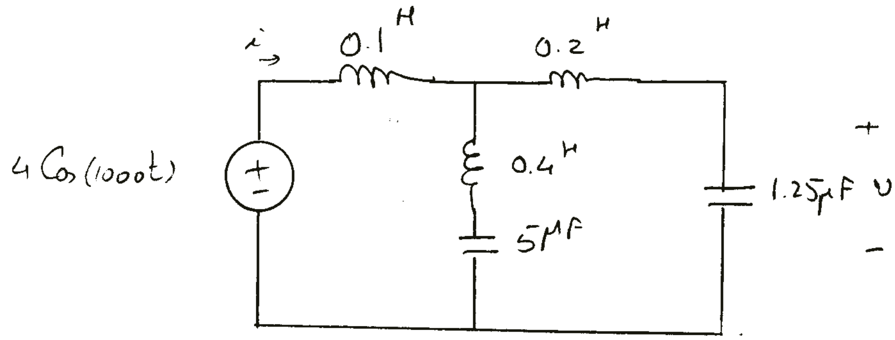
Example, Find i & v



$\omega = 1000$

$4\cos(1000t) \rightarrow 4$

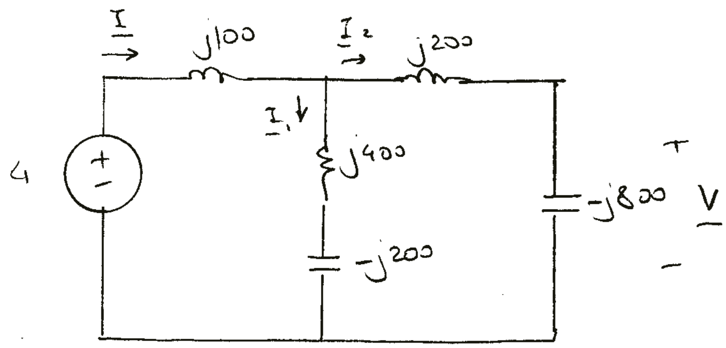
$0.1\ H \rightarrow j\omega L = j(1000)(0.1) = j100$

$0.2\ H \rightarrow j200$

$0.4\ H \rightarrow j400$

$1.25\mu F \rightarrow \frac{-j}{\omega C} = \frac{-j}{1000 \times 1.25 \times 10^{-6}} = -j800$

$5\mu F \rightarrow -j200$



KCL : $\underline{I} = \underline{I}_1 + \underline{I}_2$

KVL : $\underline{I}(j100) + \underline{I}_1(j400 - j200) - 4 = 0$

KVL : $\underline{I}_2(j200 - j800) - \underline{I}_1(j400 - j200) = 0$

$$\begin{cases} \underline{I} = \underline{I}_1 + \underline{I}_2 \\ j100\,\underline{I} + j200\,\underline{I}_1 = 4 \\ -j600\,\underline{I}_2 - j200\,\underline{I}_1 = 0 \end{cases} \Rightarrow \underline{I}_1 = -3\underline{I}_2$$

$$\begin{cases} \underline{I} = -3\underline{I}_2 + \underline{I}_2 = -2\underline{I}_2 \\ j100\,\underline{I} - j600\,\underline{I}_2 = 4 \end{cases} \Rightarrow -j200\,\underline{I}_2 - j600\,\underline{I}_2 = 4$$

$$-j800\,\underline{I}_2 = 4$$

$$\underline{I}_2 = \frac{4}{-j800} = j\frac{1}{200} = 0.005j = 0.005\angle 90$$

$$\underline{I} = -2\underline{I}_2 = -0.01j = 0.01\angle -90$$

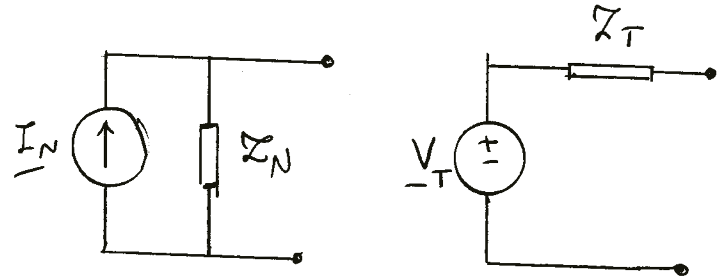
$$\underline{V} = -j800\,\underline{I}_2 = -j800(0.005j) = 4 = 4\angle 0$$

$$\Rightarrow \begin{cases} i_2 = 0.005\cos(1000t + 90^\circ) \\ i = 0.01\cos(1000t - 90^\circ) \\ v = 4\cos(1000t) \end{cases}$$

Thevenin / Norton Transformation

$\underline{Z}_T = \underline{Z}_N$

$\underline{V}_T = \underline{Z}_N \, \underline{I}_N$

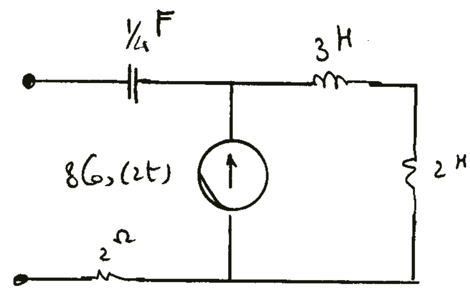


Also $\underline{V}_T = \underline{V}_{oc}$

$\underline{I}_N = \underline{I}_{sc}$

$\underline{Z}_T = \underline{Z}_N = \dfrac{\underline{V}_{oc}}{\underline{I}_{sc}}$

Example.



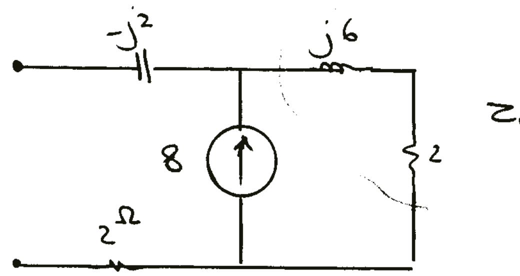
$\omega = 2$

$8\cos(2t) \rightarrow 8$

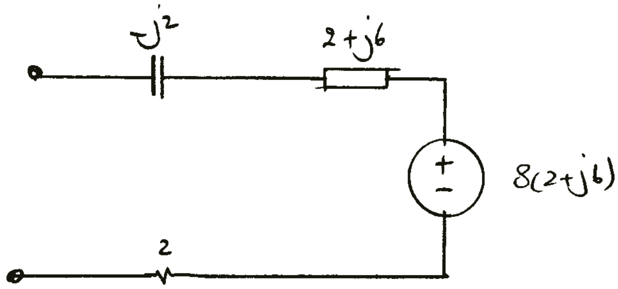
$\frac{1}{4}F \rightarrow \dfrac{-j}{2\times(\frac{1}{4})} = -j2$

$2H \rightarrow j\omega(2) = j4$

$3H \rightarrow j6$



$Z_1 = 2 + j6$



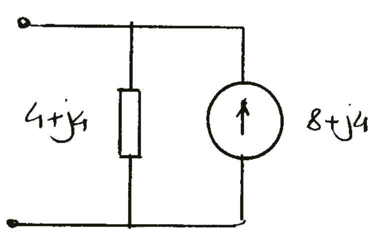
$\underline{Z}_T = 4 + j4$, $\underline{V}_T = 16 + j48$

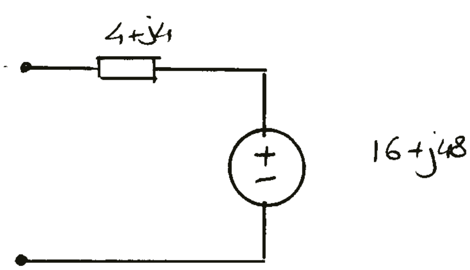
$\underline{Z}_N = 4 + j4$

$\underline{I}_N = \dfrac{\underline{V}_T}{\underline{Z}_N} = \dfrac{16+j48}{4+j4}$

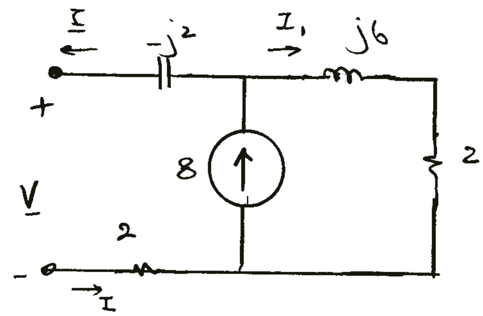
$= \dfrac{4+j12}{1+j} = \dfrac{(4+j12)(1-j)}{2}$

$= 8 + j4$





Method 2:



① $\underline{V} = \underline{V}_{oc}$, $\underline{I} = 0$

$KCL \rightarrow I_1 = 8$

KVL: $-\cancel{I}(-j2) + I_1(j6+2) - \cancel{2I} - \underline{V}_{oc} = 0$

$\underline{V}_{oc} = I_1(j6+2) = 8(j6+2) = 16+j48$ ←

② $\underline{V} = 0$, $\underline{I} = \underline{I}_{sc}$

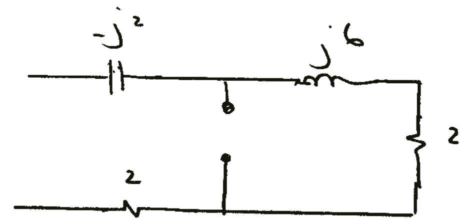
using current divider formula

$$\underline{I}_{sc} = \frac{\underline{V}}{-j2} = \frac{\underline{I}\,(Z_{eq})}{-j2+2} = \frac{\frac{1}{-j2+2}}{1/Z_{eq}}\,\underline{I}$$

$$\underline{I}_{sc} = \frac{\frac{1}{2-j2}}{\frac{1}{2-j2} + \frac{1}{2+j6}}\,8 = \frac{2+j6}{2+j6+2-j2} \times 8 = \frac{8(2+j6)}{4+j4} = \frac{4+j12}{1+j} = 8+j4$$

$\underline{I}_{sc} = 8+j4$ ←

③ Z_T (kill all independent source)



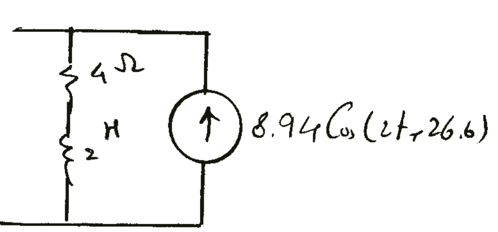
$Z_T = -j2+j6+2+2 = 4+j4$

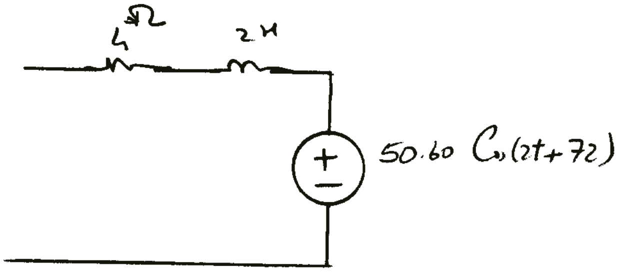
Conversion to time domain

$\underline{V}_T = 16+j48 = 50.60 \angle 71.57$ $\qquad v_T = 50.60\, \cos(2t + 71.57)$

$\underline{I}_{sc} = 8+j4 = 8.94 \angle 26.57$ $\qquad i_{sc} = 8.94\, \cos(2t + 26.57)$

$Z_T = 4+j4 = \underbrace{4}_{R} + j\omega \underbrace{2}_{L}$

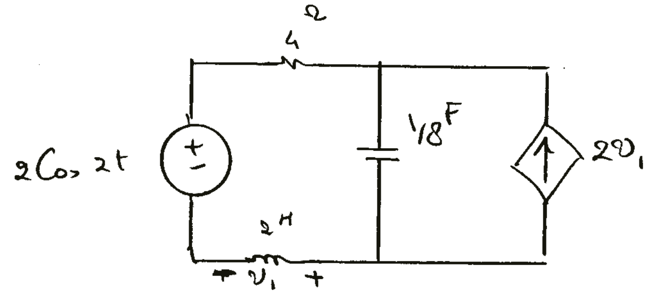




★ Norton / Thevenin Transformation is frequency dependent

Dependent sources

Example Find v_1

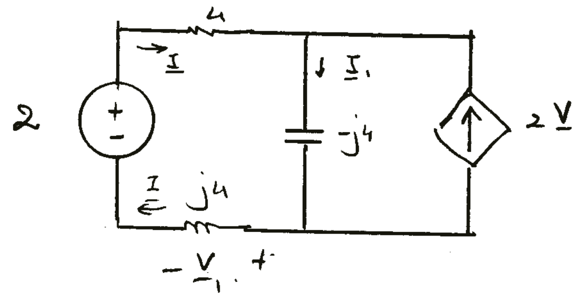


$\omega = 2$

$2 \cos 2t \rightarrow 2$

$\frac{1}{8} F \rightarrow \frac{-j}{\omega C} = \frac{-j}{2 \times 1/8} = -j4$

$2H \rightarrow j\omega L = j(2)(2) = j4$



KCL : $-\underline{I} - 2\underline{V}_1 + \underline{I}_1 = 0 \Rightarrow \underline{I}_1 = \underline{I} + 2\underline{V}_1$

KVL: $4\underline{I} + \underline{I}_1(-j4) + \underline{V}_1 - 2 = 0 \Rightarrow 4\underline{I} - j4(\underline{I} + 2\underline{V}_1) + \underline{V}_1 = 2$

Aux: $\underline{V}_1 = +j4\,\underline{I}$

$$\begin{cases} \underline{I}_1 = \underline{I} + 2\underline{V}_1 \\ \underline{I}(4 - j4) + \underline{V}_1(1 - j8) = 2 \\ \underline{V}_1 = +j4\,\underline{I} \end{cases}$$

$\Rightarrow \underline{I}\left(4 - j4 + j4(1 - j8)\right] = 2 \Rightarrow \underline{I} = \frac{2}{36}$ A

$4 - \cancel{j4} + \cancel{j4} + 32$

$\underline{V}_1 = j4\underline{I} = \frac{j8}{36}$ A

$$\begin{cases} \underline{I} = 0.0566 \text{ A} = 56.6 \angle 0 \text{ mA} \\ \underline{V}_1 = j\,0.224 \text{ v} = 0.224 \angle 90^\circ \text{ v} \end{cases}$$

$\Rightarrow$ $i(t) = 56.6 \cos(2t)$

$v(t) = 0.224 \cos(2t + 90^\circ)$

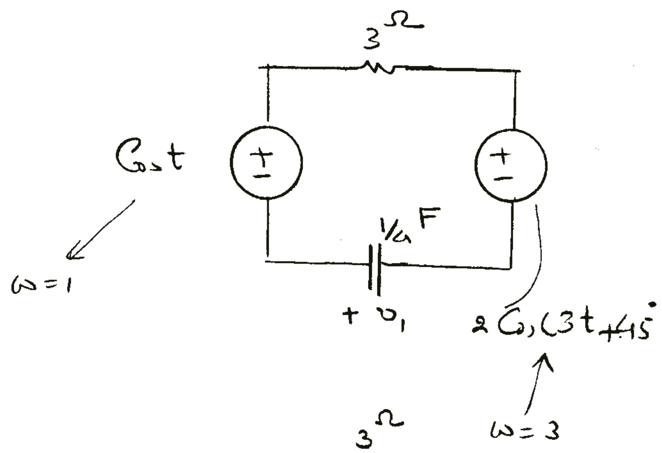
$= -0.224 \sin(2t)$

SUPERPOSITION

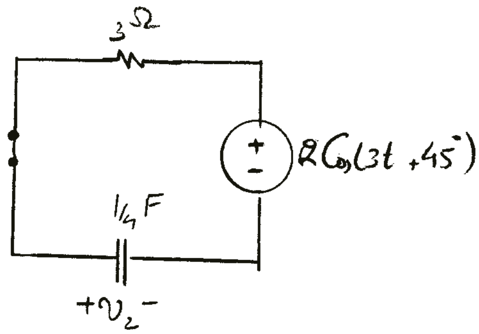
- If all sources have the same frequency, superposition is an alternative to other analysis method
- If a circuit contains sources with <u>different frequencies</u>, superposition is the only applicable method

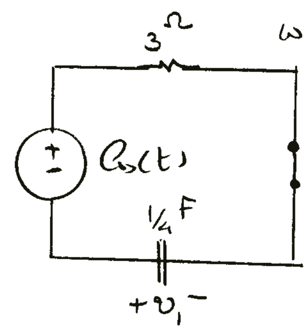
Method:
1. Write the response in terms of circuit response to sources of the same frequency (kill other sources)
2. Solve each circuit in the frequency domain
3. transfer the solution to time domain.
4. Sum over all sources.

Example. Find v



① Superposition in time domain

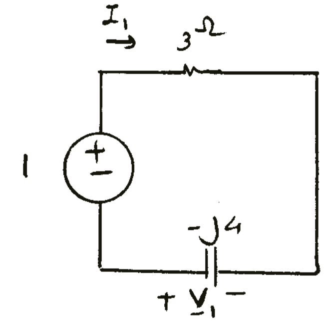




② Solve circuits 1 & 2 in frequency domain

$\omega = 1$, $\quad \frac{1}{4}F \to -\frac{j}{\omega C} = -\frac{j}{1/4} = -j4$

$\underline{V}_1 = -\underline{I}_1 (j4) = -j4 \dfrac{1}{3-j4} = \dfrac{4}{25}(4-j3)$



$$\underline{V}_1 = \frac{4}{25}(4-j3) = 0.8 \angle -37^\circ$$

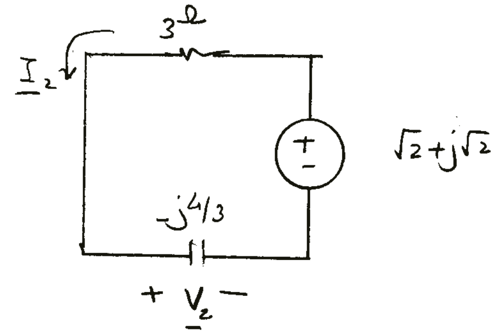
$$v_1(t) = 0.8 \cos(t - 37^\circ)$$

Circuit #2

$\omega = 3$

$$2\cos(3t+45^\circ) \rightarrow 2e^{j45} = 2\left(\frac{\sqrt{2}}{2} + j\frac{\sqrt{2}}{2}\right)$$

$$\frac{1}{4}F \rightarrow -\frac{j}{\omega C} = -\frac{j}{3 \times 1/4} = -j\frac{4}{3}$$



$$\underline{V}_2 = \underline{I}_2\left(-j\frac{4}{3}\right) = \left(-j\frac{4}{3}\right)\frac{\sqrt{2}+j\sqrt{2}}{3-j4/3}$$

$$\underline{V}_2 = \frac{-j4(\sqrt{2}+j\sqrt{2})}{9-j4} = 4\sqrt{2}\,\frac{1-j}{9-j4} = 4\sqrt{2}\,\frac{\sqrt{2}\angle -45^\circ}{\sqrt{97}\angle -24^\circ} = \frac{8}{9.85}\angle -45+24$$

$$\underline{V}_2 = 0.81 \angle -21$$

$$v_2 = 0.81 \cos(3t - 21^\circ)$$

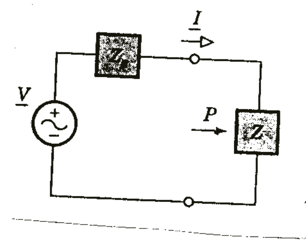
(3) Add in time domain.

$$v(t) = v_1(t) + v_2(t) = 0.80\cos(t-37^\circ) + 0.81\cos(3t-21^\circ)$$

Notes:
- You do not need to divide the circuit into sub circuits each containing only one source. Divide the circuit into sub circuits containing sources of same frequency (or DC, i.e $\omega = 0$). Then solve each circuit with an approriate analysis method in the frequency domain
- Always add responses in The time domain & not the frequency domain.

MAXIMUM POWER TRANSFER

$$\begin{cases} Z_s = R_s + jX_s \\ Z = R + jX \end{cases}$$



$$\underline{I} = \frac{\underline{V}}{Z + Z_s}$$

$$P = \frac{1}{2} R I_m^2 = \frac{R}{2} \frac{V_m^2}{|Z+Z_s|^2} = \frac{V_m^2}{2} \cdot \frac{R}{\left[(R+R_s)^2 + (X+X_s)^2\right]}$$

To find Z (R & X) values which maximize P, set $\frac{\partial P}{\partial R} = 0$, $\frac{\partial P}{\partial X} = 0$

$$\frac{\partial P}{\partial X} = \frac{V_m^2 R}{2} \cdot \frac{-2(X+X_s)}{\left[(R+R_s)^2 + (X+X_s)^2\right]^2} = 0 \implies X = -X_s$$

$$\frac{\partial P}{\partial R} = \frac{V_m^2}{R} \frac{(R+R_s)^2 + (X+X_s)^2 - 2R(R+R_s)}{\left[(R+R_s)^2 + (X+X_s)^2\right]^2} = 0$$

$$\rightarrow (R+R_s)^2 + \underbrace{(X+\cancel{X_s})^2}_{0} - 2R(R+R_s) = 0 \implies R + R_s - 2R = 0 \implies R = R_s$$

Maximum Power Transfer $\rightarrow$ $\begin{cases} R = R_s \\ X = -X_s \\ Z = Z^* \end{cases}$

Maximum Available Power $P_{max} = \frac{V_m^2}{2} \cdot \frac{R_s}{(2R_s)^2} = \frac{V_m^2}{8R_s} = \frac{V_{rms}^2}{4R_s}$