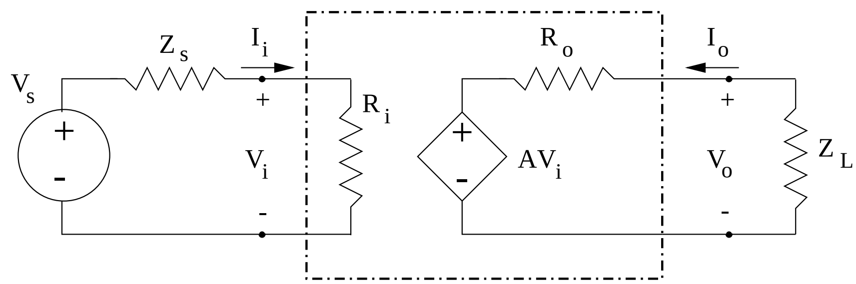
Operational Amplifiers

Amplifiers are two-port networks in which the output voltage or current is directly proportional to either input voltage or current. Four differnet kind of amplifiers exits. :

Voltage amplifier:	$A_v = V_o/V_i$
Current amplifier:	$A_i = I_o/I_i$
Transconductance amplifier:	$G_m = I_o/V_i$
Transresistance amplifier:	$R_m = V_o/I_i$

We will examine several voltage amplifiers utilizigng operational amplifier as a building block. Controlled or dependent sources can be used to model practical amplifiers quite accurately. Circuit below shows a simple but quite accurate for a typical linear voltage amplifier. An important parameter for these amplifiers is the voltage gain, $A_v = V_o/V_i$.



Voltage Amplifier Model

Practical Amplifiers

Real voltage amplifiers differ from the ideal amplifiers. One should always be aware of the range where the circuit acts as a linear amplifier, *i.e.,* the output is proportional to the input with the ratio of $A_v = V_o/V_i = constant$ (exactly the same waveform) .

Amplifier Saturation: Amplifiers do not create power. Rather, they act as a "valve" adjusting the power flow from the power supply into the load according to the input signal. As such, the output voltage amplifier cannot exceed the power supply voltage (it can be lower because of voltage drop on some active elements). The fact that the output voltage of a practical amplifier cannot exceed certain threshold value is called saturation. A voltage amplifier behaves linearly, *i.e.,* $V_o/V_i = A_v = constant$ as long as the output voltage remains below the "saturation" voltage,

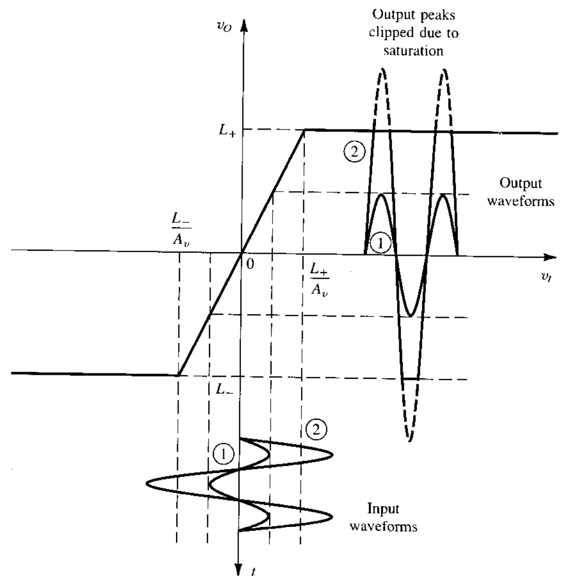
$$-V_{sat} < V_o < V_{sat}$$

Note that the saturation voltage, in general, is not symmetric, *i.e.,* $-V_{sat,1} < V_o < V_{sat,2}$.

For an amplifier with a given gain, A_v, the above range of V_o translate into a certain range for V_i

$$-V_{sat} < V_o < V_{sat}$$
$$-V_{sat} < A_v V_i < V_{sat}$$
$$-\frac{V_{sat}}{A_v} < V_i < \frac{-V_{sat}}{A_v}$$

i.e., any amplifier will enter its saturation region if V_i is raised above certain limit. The figure shows how the amplifier output clips when amplifier is not in the linear region.



Feedback

Not only a good amplifier should have sufficient gain, its performance should be insensitive to environmental and manufacturing conditions. It is easy to make an amplifier with a very large gain. A typical transistor circuit can easily have a gain of 100 or more. A three-stage transistor amplifier can easily get gains of 10^6. Other characteristics of a good amplifier is hard to achieve. For example, the gain of a transistor changes with operating temperature making the gain of the three-stage amplifier vary widely. The system can be made to be insensitive to environmental and manufacturing conditions by the use of feedback. Feedback also helps in other regards.

Principle of feedback: The input to the circuit is modified by "feeding" a signal proportional to the output value "back" to the input. There are two types of feedback (remember the example of a car in the freeway discussed in the class):

1. Negative feedback: As the output is increased, the input signal is decreased and *vice versa.* Negative feedback stabilizes the output to the desired level. Linear system employs negative feedback.

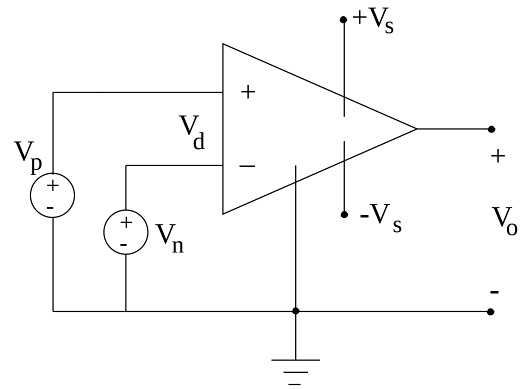
2. Positive feedback: As the output is increased, the input signal is increased and *vice versa.* Positive feedback leads to instability. (But, it has its uses!)

We will explore the concept of feedback in the context of operational amplifiers.

OpAmps as linear amplifiers

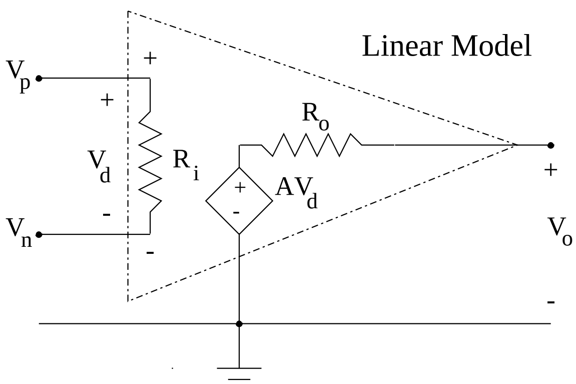
Operational amplifiers are general purpose voltage amplifiers employed in a variety of circuits. They are "DC" amplifiers with a very large gain (10^5 to 10^6), high input impedance ($> 1 - 10$ MΩ), and low output resistance ($< 100\ \Omega$). They are constructed as a "difference" amplifier, *i.e.,* the output signal is proportional to the difference between the two input signals.

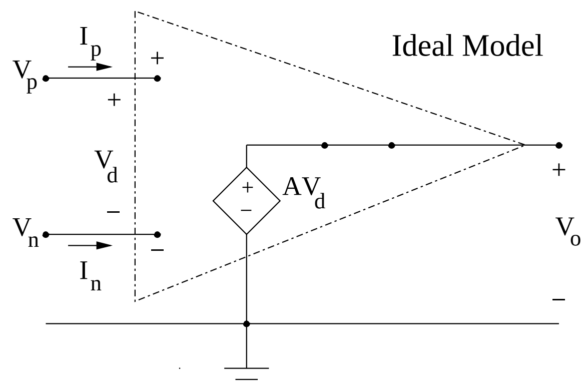
$$V_o = A_o V_d = A_o (V_p - V_n)$$



V_s and $-V_s$ are power supply attachments. They set the saturation voltages for the OpAmp circuit (within 0.2 V). Power supply ground should also be connected to the OpAmp ground. $+$ and $-$ terminals of the OpAmp are called, respectively, non-inverting and inverting terminals.

OpAmp Models



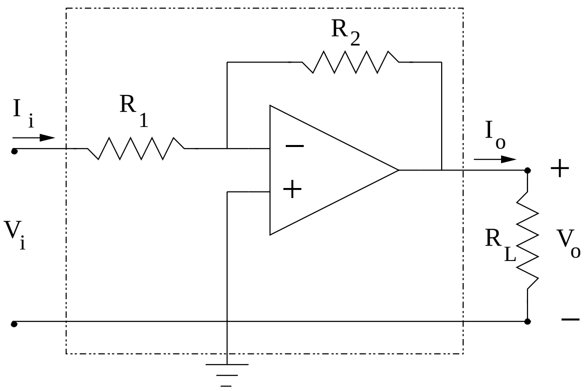


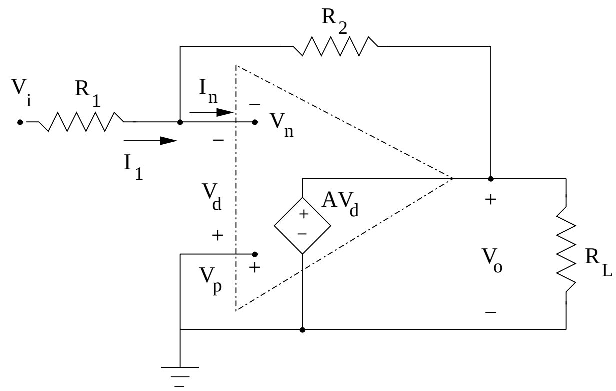
Because R_i is very large and R_o is very small, ideal model of the OpAmp assumes $R_i \to \infty$ and $R_o \to 0$. Ideal model is usually a very good model for OpAmp circuits. Very large input resistance also means that the input current into an OpAmp is very small:

First Golden Rule of OpAmps: $I_p \approx I_n \approx 0$ (Also called "Virtual Open Principle")

Another important feature of OpAmp is that because the gain is very high, without negative feedback, the OpAmp will be in the saturation region. For example, take an OpAmp with a gain of 10^5 and $V_{sat} = 15$ V. Then, $V_i \leq 15 \times 10^{-5} = 150\ \mu$V (a very small value) for OpAmp to be in linear region. Example below shows how feedback is utilized.

Inverting Amplifier





The first step in solving OpAmp circuits is to replace the OpAmp with its circuit model (ideal model is usually very good). Then, using node-voltage method and noting $I_n \approx 0$:

$$V_p = 0, \qquad V_o = A_o V_d = A_o(V_p - V_n) = -A_o V_n$$
$$\frac{V_n - V_i}{R_1} + \frac{V_n - V_o}{R_2} = 0$$

Substituting for $V_n = -V_o/A_o$ in the second equation and dividing the equation by R_2, we have:

$$-\frac{R_2}{A_o R_1} V_o - \frac{R_2}{R_1} V_i - \frac{V_o}{A_o} - V_o = 0 \qquad \rightarrow \qquad V_o \left[1 + \frac{1}{A_o} + \frac{R_2}{A_o R_1}\right] = -\frac{R_2}{R_1} V_i$$
$$\frac{V_o}{V_i} = -\frac{R_2}{R_1} \frac{1}{1 + \dfrac{1}{A_o} + \dfrac{R_2}{A_o R_1}}$$

Since OpAmp gain is very large, $1/A_o \ll 1$. Also if R_2 and R_1 are chosen such that their ratio is not very large, $R_2/R_1 \ll A_o$ or $R_2/(A_o R_1) \ll 1$, then the voltage transfer function of the OpAmp is

$$\frac{V_o}{V_i} = -\frac{R_2}{R_1}$$

The circuit is called an inverting amplifier because the voltage transfer function is "negative." (A "negative" sinusoidal function looks inverted.) The negative sign means that there is 180° phase shift between input and output signals.

Note that the voltage transfer function is "independent" of the OpAmp gain, A_o, and is only set by the values of the resistors R_1 and R_2. While A_o is quite sensitive to environmental and manufacturing conditions (can vary by a factor of 10 to 100), the resistor values are not very sensitive and, thus, the gain of the system is quite stable. This stability is achieved by negative feedback, the output of the OpAmp is connected via R_2 to the inverting terminal of OpAmp. If V_o increases, this resistor forces V_n to increase, reducing $V_d = V_p - V_n$ and stabilizes the OpAmp output $V_o = A_o V_d$.

An important feature of OpAmp circuits with negative feedback is that because the OpAmp is NOT saturated, $V_d = V_o/A_o$ is very small (because A_o is very large). As a result,

$$\text{Negative Feedback} \qquad \rightarrow \qquad V_d \approx 0 \qquad \rightarrow \qquad V_n \approx V_p$$

Second Golden Rule of OpAmps: For OpAmps circuits with negative feedback, the OpAmp adjusts its output voltage such that $V_d \approx 0$ or $V_n \approx V_p$ (Also called "Virtual Short Principle")

The above rule simplifies solution to OpAmp circuits dramatically. For example, for the inverting amplifier circuit above, we will have:

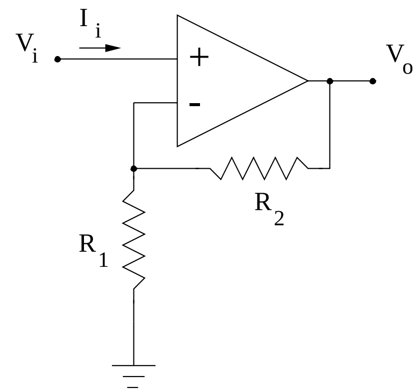
$$\text{Negative Feedback} \qquad \rightarrow \qquad V_n \approx V_p \approx 0$$
$$\frac{V_n - V_i}{R_1} + \frac{V_n - V_o}{R_2} = 0 \quad \rightarrow \quad \frac{V_i}{R_1} + \frac{V_o}{R_2} = 0 \quad \rightarrow \quad \frac{V_o}{V_i} = -\frac{R_2}{R_1}$$

OpAmps are used in many configurations (with different feedback arrnagments) to build various forms of amplifiers. Several of these are discussed below.

How to solve OpAmp circuits:

1) Replace the OpAmp with its circuit model.
2) Check for negative feedback (connection from output to the inverting terminal), if so, write down $V_p \approx V_n$.
3) Solve. Best method is usually node-voltage method. You can solve simple circuits with KVL and KCLs. Do not use mesh-current method.

Non-inverting Amplifier

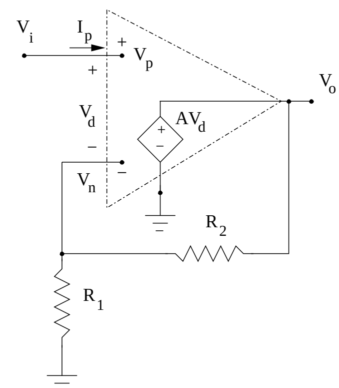


$$V_p = V_i$$

$$\text{Negative Feedback} \qquad \rightarrow \qquad V_n \approx V_p = V_i$$

$$\frac{V_n - 0}{R_1} + \frac{V_n - V_o}{R_2} = 0$$

Note that you should not write a node equation at OpAmp output as its a node attached to a voltage source. The value of V_o is $A_o V_d$. However, instead of using this equation, we set $V_d \approx 0$ (Here, we are assuming $A_o \rightarrow \infty$ and $V_d \rightarrow 0$, so $v_o = A_o V_d = \infty$ is undefined)



Substituting for $V_n = V_i$, we get

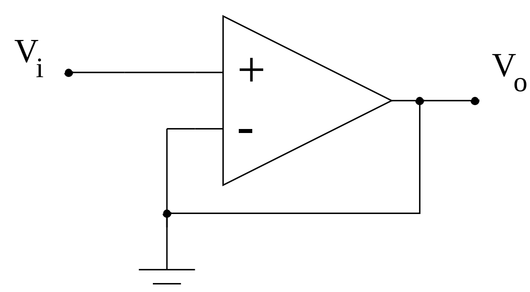
$$\frac{R_2}{R_1} V_i + V_i - V_o = 0$$

$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1}$$

Voltage Follower

In some cases, we have two-terminal networks which do not match well, *i.e,* the input impedance of the later stage is not very large, or the output impedance of preceding stage is not low enough. A "buffer" circuit is usually used in between these two circuits to solve the matching problem. These "buffer" circuit typically have a gain of 1 but have a very large input impedance and a very small output impedance. Because their gains are 1, they are also called "voltage followers."

The non-inverting amplifier above can be turned into a voltage follower (buffer) by adjusting R_1 and R_2 such that the gain is 1.



$$\frac{V_o}{V_i} = 1 + \frac{R_2}{R_1} = 1 \qquad \rightarrow \qquad R_2 = 0$$

So by setting $R_2 = 0$, we have $V_o = V_i$ or a gain of unity. We note that this expression is valid for any value of R_1. As we want to minimize the number of components in a circuit as a rule (cheaper circuits!) we set $R_1 = 0$ and also remove R_1 from the circuit.

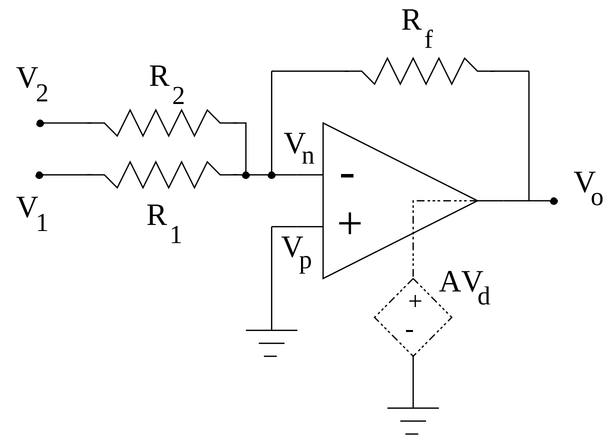
Inverting Summer

$V_p = 0$

Negative Feedback: $\quad V_n \approx V_p = 0$

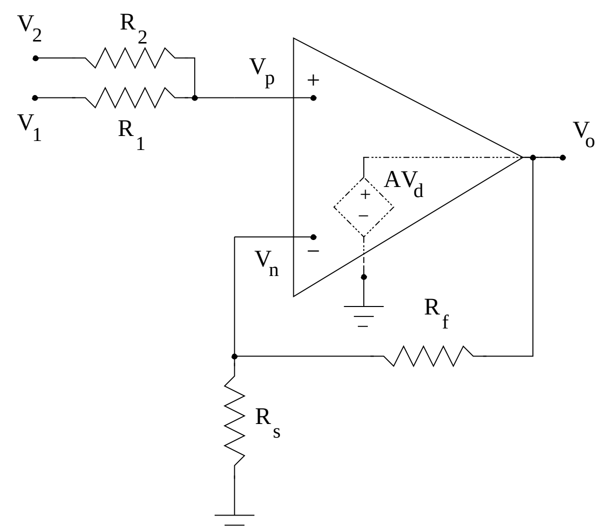
$$\frac{V_n - V_1}{R_1} + \frac{V_n - V_2}{R_2} + \frac{V_n - V_o}{R_f} = 0$$

$$V_o = -\frac{R_f}{R_1}V_1 - \frac{R_f}{R_2}V_2$$



So, this circuit adds (sums) two signals. An example of the use of this circuit is to add a DC offset to a sinusoidal signal.

Non-Inverting Summer



Negative Feedback: $\quad V_n \approx V_p$

$$\frac{V_p - V_1}{R_1} + \frac{V_p - V_2}{R_2} = 0 \quad \longrightarrow$$

$$V_p\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{V_1}{R_1} + \frac{V_2}{R_2}$$

$$\frac{V_n - 0}{R_s} + \frac{V_n - V_o}{R_f} = 0 \longrightarrow$$

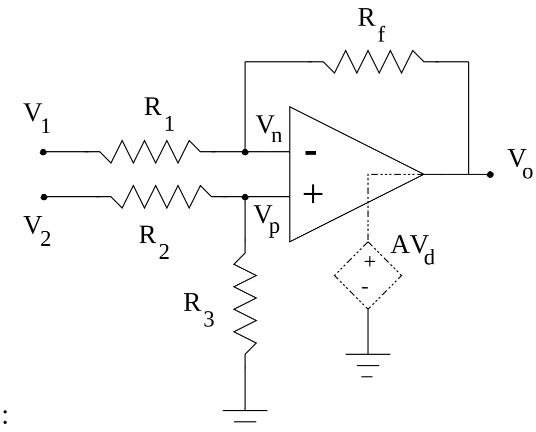
$$V_o = \left(1 + \frac{R_S}{R_f}\right) V_n$$

Substituting for V_n in the second equation from the first (noting $V_p = V_n$):

$$V_o = \frac{1 + R_S/R_f}{1/R_1 + 1/R_2}\left(\frac{V_1}{R_1} + \frac{V_2}{R_2}\right)$$

So, this circuit also signal adds (sums) two signals. It does not, however, inverts the signals.

Difference Amplifier



Negative Feedback: $V_n \approx V_p$

$$\frac{V_p - V_2}{R_2} + \frac{V_p - 0}{R_3} = 0 \quad \longrightarrow$$

$$V_n \approx V_p = \frac{R_3}{R_2 + R_3} V_2$$

$$\frac{V_n - V_1}{R_1} + \frac{V_n - V_o}{R_f} = 0$$

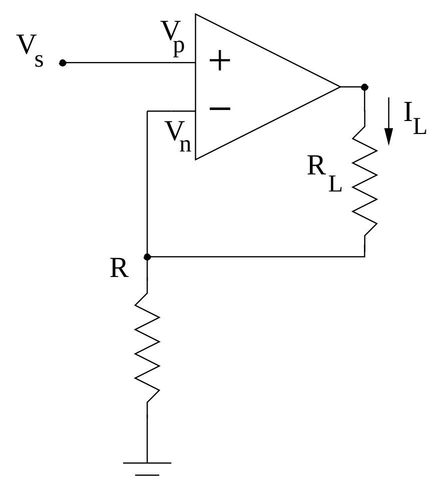
Substituting for V_n in the 2nd equation, one can get:

$$V_o = -\frac{R_f}{R_1} V_1 + \left(1 + \frac{R_f}{R_1}\right)\left(\frac{R_3}{R_2 + R_3}\right) V_2$$

If one choose the resistors such that $\dfrac{R_3}{R_2} = \dfrac{R_f}{R_1}$, then

$$V_o = \frac{R_f}{R_1}(V_2 - V_1)$$

Current Source



Negative Feedback: $V_n \approx V_p = V_s$

$$i_L = \frac{V_n}{R} = \frac{V_s}{R} = constant$$

Note the current I_L is independent of value of R_L and, therefore, indepdent of voltage V_L. As such this circuit (without resistor R_L is an independent current source.

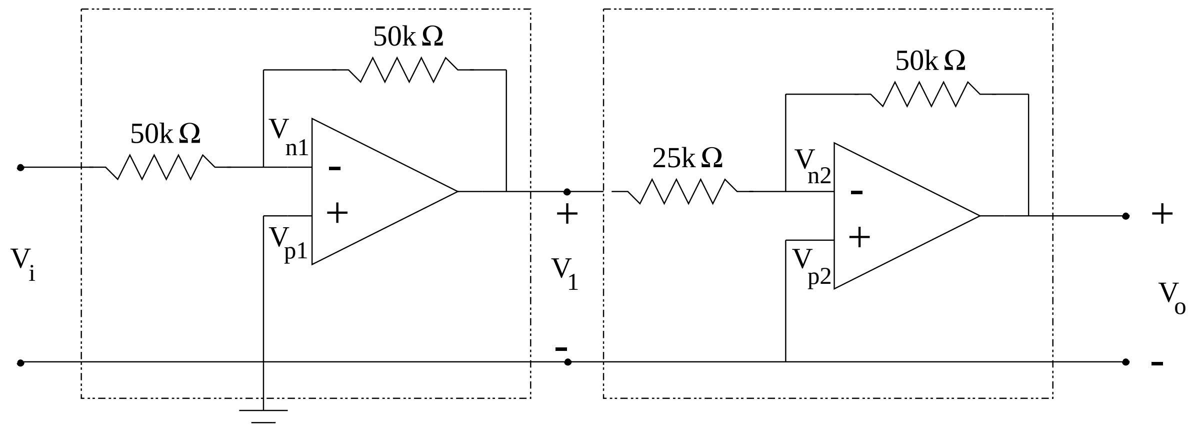
The value of the current can be adjusted by changing V_s. Therefore, this circuit is also a "voltage to current" convertor.

Cascading OpAMp circuits

An importnant property of the OpAmp circuits discussed above (with the exception ofthe current source) is that the circuit gain, v_o/v_i is independent of the load. To see that, attached a resistance R_L between the output and ground (voltage v_o appears across this resistor) and solve the circuit. One finds that v_o/v_i is indepdent of R_L.

This observation means that we can attach any sub-circuit to the OpAmp output with affecting v_o/v_i ratio. As such, analysis of cascading OpAmp circuit is easy in most cases. Alternatively, a circuit can be designed with the above opAmp building blocks to have a desired value of v_o/v_i. Examples below highlight these two features.

Example: Find v_o/v_i



The circuit is made of two OpAmp configured as an inverting amplifiers (see inside each dashed box). Thus,

$$\text{First stage is an inverting Amp.:} \quad \frac{v_1}{v_i} = -\frac{R_2}{R_1} = -\frac{50 \times 10^3}{50 \times 10^3} = -1$$

$$\text{Second stage is an inverting Amp.:} \quad \frac{v_o}{v_1} = -\frac{R_2}{R_1} = -\frac{50 \times 10^3}{25 \times 10^3} = -2$$

$$\text{System gain:} \quad \frac{v_o}{v_i} = \frac{v_1}{v_i} \times \frac{v_o}{v_1} = (-1) \times (-2) = 2$$

This method is much easier than solving the circuit as is shown below using node-voltage method:

OpAmp 1 has negative feedback: $v_{n1} \approx v_{p1} = 0$

OpAmp 2 has negative feedback: $v_{n2} \approx v_{p2} = 0$

$$\text{KCL on node } v_{n1}\text{:} \quad \frac{v_{n1} - v_i}{50 \times 10^3} + \frac{v_{n1} - v_1}{50 \times 10^3} = 0$$

$$\text{KCL on node } v_{n2}\text{:} \quad \frac{v_{n2} - v_1}{25 \times 10^3} + \frac{v_{n2} - v_o}{50 \times 10^3} = 0$$

Now substiuting for $v_{n1} = v_{n2} = 0$ in the two node equations, we get: $v_1 = -v_i$ and $v_o = -2v_1$ and, therefore, $v_o = 2v_i$

Example: Design an OpAmp circuit such that $v_o = 200v_a - 40v_b$ (v_a and v_b are given inputs)

We need first to write the expression for v_o in a form that can be broken into expressions for our 4 OpAmp building blocks. Many variations are possible. For example, we can let

$$\begin{aligned} \text{1st Stage:} \quad & v_1 = -5v_a \\ \text{2st Stage:} \quad & v_o = -40v_a - 40v_b \\ \text{System Gain:} \quad & v_o = -40(-5v_a) - 40v_b = 200v_a - 40v_b \end{aligned}$$

As can be seen, the first stage is an iverting amplifier with a gain of five. The second stage is an inverting summer with a gain of 40. Thus, the circuit will be