ECE 107 HW8 solutions, Spring 2010

1 8.4

a) $\tilde{\vec{E}}^i(z) = E_0(\hat{x}+j\hat{y})e^{-jkz}$

$= 5\frac{V}{m}(\hat{x}+j\hat{y})e^{-jkz}$

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{c} = \frac{2\pi \cdot 200\times10^6\,\text{Hz}}{3\times10^8\,\frac{m}{s}}$$

$= 4.18\,m^{-1}$

b) $$\Gamma = \frac{\eta_2-\eta_1}{\eta_2+\eta_1} = \frac{\eta_0/\sqrt{\varepsilon_r}-\eta_0}{\eta_0/\sqrt{\varepsilon_r}+\eta_0}$$

$$= \frac{-\sqrt{\varepsilon_r}+1}{\sqrt{\varepsilon_r}+1}$$

$$= -\frac{1}{3}$$

$$\tau = \frac{2\eta_2}{\eta_1+\eta_2} = \frac{2\,\eta_2/\eta_1}{\eta_2/\eta_1+1} = \frac{2/\sqrt{\varepsilon_r}}{1/\sqrt{\varepsilon_r}+1} = \frac{2}{1+\sqrt{\varepsilon_r}} = \frac{2}{3}$$

c) Transmitted field

$\tilde{\vec{E}}^T(z) = 6.67\frac{V}{m}(\hat{x}+j\hat{y})e^{-jk_t z}$ $\quad k_t = \frac{2\pi f}{c/\sqrt{\varepsilon_r}} = \sqrt{\varepsilon_r}\,k = 8.38\,m^{-1}$

Reflected field

$\tilde{\vec{E}}^R(z) = 1.67\frac{V}{m}(\hat{x}+j\hat{y})e^{jkz}$

Total field for $z \le 0$

$$\tilde{\vec{E}}^{tot}(z) = \tilde{\vec{E}}^i(z) + \tilde{\vec{E}}^R(z)$$

$$= \hat{x}\left(5e^{-jkz}+1.67e^{jkz}\right) + j\hat{y}\left(5e^{-jkz}+1.67e^{jkz}\right)$$

d) Incident power

$$P^i = \frac{|\tilde{E}^i|^2}{2\eta_1}$$

Reflected power

$$P^r = \frac{|\tilde{E}^r|^2}{2\eta_1}$$

transmitted power

$$P^t = \frac{|\tilde{E}^t|^2}{2\eta_2}$$

$$\frac{P^r}{P^i} = \frac{|\tilde{E}^R|^2}{|\tilde{E}^i|^2} = |\Gamma|^2 = \frac{1}{9}$$

$$\frac{P^t}{P^i} = \frac{|\tilde{E}^t|^2}{|\tilde{E}^i|^2}\frac{\eta_1}{\eta_2} = |\tau|^2 \cdot \sqrt{\varepsilon_r} = \frac{8}{9}$$

2 8.9

Use $\lambda/4$ transformer:

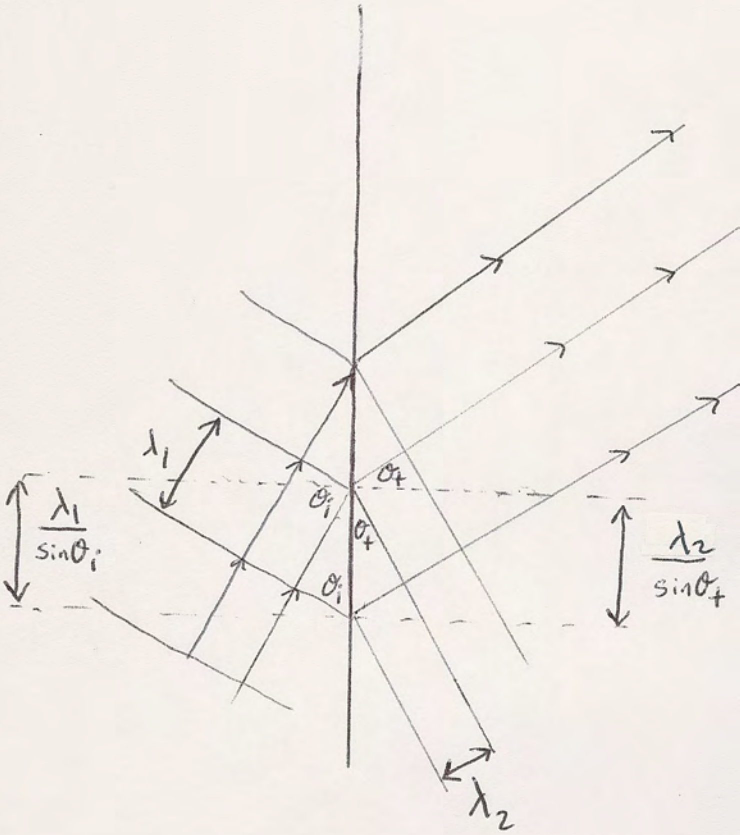
$$d = \frac{\lambda_2}{4} = \frac{c/\sqrt{\varepsilon_{r2}}}{4f}$$

and $\eta_2 = \sqrt{\eta_1 \eta_3} \implies \sqrt{\frac{\mu_0}{\varepsilon_2}} = \sqrt{\sqrt{\frac{\mu_0}{\varepsilon_1}}\sqrt{\frac{\mu_0}{\varepsilon_3}}}$

or $\varepsilon_{r2} = \sqrt{\varepsilon_{r1}\varepsilon_{r3}}$

so $d = \dfrac{c}{4f\,(\varepsilon_{r1}\varepsilon_{r3})^{1/4}}$

③



The distance between successive wavefronts, as drawn, is simply the wavelength of the medium.

With the angles drawn as shown, matching the incident and transmitted waves at the boundary requires

$$\frac{\sin\theta_i}{\lambda_1} = \frac{\sin\theta_t}{\lambda_2}$$

but $\lambda_1 = \frac{c_1}{f} = \frac{c_0}{n_1 f}$ where $c_0 = 3\times10^8\ m/s$

= vacuum speed of light

and $\lambda_2 = \frac{c_2}{f} = \frac{c_0}{n_2 f}$

Hence, $n_1 \sin\theta_i \frac{f}{c_0} = n_2 \sin\theta_t \frac{f}{c_0}$

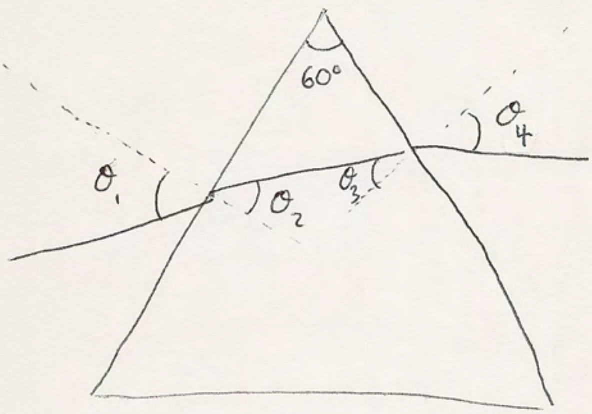
$$\boxed{n_1 \sin\theta_i = n_2 \sin\theta_t}$$

A similar analysis of the reflected wave yields $\theta_i = \theta_r$

(4) 8.18

$$\sin\theta_1 = n\sin\theta_2$$

$$\sin\theta_4 = n\sin\theta_3$$



$$60° + 90° - \theta_2 + 90° - \theta_3 = 180°$$

$$\theta_3 = 60° - \theta_2 = \frac{\pi}{3} - \theta_2$$

so $\theta_4 = \sin^{-1}(n\sin\theta_3)$

$$= \sin^{-1}\left(n\sin\left(\frac{\pi}{3} - \theta_2\right)\right)$$

$$= \sin^{-1}\left(n\left[\frac{\sqrt{3}}{2}\cos\theta_2 - \frac{1}{2}\sin\theta_2\right]\right) \qquad \cos\theta_2 = \sqrt{1-\sin^2\theta_2}$$

$$= \sin^{-1}\left(n\left[\frac{\sqrt{3}}{2}\sqrt{1-\frac{\sin^2\theta_1}{n^2}} - \frac{1}{2}\frac{\sin\theta_1}{n}\right]\right)$$

$$= \sin^{-1}\left(\frac{\sqrt{3}}{2}\sqrt{n^2-\sin^2\theta_1} - \frac{1}{2}\sin\theta_1\right)$$

$$n_{red} = 1.71 - \frac{4}{30}(.7\mu m) = 1.617$$

$$\theta_{4red} = 58.2°$$

$$n_{violet} = 1.71 - \frac{4}{30}(.4\mu m) = 1.66$$

$$\theta_{4violet} = 62.8°$$

Angular Dispersion $\Delta\theta = 62.7° - 58.2°$

$$= 4.56°$$

(5) 8.30

$$\Gamma_\perp = \frac{\eta_2 \cos\theta_i - \eta_1 \cos\theta_t}{\eta_2 \cos\theta_i + \eta_1 \cos\theta_t} = -.55 \qquad \theta_i = 70^\circ$$

$$\theta_t = \sin^{-1}\left(\frac{\sin 70^\circ}{1.5}\right) =$$

$$\Gamma_{\parallel} = \frac{\eta_2 \cos\theta_t - \eta_1 \cos\theta_i}{\eta_2 \cos\theta_t + \eta_1 \cos\theta_i} = .21 \qquad \frac{\eta_1}{\eta_2} = \sqrt{\varepsilon_r} = n = 1.5$$

Power reflected

$$\frac{P^R_{\parallel}}{P^i_{\parallel}} = |\Gamma_{\parallel}|^2 = .04 \qquad \frac{P^R_{\perp}}{P^i_{\perp}} = |\Gamma_\perp|^2 = .30$$

Total reflected power: $\frac{1}{2}\left(|\Gamma_{\parallel}|^2 + |\Gamma_\perp|^2\right) = 0.171 \rightarrow \boxed{17.1\%}$

(6) $\theta_i = \theta_B = \tan^{-1}(\sqrt{\varepsilon_r}) = \tan^{-1}(3) = 71.6^\circ$

$$\theta_t = \sin^{-1}\left(\frac{\sin\theta_i}{n}\right) \qquad n = \sqrt{\varepsilon_r} = 3$$

$$= \sin^{-1}\left(\frac{\sin(71.6^\circ)}{3}\right)$$

$$= 18.4^\circ$$