HW 7 solutions ECE 107, Spring 2010

(1) 7.1 $\vec{H} = 30\cos(10^8 t - 0.5y)\,\hat{z}$

a) the wave propagates in the positive $\hat{y}$ direction

b) $V_{phase} = \dfrac{\omega}{k} = \dfrac{10^8 s^{-1}}{.5 m^{-1}} = 2\times10^8 m/s$

c) $\lambda = \dfrac{2\pi}{k} = \dfrac{2\pi}{.5 m^{-1}} = 4\pi$

d) $V_{phase} = \dfrac{c}{\sqrt{\varepsilon_r}} \Rightarrow \varepsilon_r = \left(\dfrac{c}{V_{phase}}\right)^2 = \left(\dfrac{3\times10^8 m/s}{2\times10^8 m/s}\right)^2 = 2.25$

e) $\nabla\times\vec{H} = -\varepsilon\dfrac{\partial\vec{E}}{\partial t}$

$$\vec{E} = -\frac{1}{\varepsilon}\int \vec{\nabla}\times\vec{H}\,dt$$

$$= -\frac{1}{\varepsilon}\int \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 0 & 0 & H_z \end{vmatrix} dt$$

$$= -\frac{1}{\varepsilon}\int \hat{x}\frac{\partial H_z}{\partial y}\,dt$$

$$= -\frac{1}{\varepsilon}\int \hat{x}\,30\cdot 0.5\sin(10^8 t - 0.5y)$$

$$= -\frac{15}{\varepsilon}\hat{x}\,\frac{1}{10^8}\left[-\cos(10^8 t - 0.5y)\right]$$

$$= \hat{x}\frac{15}{(2.25\times 8.85\times10^{-12})10^8}\cos(10^8 t - 0.5y)$$

$$= \hat{x}\,7{,}533\,\tfrac{V}{m}\cos(10^8 t - .5y)$$

(2) 7.6 a) $f = \dfrac{\omega}{2\pi} = \dfrac{6\pi\times10^9}{2\pi} = 3\times10^9 Hz$ $k = \dfrac{2\pi}{\lambda} = 101\ m^{-1}$

$u_p = \dfrac{c}{\sqrt{\varepsilon_r}} = \dfrac{3\times10^8 \frac{m}{s}}{\sqrt{2.56}} = 1.875\times10^8 \frac{m}{s}$ $\eta = \sqrt{\dfrac{\mu_0}{\varepsilon_r\varepsilon_0}} = \dfrac{\eta_0}{\sqrt{\varepsilon_r}} = \dfrac{120\pi}{\sqrt{2.56}} = 236\,\Omega$

$\lambda = \dfrac{u_p}{f} = \dfrac{1.875\times10^8 \frac{m}{s}}{3\times10^9 Hz} = 6.25 cm$

(2) 7.6

$$\vec{E} = 20\cos(6\pi\times10^9 t - kz)$$
$$\tilde{E} = 20 e^{-jkz}\hat{y}$$
$$\tilde{H} = \frac{1}{j\omega\mu}\nabla\times\vec{E} = \frac{1}{j\omega\mu}\begin{vmatrix}\hat{x} & \hat{y} & \hat{z}\\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z}\\ 0 & E_y & 0\end{vmatrix} = \frac{1}{j\omega\mu}\left(-\frac{\partial E}{\partial z}\right)\hat{x}$$
$$= \frac{1}{j\omega\mu}(20kje^{-jkz})\hat{x}$$
$$= \frac{20k}{\omega\mu}e^{-jkz}\hat{x} \qquad \text{but } \frac{k}{\omega\mu} = \frac{1}{\mu c} = \frac{\sqrt{\varepsilon\mu}}{\mu} = \sqrt{\frac{\varepsilon}{\mu}} = \frac{1}{\eta}$$
$$\vec{H} = \frac{20}{\eta}\cos(6\pi\times10^9 t - kz)\hat{x}$$
$$= 0.085\,\tfrac{A}{m}\cos(6\pi\times10^9 t - 101z)\hat{x}$$

(3) 7.8

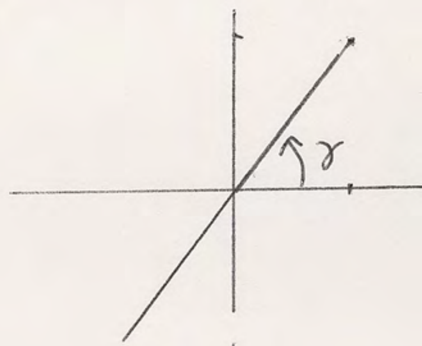
$$\tilde{E} = \frac{E_0}{\sqrt{2}}(\hat{x} + j\hat{y})e^{jkz}$$
$$\vec{E} = \mathrm{Re}\{\tilde{E}e^{j\omega t}\}$$

$$E_0 = |\tilde{E}| = 2\,\tfrac{V}{m}$$
$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{.06} = 105\text{ m}^{-1}$$
$$\omega = kc = 105\text{ m}^{-1}\cdot 3\times10^8\tfrac{m}{s} = 3.14\times10^{10}\tfrac{1}{s}$$

$$= \frac{E_0}{\sqrt{2}}(\hat{x}\cos(\omega t + kz) - \hat{y}\sin(\omega t + kz))$$
$$= 1.4\left(\hat{x}\cos(3.14\times10^{10}t + 105z) - \hat{y}\sin(3.14\times10^{10}t + 105z)\right)$$

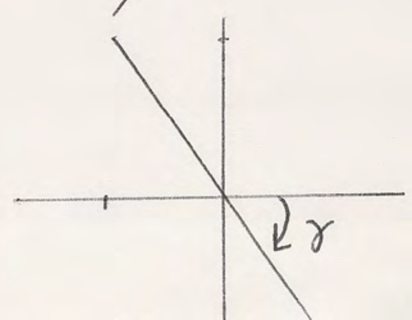
(4) 7.9

(a) $\delta = 0 \Rightarrow$ linear polarization



$$\vec{E}(0,t) = (3\hat{x} + 4\hat{y})\cos(\omega t)$$
$$\chi = 0,\ \gamma = \psi_0 = \tan^{-1}\tfrac{4}{3} = 53.1^\circ$$

(b) $\delta = 180^\circ \Rightarrow$ linear polarization



$$\vec{E}(0,t) = 3\hat{x}\cos(\omega t) + 4\hat{y}\cos(\omega t + \pi)$$
$$= (3\hat{x} - 4\hat{y})\cos(\omega t)$$
$$\chi = 0 \quad \gamma = -\psi_0 = -53.1^\circ$$

(4) 7.9

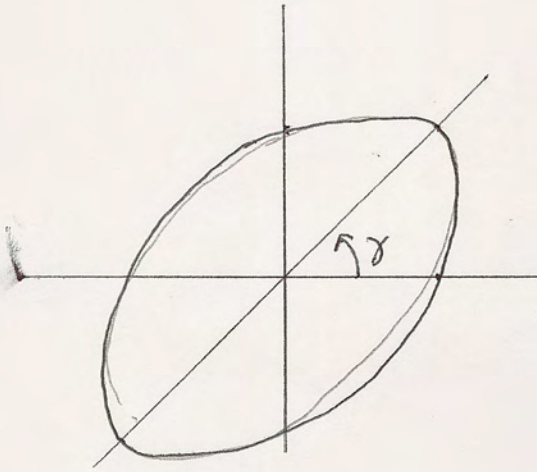
(c) $\vec{E}(0,t) = 3\left[\hat{x}\cos(\omega t) + \hat{y}\cos(\omega t + \pi/4)\right]$

$$\tan(2\gamma) = \tan(2\psi_0)\cos\delta$$

$$= \tan(2\cdot\tan^{-1}(1))\cos\frac{\pi}{4}$$

$$= \tan(\pi/2)(\cos\pi/4)$$

$$= \infty$$



so $2\gamma = \pi/2 \Rightarrow \boxed{\gamma = \frac{\pi}{4}}$

$$\sin(2\chi) = \sin(2\psi_0)\sin\delta$$

$$= \sin(\pi/2)\sin\pi/4$$

$$= \frac{1}{\sqrt{2}} > 0 \Rightarrow \underline{\text{Left handed}}$$

$$\chi = \frac{1}{2}\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = \frac{\pi}{8} \Rightarrow \underline{\text{Elliptical polarization}}$$

(d) $\delta = -135°$

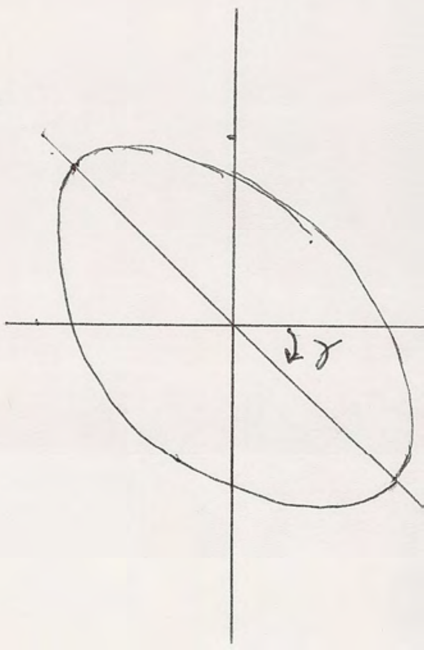
Following the same procedure, with $\psi_0 = \frac{\pi}{4}$ again,

$$\tan(2\gamma) = -\infty \Rightarrow \boxed{\gamma = -\frac{\pi}{4}}$$

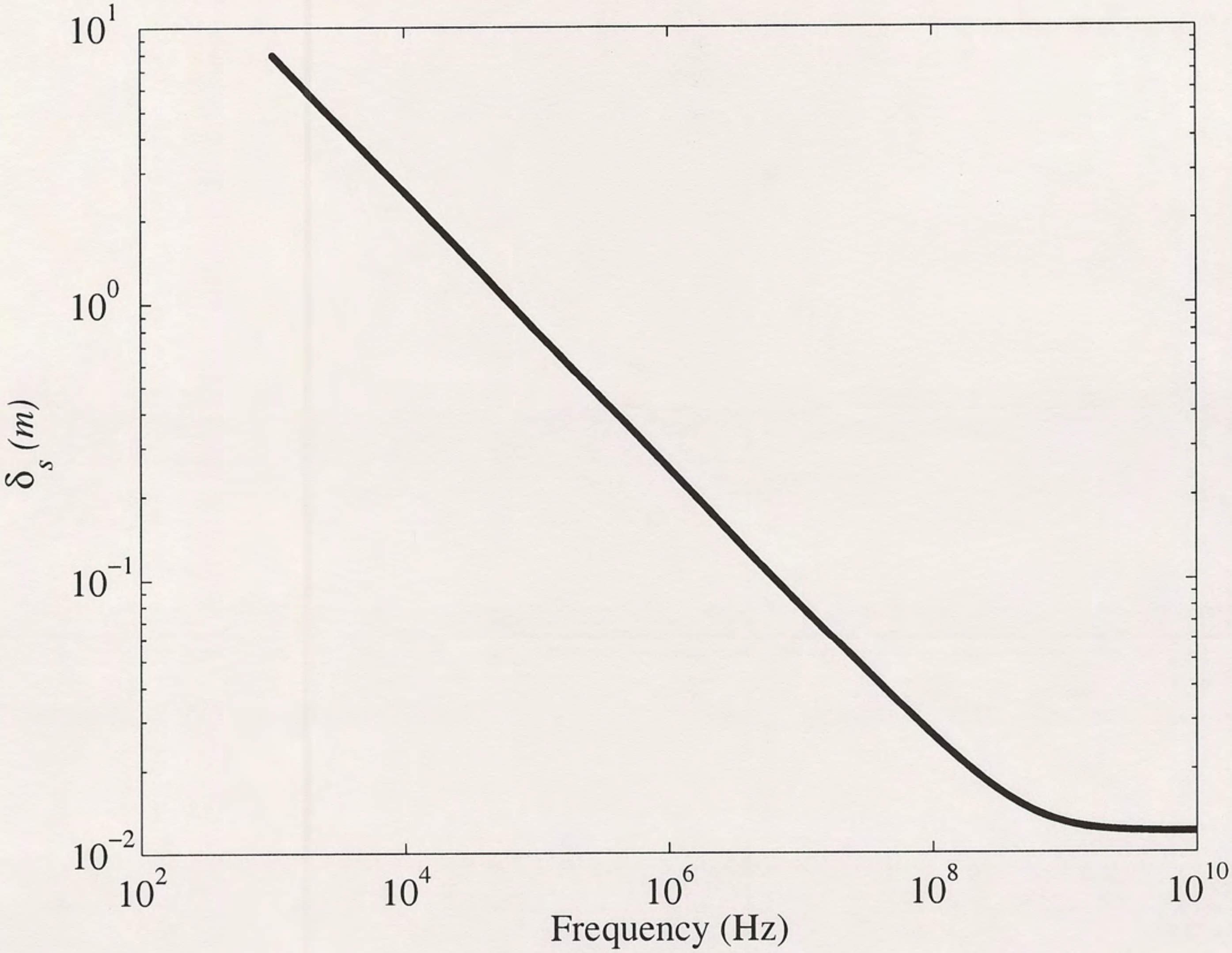
$$\sin(2\chi) = \sin\frac{\pi}{2}\sin\frac{5\pi}{4}$$

$$= -\frac{1}{\sqrt{2}} < 0 \Longrightarrow \underline{\text{Right handed}}$$

$$\chi = -\frac{\pi}{8} \Longrightarrow \underline{\text{Elliptical polarization}}$$



(5) 7.18



$$\alpha = \omega\left[\frac{\mu\varepsilon'}{2}\left(\sqrt{1+\left(\frac{\varepsilon''}{\varepsilon'}\right)^2}-1\right)\right]^{1/2}$$

$$\omega = 2\pi f$$

$$\mu = 4\pi\times10^{-7}\ \frac{H}{m}$$

$$\varepsilon' = \varepsilon_r\varepsilon_0 = 7.08\times10^{-10}\ \frac{F}{m}$$

$$\delta_s = \frac{1}{\alpha}$$

$$\varepsilon'' = \frac{\sigma}{\omega} = \frac{4\ S/m}{2\pi f}$$

(6) 7.26

a) For this good conductor,

$$\delta_s \approx \frac{1}{\sqrt{\pi f \mu \sigma}} = \frac{1}{\sqrt{\pi \cdot 10^7 \text{Hz} \cdot 4\pi \times 10^{-7} \frac{H}{m} \cdot 5.8\times 10^7 \frac{S}{m}}} = 20.9\,\mu m$$

$\delta_s << .5mm$, so the conductors may be viewed as infinitely thick.

b) The surface resistance is

$$R_s = \frac{1}{\sigma \delta_s} = 8.25\times 10^{-4}\,\Omega$$

c) $$R' = \frac{R_s}{2\pi}\left(\frac{1}{a}+\frac{1}{b}\right)$$

$$= \frac{8.85\times 10^{-4}\,\Omega}{2\pi}\cdot\left[\frac{1}{.005m}+\frac{1}{.01m}\right]$$

$$= .039\,\frac{\Omega}{m}$$

(7) 7.30

$$|\vec{H}| = \sqrt{H_{y0}^2 + H_{z0}^2} = \sqrt{(3\tfrac{mA}{m})^2 + (4\tfrac{mA}{m})^2} = 5\,\frac{mA}{m}$$

$$\vec{S}_{av} = \frac{1}{2} R_e\{\tilde{E}\times\tilde{H}\} = \frac{1}{2}\eta\,|\tilde{H}|^2\,\hat{x}$$

(Note that the direction of propagation ($\hat{x}$) must be the direction of power flow)

$$\eta = \frac{\eta_0}{\sqrt{\varepsilon_r}} = \frac{120\pi}{\sqrt{4}} = 60\pi\,\Omega$$

$$P_{av} = \int \vec{S}_{av}\cdot d\vec{A} = \int \vec{S}_{av}\cdot dy\,dz\,\hat{x} = |\vec{S}_{av}|\cdot A$$

$$P_{av} = \frac{1}{2}\eta\,|\tilde{H}|^2 A = \frac{1}{2}\cdot(60\pi\,\Omega)\cdot(.005\,\tfrac{A}{m})^2\cdot(20m^2) = .047\,W$$

8 7-32

Need to find the distance R_o at which

$$|\vec{S}_{av}| = \frac{1}{2\eta}|E(R)|^2 = 1\ \frac{mW}{cm^2} \cdot \frac{1\,W}{10^3\,mW} \cdot \frac{10^4\,cm^2}{1\,m^2} = 10\ \frac{W}{m^2}$$

$$\frac{1}{2(120\pi)} \cdot \left(\frac{3000\,V}{R_o}\right)^2 = 10\ \frac{W}{m^2}$$

$$R_o = \sqrt{\frac{\frac{1}{240\pi\,\Omega}\,9\times10^6\,V^2}{10\ \frac{W}{m^2}}} = 34.5\,m$$