text{ m})\ln\left(\frac{6}{5}\right)\sin(120\pi t)$$

$$= 5.50\text{ V}\sin(120\pi t)$$

(4) 6.11 $$V_{emf} = -\frac{d}{dt}\int \vec{B}\cdot d\vec{s} = -\frac{d}{dt}\int\left(-\frac{\mu_0 I}{2\pi y}\hat{x}\right)\cdot\left(dy\,dz\,\hat{x}\right)$$

this choice of sign, or orientation, is consistent with the direction of I_2, which is chosen as CW

$$= -\frac{\mu_0 I}{2\pi}\int_0^{20cm} dz\frac{d}{dt}\int_{ut}^{ut+10cm}\frac{dy}{y}$$

$$= -.2m\,\frac{\mu_0 I}{2\pi}\frac{d}{dt}\left(\ln\left(\frac{ut+.1m}{ut}\right)\right)$$

$$= -.2\text{ m}\,\frac{\mu_0 I}{2\pi}\,\frac{.1m}{u}\cdot\frac{-1}{t^2}\,\frac{ut}{ut+.1m}$$ but $ut = y_0$

$$= +.02m\,\frac{\mu_0 I}{2\pi}\,\frac{u}{y_0(y_0+.1m)}$$

$$I_2 = \frac{V_{emf}}{2R} = \frac{.2m}{2\cdot 10\Omega}\,\frac{4\pi\times10^{-7}\frac{H}{m}\cdot 10A}{2\pi}\,\frac{7.5\,m/s}{y_0(y_0+.1m)}$$

$$= \frac{1.5\times10^{-8}}{y_0(y_0+0.1m)}$$

5 6.16

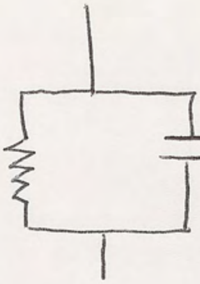
(a) $I_c = \int \vec{J} \cdot d\vec{s}$, where $\vec{J} = \sigma \vec{E}$ by ohm's law

$|\vec{E}| = \frac{V}{d}$

So $I_c = \int \sigma \frac{V}{d} \, ds = \frac{\sigma A V(t)}{d}$

(b) $I_D = \int \frac{\partial \vec{D}}{\partial t} \cdot d\vec{s} = \frac{\varepsilon \frac{\partial}{\partial t}(V(t)) A}{d}$

(c) The equivalent circuit representation is an ideal capacitor in parallel with a resistor. This allows current I_c to flow when a voltage $V(t)$ is applied, and displacement current I_D to account for the reactive fields.



(d) $C = \frac{\varepsilon A}{d} = 2.83 \text{pF}$ $R = \frac{d}{\sigma A} = 5\Omega$

6 6.22 The relaxation time constant is

$$\tau_{soil} = \frac{\varepsilon_{soil}}{\sigma_{soil}} = \frac{2.5 \varepsilon_0}{10^{-4} \text{ S/m}} = 2.5 \times 10^4 \varepsilon_0$$

$$\tau_{H_2O} = \frac{\varepsilon_{H_2O}}{\sigma_{H_2O}} = \frac{80 \varepsilon_0}{10^{-3} \text{ S/m}} = 8.0 \times 10^4 \varepsilon_0$$

$\tau_{H_2O} > \tau_{soil}$, so fresh water is a better insulator

7 6.25

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} = -\mu_0 \frac{\partial \vec{H}}{\partial t}$$

$$\vec{H} = \frac{-1}{\mu_0} \int \nabla \times \vec{E} \, dt$$

$$= -\frac{1}{\mu_0} \int \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ E_x & 0 & 0 \end{vmatrix} dt$$

$$= -\frac{1}{\mu_0} \int \left(\frac{\partial E_x}{\partial z} \hat{y} - \frac{\partial E_x}{\partial y} \hat{z} \right) dt$$

$$= -\frac{1}{\mu_0} \int \left(E_0 \sin(ay)(-k)(-\sin(\omega t - kz)) \hat{y} - E_0 a \cos(ay) \cos(\omega t - kz) \hat{z} \right) dt$$

$$= -\frac{E_0}{\mu_0} \left[k \sin(ay) \left(\int \sin(\omega t - kz) \, dt \right) \hat{y} - a \cos(ay) \int \cos(\omega t - kz) dt \, \hat{z} \right]$$

$$= -\frac{E_0}{\mu_0} \left[-\frac{k}{\omega} \sin(ay) \cos(\omega t - kz) \hat{y} - \frac{a}{\omega} \sin(\omega t - kz) \hat{z} \right]$$

$$= \frac{E_0}{\omega \mu_0} \left[k \sin(ay) \cos(\omega t - kz) \hat{y} + a \cos(ay) \sin(\omega t - kz) \hat{z} \right]$$