ECE 107 Homework 9

(1) 8.38

$$\frac{\partial \tilde{E}_z}{\partial y} + j\beta \tilde{E}_y = -j\omega\mu \tilde{H}_x$$

$$= -j\omega\mu \left[j\omega\varepsilon \tilde{E}_y + \frac{\partial \tilde{H}_z}{\partial x} \right] \cdot \frac{1}{j\beta}$$

$$= -\frac{\omega}{\beta}\mu \left[j\omega\varepsilon \tilde{E}_y + \frac{\partial \tilde{H}_z}{\partial x} \right]$$

$$\tilde{E}_y \left[j\beta - j\frac{\omega^2}{\beta}\mu\varepsilon \right] = -\frac{\omega}{\beta}\mu \frac{\partial \tilde{H}_z}{\partial x} + \frac{\partial \tilde{E}_z}{\partial y}$$

$$\frac{j}{\beta}\underbrace{\left(\beta^2 - \frac{\omega^2}{c^2}\right)}_{k_c^2} \tilde{E}_y = -\frac{\omega}{\beta}\mu \frac{\partial \tilde{H}_z}{\partial x} + \frac{\partial \tilde{E}_z}{\partial y}$$

$$\tilde{E}_y = \frac{j}{k_c^2}\left(\omega\mu \frac{\partial \tilde{H}_z}{\partial x} - \beta \frac{\partial \tilde{E}_z}{\partial y} \right)$$

(2) 8.40

(a) $\frac{m\pi}{a} = 40\pi \Rightarrow m = 40a = 40 \cdot .05\,m$

$$\boxed{m = 2}$$

$\frac{n\pi}{b} = 100\pi \Rightarrow n = 100b = 100 \cdot .03_m$

$$\boxed{n = 3}$$

(b) $\beta = 52.9\pi\ m^{-1}$ $\omega = 2.4\pi \times 10^{10}\ s^{-1}$

$$\beta^2 = \frac{\omega^2}{c_0^2}\varepsilon_r - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2 \quad \text{where } c_0 = 3\times10^8\ m/s$$

$$\varepsilon_r = \left(\frac{c_0}{\omega}\right)^2 \left[\beta^2 + \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 \right] = 2.25$$

(c) cutoff frequency

$$f_{mn} = f_{23} = \frac{c_0}{2\sqrt{\varepsilon_r}}\sqrt{\left(\frac{m}{a}\right)^2+\left(\frac{n}{b}\right)^2} = 10.77\,GHz$$

(d) $$36\tfrac{V}{m} = \frac{\omega\mu}{k_c^2}\,\frac{n\pi}{b}\,H_0$$

$$\frac{H_0}{k_c^2}\,\frac{n\pi}{b} = \frac{36\frac{V}{m}}{\omega\mu}$$

$$H_y = \left(\frac{H_0}{k_c^2}\,\frac{n\pi}{b}\right)\beta\cos\left(\frac{m\pi x}{a}\right)\sin\left(\frac{n\pi y}{b}\right)\sin(\omega t-\beta z)$$

$$= 0.063\tfrac{A}{m}\cos(40\pi x)\sin(100\pi y)\sin(2.4\pi\times10^{10}t-5.29\pi z)$$

(3) 9.1

$$S_{max} = \frac{15\pi I_0^2}{R^2}\left(\frac{\ell}{\lambda}\right)^2$$

$$= \frac{15\pi\,(20A)^2}{(10^3\,m)^2}\left(\frac{1}{50}\right)^2$$

$$\boxed{= 7.54\times10^{-6}\,\tfrac{W}{m^2}}$$

(4) 9.2

$$\lambda = \frac{c}{f} = \frac{3\times10^8\,m/s}{10^6\,Hz} = 300m$$

$$S = S_{max}\sin^2\theta = \frac{15\pi\,(12A)^2}{(5000m)^2}\left(\frac{1m}{300m}\right)^2\left(\frac{1}{\sqrt{2}}\right)^2$$

$$= 1.51\times10^{-9}\,\tfrac{W}{m^2}$$

(5) 9.5 (a) $R_{rad} = 80\pi^2\left(\frac{\ell}{\lambda}\right)^2 = 80\pi^2\left(\frac{2}{300}\right)^2 = 0.351\Omega$

$$R_{loss} = \frac{\ell}{2\pi a}\sqrt{\frac{\pi f \mu_c}{\sigma_c}} = \frac{2m}{2\pi \cdot 10^{-3}m}\sqrt{\frac{\pi \cdot 10^6 Hz \cdot 4\pi\times10^{-7}\frac{H}{m}}{5.8\times10^7 S/m}} = 0.083\Omega$$

$$\xi = \frac{R_{rad}}{R_{rad}+R_{loss}} = 0.297$$

(b) $G = \xi D = 0.45$ $\quad D = 1.5$

$G(dB) = 10\log_{10} G = -3.5dB$

(c) $P_{rad} = \frac{4\pi R^2}{D}\cdot\frac{15\pi I_0^2}{R^2}\left(\frac{\ell}{\lambda}\right)^2 = 80W$

$$80W = \frac{60\pi^2}{1.5}\left(\frac{2}{300}\right)^2 I_0^2$$

$I_0 = 67.5A$

$$P_{generator} = \frac{P_{rad}}{\xi} = \frac{80W}{.297} = 269W$$

(6) 9.8

$D(dB) = 10\log_{10} D = 7dB$

$D = 10^{0.7} = 5.01$

$G = \xi D = .9\cdot 10^{.7} = 4.51$

$G(dB) = 10\log_{10}(G) = 6.54dB$

(7) 9.9

$$3^\circ \cdot \frac{\pi}{180^\circ} = \frac{\pi}{60} \text{ radians}$$

$$D = \frac{4\pi}{(\pi/60)^2}$$

$$= \frac{4\pi \cdot 3600}{\pi^2}$$

$$= 4584$$

$$D(dB) = 10 \log_{10}(D) = 36.6\,dB$$