Experimental Validation of Performance Limits and Design Guidelines for Small Antennas

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***Abstract*—The theoretical limit for small antenna performance that was derived decades ago by Wheeler and Chu governs design tradeoffs for size, bandwidth, and efficiency. Theoretical guidelines have also been derived for other details of small antenna design such as permittivity, aspect ratio, and even the nature of the internal structure of the antenna. In this paper, we extract and analyze experimental performance data from a large body of published designs to establish several facts that have not previously been demonstrated: (1) The theoretical performance limit for size, bandwidth, and efficiency are validated by all available experimental evidence. (2) Although derived for electrically small antennas, the same theoretical limit is also generally a good design rule for antennas that are not electrically small. (3) The theoretical predictions for the performance due to design factors such as permittivity, aspect ratio, and the internal structure of the antenna are also supported by the experimental evidence. The designs that have the highest performance are those that involve the lowest permittivity, have an aspect ratio close to unity, and for which the fields fill the minimum size enclosing sphere with the greatest uniformity. This work thus validates the established theoretical design guidelines.**

***Index Terms*—Small antenna, planar antenna, slot antenna, fractal, metamaterial, dielectric resonator antenna, bandwidth, quality factor, efficiency.**

I. INTRODUCTION

SMALL antennas have been an important topic of research for many decades, and interest in the field is increasing with the development of new systems that require broadband antennas with a small form factor. The analysis of small antennas is generally considered to have begun with the work of Wheeler [1] and Chu, [2] who established the theoretical limits that show how electrical size and bandwidth are related. Since this early work, numerous authors have revisited these theories, and have suggested further refinements. Although slightly more accurate, all of these new theories share the same basic conclusions established in the 1940s – that size can only be reduced at the expense of bandwidth or efficiency. Furthermore, the early papers as well as others that followed have provided theoretical guidelines for other aspects of small antenna design. In general, the best performance will be achieved if the dielectric constant is as low as possible, if the aspect ratio is close to unity, and if the internal structure of the antenna is such that the fields fill the minimum size enclosing sphere with the greatest possible uniformity.

Along with the work that has been done to develop theoretical limits, a large amount of effort has been put into developing specific antenna designs in an attempt to optimize the relationship between size and bandwidth. In the 64 years since Wheeler's first paper, thousands of new antenna designs have been published, and each year we continue to see new publications exploring every conceivable arrangement of metal shapes and dielectric regions. However, many of these designs have sub-optimum performance, and could have been predicted to perform poorly if the theoretical design guidelines were more clearly understood from the start. In addition, many antenna designs are proposed for which performance is overestimated, such as by ignoring losses or incorrectly calculating the true electrical size of the antenna. This challenges not only antenna engineers, who must address these unphysical performance claims, but also system engineers, who end up relying on performance metrics that are ultimately unachievable. These issues may be caused in part because the theoretical design guidelines are not widely understood, and in fact have never been rigorously validated experimentally.

Unfortunately, it is impossible to experimentally "prove" a physical theory - it can only be disproven by contradictory experimental evidence. Nonetheless, a theory that has been tested extensively and found to be true in all tests is generally accepted as correct, at least until contradictory evidence is found. In the field of small antennas, there have been many attempts to optimize antenna designs to get as close as possible to the theoretical limits. However, each of these antennas represents a local optimization, and in each case it is possible that the authors have simply not chosen the best design, and perhaps another one may be found that could exceed the theoretical limit. The purpose of this paper is to systematically extract experimental results from a sufficiently large sample of existing designs to demonstrate that the Wheeler-Chu limit is valid and correct across a broad range of

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electrical sizes and bandwidths. We furthermore show that the design guidelines that have been established for permittivity, aspect ratio, and the internal structure of the antenna are also supported by the experimental evidence.

This paper establishes several important facts: (1) When the size and efficiency are correctly calculated, the measured bandwidth of an electrically small antenna does not exceed the theoretical limit, regardless of the design. (2) The theoretical limit for small antennas is also a good design guideline even for antennas that are not electrically small. (3) The experimental evidence supports the theoretical predictions that performance of a small antenna is maximized with low permittivity, low aspect ratio antennas, in which the fields fill the smallest enclosing sphere as uniformly as possible. For electrically small antennas, a class of wire cage designs appears to have a performance advantage compared to other types, while some of the new and popular concepts, such as fractals and metamaterials, do not appear to provide a performance advantage compared to conventional designs.

There have been other studies comparing various specific antenna types to the theoretical limit, such as Best and Hanna's recent paper in which they compared several different designs, [3] and Best's paper involving specifically planar designs. [4] However, to date there has not yet been a study which has systematically examined the body of experimental data to validate the theoretical limits over a wide variety of antenna types. Thus, the value of this paper is to demonstrate that the Wheeler-Chu limit has been extensively tested using 64 years of small antenna performance data, and has been found to be valid in all cases. The results shown here are also consistent with the design guidelines established decades ago. [1] It is expected that this will provide useful guidance for future small antenna designers.

II. BACKGROUND ON SMALL ANTENNA THEORY

In this section we give a brief overview of the theoretical analysis of small antennas and the results that are relevant to this study. For a more detailed examination of these theories, see for example the first chapter in either of the books by Hansen [5] or Volakis. [6]

The first author to establish the link between antenna, bandwidth, and efficiency was Wheeler. [1] He studied two simple small antennas, a cylindrical parallel plate capacitor and a cylindrical coil inductor. He calculated the radiation power factor for the capacitive antenna as

$$p = \frac{G}{\omega C}, \tag{1}$$

and for the inductive antenna as

$$p = \frac{R}{\omega L}, \tag{2}$$

where C or L is the capacitance or inductance, and G or R is the radiation shunt conductance or series resistance. He showed that the maximum power factor for a cylindrical antenna of either type, with circular area A and height b, is

$$p = \frac{1}{6\pi} k^3 f_s A b, \tag{3}$$

where $k=2\pi/\lambda$, and f_s denotes a shape factor that multiplies the area A to obtain the effective area, as augmented by the field outside the cylindrical volume. The shape factor approaches unity for thin, flat capacitive antennas or long, thin inductors, although it can be much larger for other shapes. For more details on the use of the shape factor, please refer to the original work. [1] Note that we have changed some variable names in order to be consistent throughout this paper.

Wheeler also introduced an ideal spherical wire coil antenna [7, 8] that has a power factor of

$$p = \frac{(ka)^3}{1 + 2/\mu_r}. \tag{4}$$

where a is the minimum radius of a sphere enclosing the antenna. Note that if the sphere is filled with a material having infinite permeability, as Wheeler explains, [9] expelling the avoidable stored energy from inside the antenna, then p can be increased by up to a factor of 3 compared to the air filled case. Its maximum value is fundamentally limited by the unavoidable stored energy outside the antenna, to

$$p = (ka)^3. \tag{5}$$

Wheeler also illustrated the relationship between power factor and bandwidth [9] which depends on the matching circuit and the allowable reflection coefficient, as described by Fano. [10] We can recognize Wheeler's definitions for the power factor in Eqs. 1 & 2, as the inverse of the quality factor Q of an RC or RL circuit. By inverting Eq. 5 we find a good approximation to the expressions for the minimum Q that are derived by other authors using more rigorous methods.

In addition to this limitation on bandwidth, Wheeler also identified in his initial paper on this subject [1] several important guidelines for small antenna design which are relevant to the present study: (1) that the addition of an electrically large ground plane can potentially double p for a given volume, (2) that increasing the permittivity ε_r inside antenna decreases p roughly in proportion to ε_r, and (3) that increasing the permeability μ_r can increase p by up to a factor of 3. (4) He also later explained [9] that for non-spherical antennas, the power factor is reduced because the spherical volume is only partially utilized.

Chu was the first to derive the minimum quality factor Q of a small antenna based on an expansion of the field in terms of spherical modes. [2] Unfortunately, he does not explicitly state a formula relating Q and size, thus requiring some further work by the reader to apply his results to antenna design. However, he does provide a plot showing a small dipole with an ideal matching circuit as described by Fano [10] that has a bandwidth approximately proportional to $(ka)^3$.

Hansen [11] and later McLean [12] followed Chu's analysis to derive an expression for the Q of the lowest order mode in terms of the antenna's electrical size.

$$Q \approx \frac{1+2(ka)^2}{(ka)^3\left[1+(ka)^2\right]} \tag{6}$$

Hansen also showed that loss can be represented as an additional resistance in series with the radiation resistance, so Q can be reduced at the expense of efficiency. Therefore a more useful quantity for comparing antenna performance is the quality factor divided by efficiency, Q/η.

Collin and Rothschild [13] approached the problem by subtracting the energy associated with radiation from the total energy to find simple expressions for the Q of each mode. They give the value for the lowest order spherical mode as

$$Q = \frac{1}{ka} + \frac{1}{(ka)^3}. \tag{7}$$

McLean [12] derived the propagating and non-propagating fields, and calculated Q from the ratio of these terms, arriving at the same result as Collin, above. He also found that the Q for circularly polarized antennas involving both TM and TE modes together is

$$Q = \frac{1}{ka} + \frac{1}{2(ka)^3} \tag{8}$$

Although this result is often associated with circularly polarized antennas, Pozar [14] clarified that this formula for Q is simply a result of using two modes, and is not specifically a function of the polarization of the antenna.

Other published papers have provided various other expressions for the radiation Q, including Fante, [15] Geyi, [16] Hansen and Collin, [17] Thal, [18] and Vandenbosch. [19] However, Eqs. 7 & 8 above are generally accepted today as correct. Furthermore, the other variations that have been explored generally deviate from the expressions above by only a small amount.

There has also been work to simultaneously optimize gain and Q, such as the preliminary work by Fante [20] which includes numerical results for maximum G/Q. This work was later contested by Thal. [18] Geyi [21] provides an analytical result, and finds that it is possible to simultaneously minimize Q to a value given in Eq. 8, while maximizing G/Q, giving a maximum gain of 1.5 for an omnidirectional antenna, or 3 for a directive antenna.

Although most studies have focused on Q, the quantity that is of most interest to antenna engineers is frequency bandwidth, B. Among others, Geyi [22] addressed this issue, concluding that B and 1/Q are equivalent for antennas with Q>>1, however, this claim has been disputed recently by Best. [23] In any case, when attempting to match a given load impedance there is a tradeoff between bandwidth and acceptable reflection coefficient. [10] Yaghjian and Best [24] derived the relationship between B and Q through the maximum allowable voltage standing wave ratio VSWR, or s,

$$B \approx \frac{1}{Q}\left(\frac{s-1}{\sqrt{s}}\right). \tag{9}$$

In most cases we are concerned with matching the first one or two modes, however Villalobos [25] has also derived limits for matching higher order modes.

Additional studies have focused on the limitations for specific types of antennas. Examples that are relevant to this study are as follows. Sten [26] studied antennas near a conducting plane and determined that the proper measure of antenna size is a sphere that encloses both the antenna and its image currents. Ida [27] studied dielectric loaded monopole antennas, showing that the efficiency-bandwidth product is reduced for large values of permittivity. Thal studied spherical wire antennas, [28] and loop antennas, [29] and concluded that the Q values are at least 3 times the theoretical limits for TE mode antennas, or at least 1.5 times the theoretical limit for TM mode antennas. Gustafsson [30] examined various shapes and determined the theoretical limits on Q, showing that it increases for any shape that deviates significantly from a sphere, and that the maximum performance appears at an aspect ratio falling in the range between 1 and 2. Ghorbani [31] studied microstrip antennas and concluded that although the addition of resonant structures within the microstrip pattern are usually added to increase bandwidth, they actually reduce the maximum bandwidth that could be achieved with an ideal matching circuit. This is consistent with the guidance given by Wheeler, [9] because the fields associated with these resonant structures are confined to a subset of the overall antenna volume.

Finally, Stuart et. al. [32] studied multi-resonant antennas. The authors showed through example that although the Q as defined by the energy stored and power radiated at a particular frequency, assuming no other losses,

$$Q = \omega \frac{W_{stored}}{P_{radiated}} \tag{10}$$

does not deviate from the fundamental limit, the Q implied by the impedance of the antenna,

$$Q_z = \frac{\omega_0}{2R_0}\left|Z_0'(\omega_0)\right| \tag{11}$$

can be significantly different if the antenna has two closely spaced resonances. Furthermore, they found that neither quantity is a good predictor of the half-power VSWR bandwidth for multiresonant antennas. Although the Q of an electrically small antenna can never be lower than the limits of equations 7 & 8, the bandwidth given by equation 9 is an approximation which assumes a matched antenna, and it is most accurate when the bandwidth is defined in terms of a low VSWR. Furthermore, as Fano shows, [10] maximizing the reflection coefficient toward a given tolerance increases the available bandwidth within that tolerance. Thus, by designing the antenna to meet a sufficiently high VSWR tolerance over the band of interest, it is possible to exceed the bandwidth predicted by Eq. 9. In summary, the equations for minimum Q are always correct, but the equation for bandwidth based on a given Q can be exceeded by making the antenna or matching circuit multi-resonant, and by maximizing the reflection

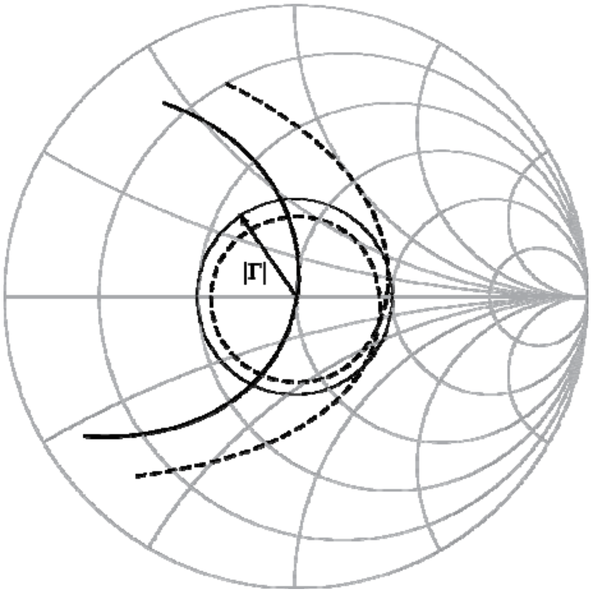

Fig. 1. Example Smith chart plot for a single-resonant, matched antenna (solid line) consistent with Eq. 9 compared to a multi-resonant, unmatched antenna (dashed line). These curves illustrate that for a given reflection coefficient tolerance |Γ|, designing an antenna with multiple resonances and avoiding a perfect match can improve bandwidth, within the Bode limit.

coefficient within that band. This is illustrated in Fig. 1.

This last discussion would seem to suggest that the fundamental limits on Q would have minimal utility for antenna design, because the quantity that system designers care about is bandwidth, not Q. However, multiple resonant modes in a single antenna must be orthogonal, either in polarization or space. For electrically small antennas, the two lowest order modes can be orthogonal in polarization, and can be designed to be close in frequency. This can indeed reduce Q and improve bandwidth, as illustrated by the fact that Eq. 8 for the case of two modes provides a Q that is one-half that of of Eq. 7 for ka<<1. However, higher spatial modes will generally occur at higher frequencies. If the structure is loaded with reactance elements to lower the frequency of the higher spatial modes, they will occupy a subset of the total antenna volume, thus further raising their Q. Finally, even if multiple tuned circuits are included in the matching network, the maximum bandwidth is ultimately governed by the Bode limit. [33]

$$B = \frac{1}{Q}\frac{\pi}{\ln\left(1/\left|\Gamma\right|\right)} \qquad (12)$$

Lopez [34, 35] and Hansen [36, 37] have explained the potential bandwidth improvement for various numbers of tuned circuits. For example, for a VSWR of 2, one additional tuned circuit can improve the bandwidth by a factor of 2.3 and an infinite number of tuned circuits would provide a theoretical bandwidth improvement of 3.8. However, in practice most of the benefit is obtained with one or two tuned circuits, and an excessively complicated matching circuit would contain substantial losses.

III. ANALYSIS PROCEDURE

To compare various electrically small antenna designs to the theoretical performance limits described above, we extracted experimental data from the published literature. A search for “small” and “antenna” using the online search engine IEEE Xplore at the end of 2010 yields 7484 papers, which is far too many to analyze. We limited our search to only publications in *IEEE Transactions on Antennas and Propagation*. This is based on the rationale that this is the premier journal for antennas, so any antenna design that has lasting impact would eventually be published here in some form. We also included the *IRE Transactions*, the predecessor to this journal, although there were no papers published in *IRE Transactions* that met the search criteria and that included sufficient data to quantify the antenna performance. This limit resulted in 763 papers, which is a reasonable number. Our approach obviously cannot find every possible small antenna design, because there are many that only appear in conference papers or other journals. However, we needed to use consistent criteria for inclusion of papers, and this sample size is sufficient for us to draw meaningful conclusions

Upon examining each of the 763 papers, we determined that many could be eliminated based on title alone. Papers that discussed small reflectors, small arrays, and other topics not related to electrically small antennas were excluded. Papers that focused on antennas embedded in materials other than free space, such as water, the human body, or other lossy media, were also excluded. Furthermore, papers that described ultrawideband (UWB) antennas were excluded, because designs aimed at multi-octave bandwidth generally are not electrically small, and usually involve different design approaches than electrically small antennas. For a similar reason, antennas focused on high frequency bands, such as millimeter wave antennas, or antennas integrated on a semiconductor substrate were excluded because those papers are generally focused on goals other than optimizing size or bandwidth. Nonetheless, published antennas that passed through our manual filter but were found to not be electrically small were still left in the data set because they provide some insight into the range of applicability of the fundamental limits as design guidelines.

In addition to the criteria described above, multiband antennas were also excluded because they are typically designed to optimize the number and spacing of bands, rather than the width of a single band. Diversity antennas were excluded for a similar reason. Tunable antennas were excluded unless we could identify one tuning point as representative of that design. In general, if a paper did not include sufficient information to evaluate the antenna, such as frequency, bandwidth, size, efficiency, gain, or plots from which this data can be extracted, then it was not included. Although active matching techniques such as non-Foster circuits can potentially exceed the limits discussed here, we considered only passive structures. Finally, since our goal was to validate the theoretical limit, we only included papers with measured data.

By manually filtering the papers as described above, we obtained 112 published antenna designs that contained sufficient experimental data for us to analyze. For each paper, we recorded the center frequency and fractional bandwidth, or extracted these from the frequencies of the band edges, or from plots of S11. For papers that included efficiency data, we used the values provided by the authors. For those that did not, we used radiation patterns and gain when these were available. We estimated the directivity using

$$D = \frac{4\pi}{\theta_x \theta_y} \tag{13}$$

where θ_x and θ_y are the 3dB beamwidths in radians in the two orthogonal planes. Although this approximation is most correct for directive antennas, it is still sufficient for our needs here, where we aim to keep the errors to within a few tens of percent or less. We then estimated the efficiency using the quoted gain and calculated directivity

$$\eta = \frac{G}{D}. \tag{14}$$

We used this approach for 19 of the papers. For published articles that provided neither efficiency nor a radiation pattern, but still quoted gain, we estimated the efficiency by using

$$\eta = \frac{G_{measured}}{G_{ideal}} \tag{15}$$

where the ideal gain depended on the type of antenna. Dipole-like antennas without a ground plane were assigned an ideal gain of 1.5, monopole-like antennas on a large ground plane that were vertically polarized with a null toward zenith were given an ideal gain of 3, and patch-like antennas on a large ground plane where there is one central lobe that rolls off rapidly toward the horizon were given an ideal gain of 6. We used this approach for 11 of the papers.

For three of the papers involving moderately high Q designs on lossy dielectrics, the efficiency was approximated using the measured antenna Q and the loss tangent of the dielectric, Tan(δ)

$$\eta = \frac{R_{rad}}{R_{rad} + R_{loss}} \approx \frac{1}{1 + Q \cdot Tan(\delta)}. \tag{16}$$

For these cases we calculated an implied Q from the bandwidth using Eq. 9. This approach was only used when efficiency could not be estimated using any other means.

For 39 of the papers, the efficiency was either quoted as nearly 100% by the author, or was assumed to be nearly 100% based on the design. That assumption was only applied if there was no other data from which to extract efficiency, and when such an assumption was considered reasonable, such as for low-Q antennas built using entirely low loss metals and dielectrics. While these methods are approximate, we expect that they will be accurate to within a few tens of percent or less, and errors of this magnitude will not have a significant effect on the overall conclusions of this paper.

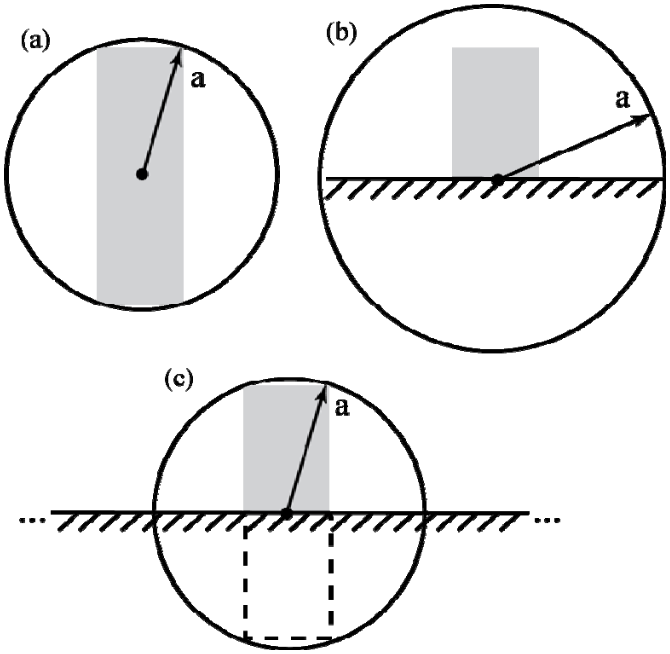

Fig. 2. The method for determining the radius a of the smallest enclosing sphere. (a) For an antenna with no ground plane, it is the smallest sphere to enclose the entire antenna. (b) For antennas on a small ground plane with less than λ/4 radius, or closer than λ/4 from an edge, the sphere encloses the entire ground plane. (c) For antennas on an electrically large ground plane, the sphere encloses the antenna and its image currents.

The electrical size of the antenna was calculated as the radius of the smallest sphere which encloses the entire antenna, as shown in Fig. 2. For antennas that do not include a ground plane, this is straightforward. For those that include an electrically large ground plane, with a radius >λ/4 at the center of the operating band, then the sphere includes the antenna and the image currents, so the radius a equals the distance to the farthest point on the antenna from the bottom center. For antennas on an electrically small ground plane, or closer than λ/4 to the edge of the ground plane, the entire ground plane was included in the size.

The value for k was taken at the center of the operating band. A maximum bandwidth efficiency product, Bη, was calculated using Eq. 7 for linearly polarized antennas, or Eq. 8 for circularly polarized antennas, and applying Eq. 9. We standardized all designs to a VSWR of 2, which is consistent with the requirements of many applications, and the vast majority of published papers. These equations were also applied as appropriate when the polarization was not stated or was indeterminate, but where a judgment of whether it involved one or two modes could be made from the symmetry of the antenna and the feed.

In addition to recording size, bandwidth, and efficiency data, we also analyzed the effects of design type, permittivity, and aspect ratio, to compare to the design guidelines discussed above. For antennas containing multiple dielectric materials, the permittivity of the material filling most of the resonant portion of the antenna was used. The aspect ratio was taken as the ratio of the largest to smallest dimension of the outside boundary of the antenna. For antennas on electrically large

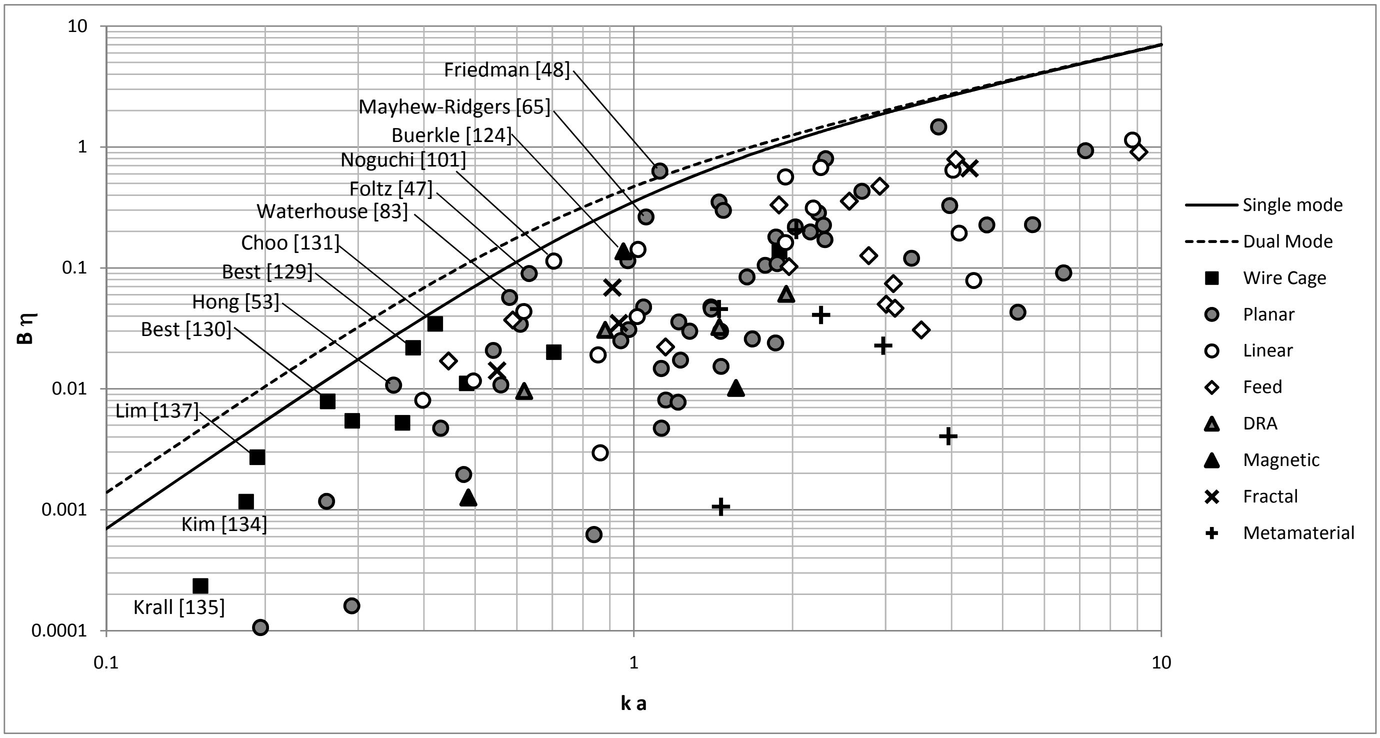

Fig. 3. The measured Bη product for 110 antenna designs published in the *IEEE Transactions on Antennas and Propagation* by the end of the year 2010. The theoretical limits are derived by applying Eq. 9 to Eqs. 7 & 8 using a VSWR of s=2. Specific references on the outer edge of the performance limit are noted.

ground planes, the dimension normal to the ground plane was doubled in calculating the aspect ratio, so as to include the effect of the image currents, as shown in Fig. 1(c).

We grouped the antennas into several categories depending on the style of design. Antennas such as patches, planar inverted F antennas (PIFAs) and other similar designs which included a ground plane and had a width greater than their height were designated as "Planar". [38-89] Dipoles and other such structures were designated as "Linear" [90-105] as well as designs that involve dipole-like modes on metal sheets, regardless of their aspect ratio. This category also included any vertically polarized antenna on an electrically large ground plane that had a height exceeding its width, and therefore behaved as a monopole. A sub-category of the linear type was a class that we called "Feed" antennas. [106-119] These were antennas on small ground planes that were typically shaped as a mobile phone or other such object, in which a small resonant structure actually serves as an exciter or feed for a mode which involves the entire ground plane. This is a category for which under-reporting the true antenna electrical size by neglecting the ground plane is common. Dielectric resonator antennas ("DRA") [120-123] also formed a separate category, as well as antennas that involved materials with relative permeability $\mu_r>1$, which were designated as "Magnetic". [124-126] One category that performed particularly well was called the "Wire Cage" [127-137] type. These antennas were generally complicated wire structures with roughly spherical shape, or having an aspect ratio close to 1. The final categories included designs which take advantage of popular trends in antenna design, and attribute their performance to "Fractal" [138-141] or "Metamaterial" [142-147] features. However, antennas that only included a single period or unit of such concepts, such as a solitary split ring resonator, were generally lumped into one of the other categories as appropriate. For papers that included multiple designs, we chose the best performing design, or the smallest. There were two papers where the measured results are so far above the theoretical limits that they were labeled as "Problematic" [148, 149] and not included in the data set. In both of those cases, the issues with the results can be traced back to problems in how the measurements were performed.

IV. RESULTS AND DISCUSSION

The centerpiece of this work is shown in Fig. 3. We plot the bandwidth efficiency product versus the electrical size for the 110 relevant small antenna papers published in the *IEEE Transactions on Antennas and Propagation*, and compare the measured results to the theoretical limits. The curves representing the theoretical limits are derived by applying Eq. 9 to Eqs. 7 & 8 using a VSWR of s=2, and including efficiency η to obtain

$$B\eta = \frac{1}{\sqrt{2}}\left(\frac{1}{ka}+\frac{1}{n(ka)^3}\right)^{-1} \tag{17}$$

where n=1 for linearly polarized or single-mode antennas, and n=2 for circularly polarized or dual-mode antennas.

Note that Eqs. 7 and 8 for the minimum Q are inviolable. However, direct data for Q is not readily available for most published antennas, so we are using measured bandwidth as a proxy for Q through Eq. 9. This is based on the assumption of a self-matched antenna without additional matching circuits,

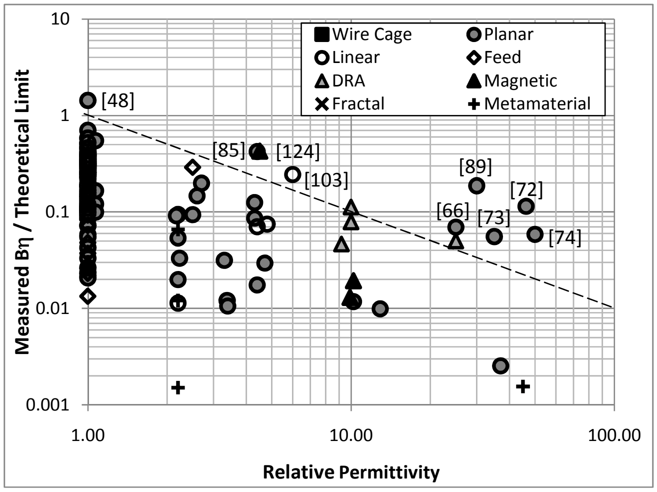

Fig. 4. The measured Bη product divided by the theoretical limit, compared to the relative permittivity of the material filling the antenna. The performance is reduced with increasing permittivity. The dashed line shows a trend of $1/\varepsilon_r$, an approximation from Wheeler's paper, for a shape factor of 1. Most of the designs that lie above the dashed line actually contained multiple materials, and we have recorded the highest permittivity value, thus overestimating the effective permittivity in these cases.

which is consistent with the vast majority of published antennas. It is possible to exceed this bandwidth-Q relationship of Eq. 9 by using additional matching circuits. However, it is never possible to exceed the Bode limit of Eq. 12 for a given Q. [33] Thus, although the Friedman antenna [48] stands out as exceeding the theoretical limits curves plotted in Fig. 3, it does not actually exceed the Wheeler-Chu limit. Furthermore, the potential benefits of each additional matching circuit are well established. [34-37] The double-tuned matching circuit of the Friedman antenna can be expected to provide a bandwidth improvement factor of up to 2.8, [34-37] and the measured Bη product of this antenna is well within this limit.

It should also be noted that the Friedman antenna is similar in concept to the widely cited Goubau antenna. [150] The two are identical in electrical size and bandwidth performance. The exceptional performance of the Goubau antenna is often credited to its multi-mode design. In fact, both antennas include complicated matching networks. While Friedman explicitly uses a separate lumped circuit at the feed, Goubau achieves the same result with inductors, slots and other features integrated directly into the antenna design.

There are several important observations to note in Fig. 3. (1) No electrically small antennas (ka<0.5) have been published in this journal that exceed the theoretical limit. (2) That limit also provides a good guideline for the maximum bandwidth even for antennas of moderate to large electrical size. (3) For electrically small antennas, wire cage designs appear to have a performance advantage compared to other types. (4) For moderate electrical size (ka~1) there are many standard planar or linear designs that can come close to the theoretical limit. (5) Dielectric resonator antennas do relatively poorly because they are based on high dielectric materials, as predicted by the design guidelines discussed above. (6) Magnetic antennas are expected to have a performance advantage of up to a factor of three compared to other types. However very few antennas based on magnetic materials appeared in our sample of papers. There is one design that does achieve good performance, so this may be a promising area for future research if low-loss magnetic materials can be realized at frequencies of interest. (7) Fractal and metamaterial designs, although popular in recent years, do not appear to provide any performance advantage compared to more conventional antenna types.

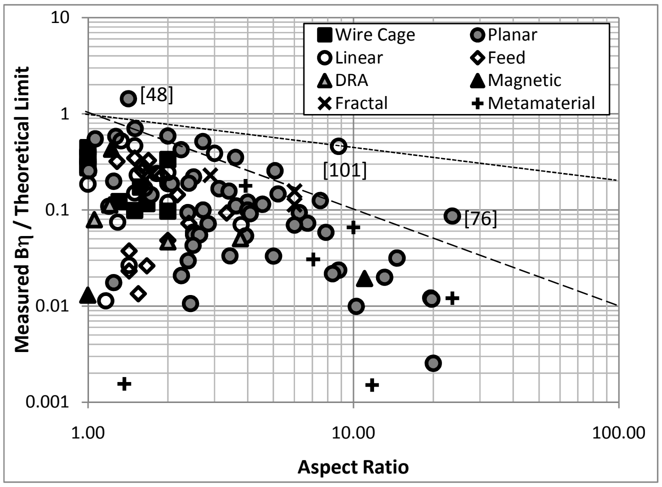

Fig. 5. The measured Bη product divided by the theoretical limit, compared to the aspect ratio of the antenna. The performance is reduced with increasing aspect ratio. The dashed line shows a good linear approximation to Gustaffson's curve for vertically polarized cylindrical antennas with diameter/height>1, roughly corresponding to some of the antennas designated here as the "Planar" type. The dotted curve is for height/diameter>1, corresponding approximately to our "Linear" type.

From these observations, we can draw several important conclusions about the relative merits of different antenna types, and the relationship between these performance variations and the established theoretical design guidelines. In general, the high-performance wire cage designs include low permittivity, low aspect ratio, and they have their fields evenly distributed throughout their volume or surface, so they are consistent with established design guidelines discussed above. Furthermore, the dielectric resonator designs involve high permittivity materials, so their poor bandwidth performance relative to their size is expected. Finally, the poor performance of metamaterial and fractal designs is consistent with the idea that the fields associated with the antenna should fill the smallest enclosing sphere as uniformly as possible. These designs typically involve highly resonant structures embedded within the antenna which tend to concentrate the fields to those regions, so they effectively use a subset of the available volume.

In addition to examining the relative performance of different antenna classes, we also specifically examined the effect of permittivity and shape. Fig. 4 shows the measured Bη product divided by the theoretical limit given by Eq. 17 for the ka value of each antenna as a function of permittivity. For antennas that include multiple materials, the permittivity corresponds to the material filling the main resonant structure of the antenna. Note that the maximum measured performance decreases with increasing dielectric constant over a wide range of antenna designs. Wheeler's original paper [1] instructed that for a capacitive antenna with a shape factor f_s, the power factor is reduced by

$$\frac{1}{1+\dfrac{\varepsilon_r-1}{f_s}}. \tag{18}$$

Thus, for a shape factor of unity, corresponding to a thin, flat capacitor, the performance is reduced approximately by the inverse of the permittivity. This performance roll-off is less severe for higher shape factors, so a $1/\varepsilon_r$ roll-off is not a strict rule, but it is still a good overall design guideline for many types of antennas. Also, from the spread of points in Fig. 4 it is obviously possible to achieve even worse performance, such as by inefficiently using the antenna volume. Although an estimate of the shape factor for each antenna would allow us to more directly compare the data with Wheeler's formula, we must remember that these antennas include a wide variety of internal structures, and many deviate significantly from an ideal small capacitor. Thus, the concept of the shape factor would be difficult to apply directly to this entire data set. It is worth noting that many of the dielectric resonator antennas lie very close to Wheeler's prediction. There are also several antennas that lie above the theoretical curve, and in most of these cases the discrepancy from the theory has to do with the choice of permittivity. These designs either contain multiple dielectric materials, [66, 72-74, 89] or the fields extend partially into air regions within or around the antenna. [85, 103] In these cases we have recorded the highest permittivity among several materials that make up the resonant portion of the antenna, and therefore have clearly overestimated the effective permittivity. Accurately determining the true effective permittivity would be difficult, so these data points are left as exceptions. There are two additional cases that either involve magnetic materials [124] which would be expected to have higher performance, or complex matching circuits [48] which have been discussed above. We can observe that as a general rule, the maximum measured performance is reduced with increasing permittivity, and that assuming a performance reduction of roughly $1/\varepsilon_r$ as a useful guideline for many designs, particularly if the effective permittivity can be accurately calculated.

The maximum performance also decreases with increasing aspect ratio, as shown in Fig. 5. For this plot, we have used the ratio of the largest to smallest exterior dimensions of the antenna, regardless of orientation, so our aspect ratio is always greater than one. For antennas that involved an electrically large ground plane, the image currents were also included in these dimensions. Gustaffson, [30] calculated the performance reduction versus aspect ratio for various ideal shapes. He showed that for most shapes the maximum performance is achieved with an aspect ratio between 1 and 2, and the performance diminished with aspect ratio at various rates depending on the antenna shape. Note that his definitions for antenna shapes do not correspond exactly to our categories. Nonetheless, some comparisons can still be made. In Fig. 5 we have added theoretical curves which are linear approximations to Gustaffson's curve for vertically antennas having a cylindrical shape. For cylindrical antennas with a diameter/height>1, this curve can be compared with some of the antennas in our "Planar" category. The theoretical performance decreases by approximately a factor of 10 for each decade increase in the diameter/height ratio. In other words, the performance is inversely proportional to the aspect ratio. Not all of these designs have a circular cross section, [76] so it is difficult to compare the aspect ratio directly with Gustaffson's ideal curves. A separate line is included for cylindrical antennas with height/diameter>1, which describes some of the antennas in our "Linear" category. A well-designed example is Noguchi's [101] dual mode helix antenna. Although a perfect match to ideal antenna types is not to be expected for so many different designs, the trends in measured antenna performance are still generally consistent with the theoretical predictions. For many designs, and particularly for planar type antennas, it is a useful guideline to assume that performance may be reduced at least in proportion to the inverse of the aspect ratio.

V. CONCLUSION

We have demonstrated that the Wheeler-Chu limit for electrically small antenna performance is supported by experimental evidence for all papers containing measured data that have been published in the *IEEE Transactions on Antennas and Propagation*. We argue that this is a sufficiently large data set to validate the theoretical limit. We have also shown that the limit serves as a good design guideline even for antennas that are not electrically small. We have further shown that the design guidelines for performance reduction with permittivity and aspect ratio agree with the theoretical predictions, to the extent that such a comparison can be made for a wide variety of antenna types. The expected performance for most antennas degrades approximately with the inverse of permittivity, and for planar type antennas it degrades approximately with the inverse of aspect ratio. Finally, we have shown that antenna types in which the resonant structure is restricted to a subset of the overall antenna volume perform poorly relative to those in which the fields are evenly distributed within the minimum size enclosing sphere. All of these conclusions are consistent with the existing theoretical design guidelines.

In general, to design an electrically small antenna with the largest possible bandwidth efficiency product, the antenna should have a low permittivity, an aspect ratio close to unity, and should have the stored fields distributed evenly throughout its volume. For this reason, in the electrically small regime, wire cage designs appear to have an advantage compared to other types. However, at larger scales there are many other linear or planar designs that perform well compared to the theoretical limits.

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