I. INTRODUCTION

1.1 Circuit Theory Fundamentals

In this course we study circuits with non-linear elements or devices (diodes and transistors). We will use circuit theory tools to analyze these circuits. Since some of tools developed in circuit theory apply only to "linear" elements (thus, linear circuit theory), let's first examine what we can use to analyze non-linear elements.

Circuit theory is an approximation to Maxwell's electromagnetic equations to simplify analysis of complicated circuit. There are nine fundamental circuit elements in circuit theory, denoted by their iv characteristics:

Resistor: $v = Ri$

Capacitor: $i = C\,\dfrac{dv}{dt}$ or $V = \dfrac{1}{j\omega C}\,I$

Inductor: $v = L\,\dfrac{di}{dt}$ or $V = j\omega L\,I$

Independent voltage source: $v = v_s = const.$ for any current

Independent current source: $i = i_s = const.$ for any voltage

and four controlled sources: voltage-controlled voltage source, (similar to an independent voltage source but with source strength depending on voltage on another element in the circuit), current-controlled voltage source, voltage-controlled current source, current-controlled current source.

There are two other circuit elements that we will use and are special cases of the above elements. They are:

Short Circuit: $v = 0$ for any current

Open Circuit: $i = 0$ for any voltage

As can be seen, "short circuit" is a special case of a resistor (with $R = 0$) or a special case of a voltage source (with $v_s = 0$) while "open circuit" is a special case of a resistor (with $R \to \infty$) or a special case of a current source (with $i_s = 0$).

It is essential to remember that the above circuit elements do NOT represent physical devices, rather they are idealized elements, "cooked" up to simplify the analysis (because their voltage-current relationship is linear).

Any two-terminal network (a box/device with two wires coming out of it) whose voltage is directly proportional to the current flowing through it, *i.e.,* the element iv characteristics equation is $v = Ri$, can be modeled as an "ideal" resistor. This means, that we can take this box/device out of the circuit and a replace it with a resistor and the response of the circuit does not change. Most importantly, we do NOT need to know what is inside the box, the only parameters we need to now is its resistance value.

Similarly, if we have a black box whose voltage is a constant for all currents, it can be modeled as an independent voltage source (without any knowledge of what is inside the box). You actually have been doing this in the lab, modeling the power supply (which includes many transistors, diodes, resistors, capacitors) with an independent voltage source.

On the other hand, physical elements (*i.e.,* resistors in the lab) can only be modeled with one of these ideal elements within a certain range of parameters and within a certain accuracy. For example take a resistor in the lab. At high enough frequencies, it will exhibit capacitance (*i.e.* its "resistance" drops as frequency increases). At high enough current, when the resistor is hot enough, the ratio of v/i is not linear anymore. So, an "ideal" resistor used in circuit theory is NOT a physical device. Rather, a resistor in the lab can be approximated by an "ideal" resistor only for a range of currents or voltages (typically rated by its maximum power), a range of frequencies, and even a range of environmental conditions (temperature, humidity, *etc.*).

The bottom line is that the iv characteristics of a two-terminal network or device is the key (not what is inside the box). If it follows the $i - v$ characteristics equation of one of the five elements above, we can use the corresponding "ideal" circuit element in our analysis.

The variables in a circuit are currents and voltages in each element. The physics of current flow is captured in the iv characteristics of each element. Two general rules governs what happens when these elements are connected to each other: Kirchhoff current low, KCL, which is conservation of electric charge and Kirchhoff voltage law, KVL, which is a topology-driven constraint (i.e., you get to the same place if you follow a closed loop). These two rules are independent of internal physics of elements and can be applied to non-linear elements.

In each circuit with N elements, we have $2N$ unknowns (i and v of each element) and we need $2N$ equations: N iv characteristics equations which describe the internal physics of each element and N KVL and KCLs which depend only on how the elements are connected to each other.

In the circuit theory, we learned that we can reduce the number of equations to be solved by a large number using "Node-Voltage" and "Mesh-Current" methods. As these method really are a compact form of writing KVL and KCLs, they equally apply to non-linear elements.

Other important circuit theory tools include: 1) Thevenin and Norton Equivalent circuits, 2) Proportionality and Superposition, and 3) Frequency domain (phasors) or s-domain analysis of circuit with sinusoidal sources. These tools, however, only apply to linear elements and we cannot use them for circuit with non-linear elements. Linear circuit analysis (including these tools) are so convenient that in a lot of cases we build approximate "linear" models for diodes and transistors so that we can apply the above rules in special cases. As such, short descriptions of frequency domain and Thevenin equivalent circuits are given below.

Which Solution Method to Use?

By looking at the circuit you should be able to decide the best method to solve the circuit. Basically, one wants to have the smallest number of equations. Assuming that we have "reduced" the circuit (*i.e.,* replaced parallel and series elements):

KVL & KCL:	$2N_{element}$ equations
Node-voltage Method:	$N_{nodes} - 1 - N_{IVS}$ equations
Mesh-Current Method:	$N_{loops} - N_{ICS}$ equations

(IVS: independent voltage source, ICS: independent current source).

Obviously, one should use KVL & KCL only if there are only a few elements. Furthermore, we are mostly interested in voltages in the circuits. As such, usually node-voltage method is preferred as we will have a fair number of voltage sources and the answer is also a voltage.

1. You CANNOT mix and match the three methods above!

2. Apply the techniques consistently *e.g.,* for example, always write KCL as sum of currents flowing out of a node = 0. This minimizes the chance for error.

1.2 Frequency Domain

In principle, the voltages and currents in analog circuits are arbitrary functions of time (we call them signals or waveforms). Analytical analysis of the circuit response to an arbitrary input waveform is difficult and requires solution to a set of differential equations. Even numerical analysis becomes difficult when there are a lot of circuit elements. Fortunately, there are ways to find the response of a linear circuit to time-dependent signal. These approaches are based on the following observations:

1. For circuits driven by sinusoidal sources, the forced response of the state variables (currents and voltages) are all sinusoidal functions with the same frequency as the source.

This is derived from the mathematical properties of sinusoidal functions. Forced response of a set of linear differential equations (circuit equations) to a sinusoidal function is a sinusoidal function. This property leads to special analysis tools for AC circuits using "phasors," or using Fourier transform. AC steady-state analysis of linear circuits are covered in ECE35/45. When we use phasors, the circuit equation do not contain time anymore, but they include frequency ω. As such, this is usually called analysis in "frequency-domain" to differentiate that from "time-domain" analysis where we solve the differential equation to find the circuit response.

2. Any arbitrary but periodic signal can be written as a sum of sinusoidal functions using Fourier series expansion.

For example, a square wave with period T or frequency $\omega_0 = 2\pi f = (2\pi)/T$ and amplitude V_m can be written as:

$$v(t) = \frac{4V_m}{\pi}\left[\sin(\omega_0 t) + \frac{1}{3}\sin(3\omega_0 t) + \frac{1}{5}\sin(5\omega_0 t) + ...\right]$$

Signals with frequencies $n\omega_0$ (n integer) are called harmonics of the fundamental frequency, ω_0. In general the amplitude of higher harmonics become smaller as n become larger. The idea of decomposition of a periodic function to a sum of sinusoidal functions can be extended to an arbitrary temporal function by using Fourier integrals. As such, in principle, any function of time can be written as a sum of (or an integral of) sinusoidal functions.

3. Proportionality and superposition principles state that the response of a linear circuit to a linear combination of sources is equivalent to the linear combination of the circuit response to each individual source.

Basically, in a circuit with several independent sources, the value of any state variable equals to the algebraic sum of the individual contributions from each independent source. So, in a circuit with a time-dependent source, we can use Fourier series decomposition and replace the source with a linear combination of several sinusoidal sources. We can then find the response of the circuit to each sinusoidal source and then use proportionality and superposition to find the response to the time-dependent source.

For example, suppose we want have a circuit driven by a source that can be decomposed into $v_i(t) = A\cos(100t) + B\cos(300t)$. We want to know the voltage across an element, $v_o(t)$. We solve the circuit with the source $\cos(100t)$ and find the voltage across the element interest, suppose $\alpha\cos(100t + \phi_\alpha)$. We then repeat the analysis with a source $\cos(300t)$ and find the voltage across the element interest, suppose $\beta\cos(300t + \phi_\beta)$. The response of the circuit to $v_i(t) = A\cos(100t) + B\cos(300t)$, then is $v_o(t) = A\alpha\cos(100t + \phi_\alpha) + B\beta\cos(300t + \phi_\beta)$.

The problem is actually much simpler than the example above. In principle, solution of AC steady-state circuit is simple and we typically find the response the circuit with frequency,

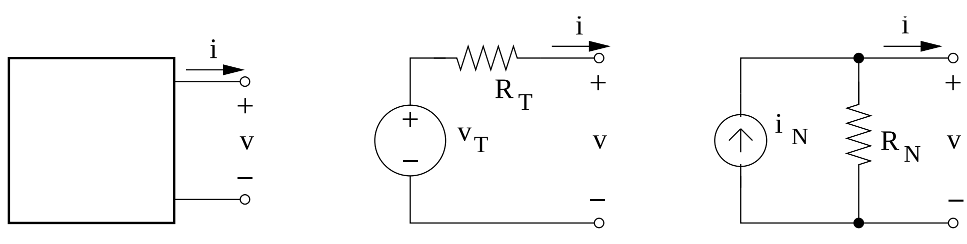
ω, as a parameter. We can then construct the response by replacing ω with frequencies of interest in the response equation (*e.g.,* set $\omega = 100$ and 300 in the above example). Another major simplification arises when the circuit response is frequency independent. In that case, the circuit response can be directly applied to any time-dependent function. For example, in the above example, if the circuit response to $\cos(100t)$ and $\cos(300t)$ sources were, respectively, $\alpha\cos(100t)$ and $\alpha\cos(300t)$ (frequency independent), then the circuit response is simply: $v_o(t) = \alpha v_i(t)$.

Therefore, in many circuit applications we focus on circuits driven by sinusoidal sources. We solve these circuits in frequency domain. We try to find circuit parameters with frequency ω as a parameter to facilitate construction of response to an arbitrary function of time.

There are several ways to solve the circuit in frequency domain, all having the same mathematical foundation. We can use phasors (which are really Fourier Transforms). Or, we can use complex frequency domain which is sometimes called "s-domain" ($s = \sigma + j\omega$). In junior level courses and beyond, you will probably use complex frequency domain mainly. Circuit analysis with phasors is sufficient for the work we do in this class (to convert from phasors to s-domain), simply replace $j\omega$ with s and $-\omega^2$ with s^2.

Analysis in frequency domain is straight-forward. Resistors, capacitors, and inductors are replaced by impedances, Z: $Z = R$ for a resistor, $Z = 1/(j\omega C)$ for a capacitor and $Z = j\omega L$ for an inductor. Impedances obey Ohm's Law: $V = ZI$. Thus, with impedances the circuit reduces to a "resistive" circuit and all analysis techniques of resistive circuits (node-voltage method, mesh-current method, Thevenin Theorem, *etc.*) apply. The only difference is that analysis is performed using complex variables.

1.3 Thevenin Theorem and Thevenin or Norton Equivalents



We know from linear circuit theory that the iv characteristics of a two-terminal network is in the form of (using active sign convention):

$$v = v_T - R_T i; \qquad R_T = R_N; \qquad i_N R_N = v_T$$

(in frequency or s domain, we should replace R_T with Z_T).

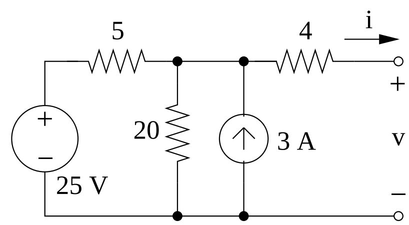
This means that we only need to solve and/or measure the Thevenin equivalent of a two-port terminal once. From then one, the two-port terminal can be replaced with either of its Thevenin or Norton equivalents without affecting the response of the rest of the circuit.

An important corollary to the Thevenin Theorem is that if a two-terminal network does not include an "independent source" it can be reduced to a single "resistance" (even if it includes dependent sources).

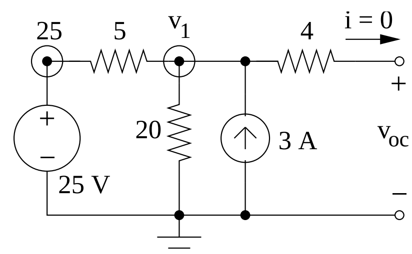
How to calculate the Thevenin equivalent

You have seen a detailed discussion of Thevenin/Norton forms in your circuit theory course(s). In summary, the best method is to calculate two of the the following three parameters: (1) Open-circuit voltage, v_{oc} (found by setting $i = 0$) , (2) Short-circuit current, i_{sc} (found by shorting the terminals of the two-terminal network, *i.e.,* setting $v = 0$), and (3) Direct calculation of R_T which is the resistance seen at the terminals with the independent sources "zeroed out" (*i.e.,* their strengths set equal to zero). Remember, you should NOT "zero out" dependent sources.

Example 1: Find the Thevenin and Norton Equivalent of this circuit:

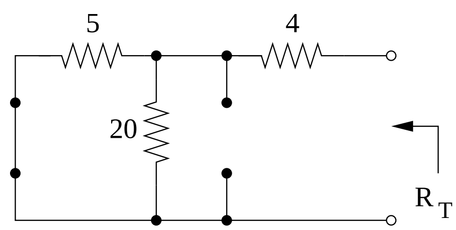


1. v_{oc}: Using node-voltage method and noting that since $i = 0$, by KVL, $v_1 = v_{oc}$.

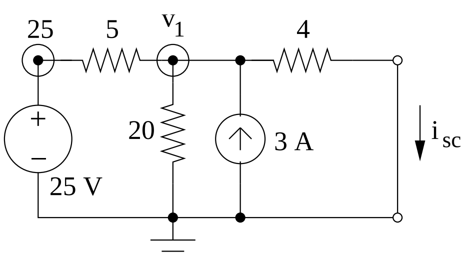


$$\frac{v_1 - 25}{5} - 3 + \frac{v_1}{20} = 0$$
$$4v_1 - 100 - 60 + v_1 = 0$$
$$v_T = v_{oc} = 32\text{V}$$

2. R_T (zeroing the independent sources): From the circuit, we have $R_T = 4 + (5 \parallel 20) = 4 + 4 = 8\ \Omega$.



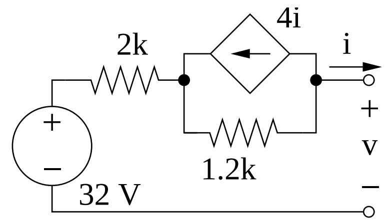
3. i_{sc}: Note that $i_{sc} = v_1/4$.



$$\frac{v_1 - 25}{5} + \frac{v_1}{4} - 3 + \frac{v_1}{20} = 0$$
$$4v_1 - 100 + 5v_1 - 60 + v_1 = 0$$
$$v_1 = 16\text{V} \qquad \rightarrow \qquad i_{sc} = i_N = 0.25 v_1 = 4\text{A}$$

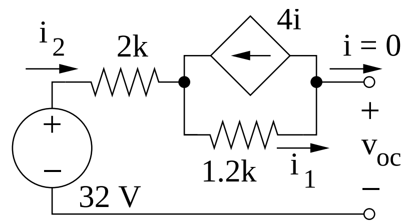
So, the Thevenin/Norton parameters are: $v_T = 32$ V, $i_N = 4$ A, and $R_T = 8\ \Omega$. (note $v_T = i_N R_T$.)

Thevenin Equivalent of two-terminal networks with controlled sources



Example: Find the Thevenin equivalent of this two-terminal network.

Finding v_{oc}: Since the circuit is simple, we proceed to solve it with KVL and KCL (noting $i = 0$):



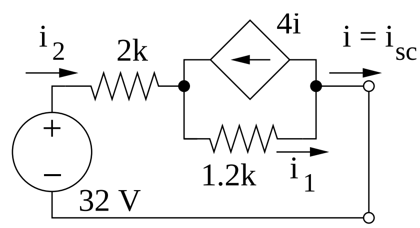
KCL: $-i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 0$

KCL: $-i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = 0$

KVL: $-32 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 + v_{oc} = 0$

$v_T = v_{oc} = 32$ V

Finding i_{sc}: Using KVL and KCL:



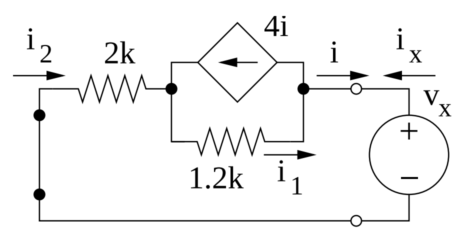
KCL: $-i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 5 i_{sc}$

KCL: $-i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = i_{sc}$

KVL: $-32 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 = 0$

$-32 + 2 \times 10^3 i_{sc} + 6 \times 10^3 i_{sc} = 0 \quad \rightarrow \quad i_N = i_{sc} = 4 \times 10^{-3} \text{ A} = 4 \text{ mA}$

Finding R_T: We "zero" out all independent sources in the circuit. The resulting circuit cannot be reduced to a simple resistor by series/parallel formulas. We can find R_T, however, by attaching a "test" source, v_x to the terminals and calculate current i_x (see figure). Since the two-terminal network should be reduced to a resistor (R_T), we should get $R_T = v_x / i_x$.



Since the circuit is simple, we proceed to solve it with KVL and KCL (note $i_x = -i$):

KCL: $-i_1 + i + 4i = 0 \quad \rightarrow \quad i_1 = 5i = -5 i_x$

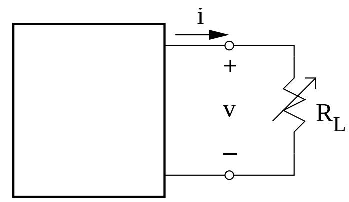
KCL: $-i_2 - 4i + i_1 = 0 \quad \rightarrow \quad i_2 = i = -i_x$

KVL: $0 + 2 \times 10^3 i_2 + 1.2 \times 10^3 i_1 + v_x = 0$

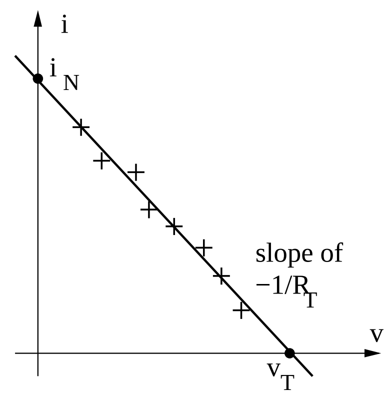
$-2 \times 10^3 i_x - 6 \times 10^3 i_x + v_x = 0 \quad \rightarrow \quad R_T = \dfrac{v_x}{i_X} = 8 \times 10^3 = 8 \text{ k}\Omega$

How to measure the Thevenin equivalent

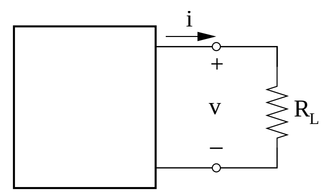
Suppose we have given a box with two terminals and want to measure the Thevenin equivalent of the circuit inside the box. In principle, we cannot use the above technique and try to measure v_{oc}, i_{sc}, and R_T. We cannot turn off the input signal and use a ohm-meter to measure R_T. Nor can we short the terminals and measure i_{sc} (there is a good chance that we are going to ruin the circuit if we do that). In principle, we can use a volt-meter (or scope) to measure v_{oc} but care should be taken as it is not known a priori if the internal resistance of the volt-meter (or scope) is large enough to act as an open circuit (there are other complications). There is also the issue of measurement error that one should consider.



The best way to measure the Thevenin Equivalent parameters (works for Resistive R_T) is to measure the iv characteristics of the two-terminal network. We can do this by attaching a variable load (a resistance) to the box, vary the load which changes the output voltage and currents, and measure several pair of i and i (here we do not use the value of R_L). Typically this is done with starting from a "large" R_L and gradually reducing the load.
These data point should lie on the iv line of the two-terminal network. Values of v_T, i_N, and R_T can be read directly from the graph as shown. This method is specially accurate as one can use a "best-fit" line to the data in order to minimize random measurement errors.

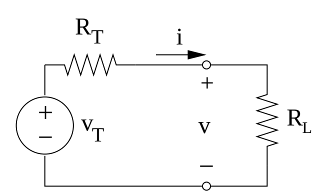


A simpler, but less accurate version of the above method is to measure the output voltage for TWO different values of R_L (*i.e.,* R_{L1} and R_{L2} with v_1 and v_2, respectively). From the circuit:



$$\frac{v}{v_T} = \frac{R_L}{R_T + R_L}$$

$$\frac{v_1}{v_T} = \frac{R_{L1}}{R_T + R_{L1}} \qquad \text{and} \qquad \frac{v_2}{v_T} = \frac{R_{L2}}{R_T + R_{L2}}$$



Dividing the two equations give:

$$\frac{v_1}{v_2} = \frac{R_{L1}}{R_T + R_{L1}} \times \frac{R_T + R_{L2}}{R_{L2}}$$

which can be solved to find R_T. Then, one of the above equations for v_1 or v_2 can then be used to find v_T. Typically, we choose R_{L2} to be very large, $R_{L2} \to \infty$ (*e.g.,* internal resistance of scope), then $v_2 = v_{oc}$ (open circuit voltage) and

$$\frac{v_1}{v_{oc}} = \frac{R_{L1}}{R_T + R_{L1}} \qquad \rightarrow \qquad \frac{R_T}{R_{L1}} = \frac{v_{oc}}{v_1} - 1$$

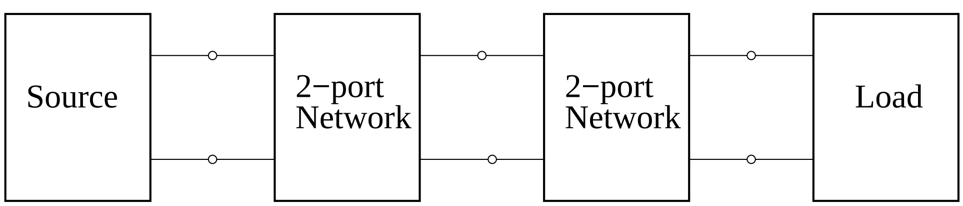
Note that we should choose R_{L1} such that v_1 is sufficiently different from v_{oc} for the measurement to be accurate. Typically, experiment is repeated for several values of R_{L1} until v_1/v_{oc} is between 0.5 to 0.8).

How to find the Thevenin equivalent using PSpice:

You can use the same technique described above for computing the Thevenin parameters with PSpice. Attach a "variable" load ("Parameter" in PSpice) to the circuit. Ask PSpice to compute output voltage V as a function of load resistance R_L. Plot the output current i versus the output voltage v and you will have the iv characteristics of the circuit similar to the figure above (Make sure that you have the current direction correctly!).

1.4 Circuit Components

It is not practical to design a complete circuit as a whole from scratch. It is usually much easier to break the circuit into components and design and analyze each component separately. In this manner we can design "building blocks" (such as amplifiers, filters, *etc.*) that can be used in a variety of devices. A typical analog circuit is composed of a "source," a "load" which is a two-terminal network (devices with two wire coming out) and several "two-port networks" (devices with two wires going in and two wires coming out). Note that these components "communicate" with each other only through the attaching wires, *i.e.,* through currents and voltages.

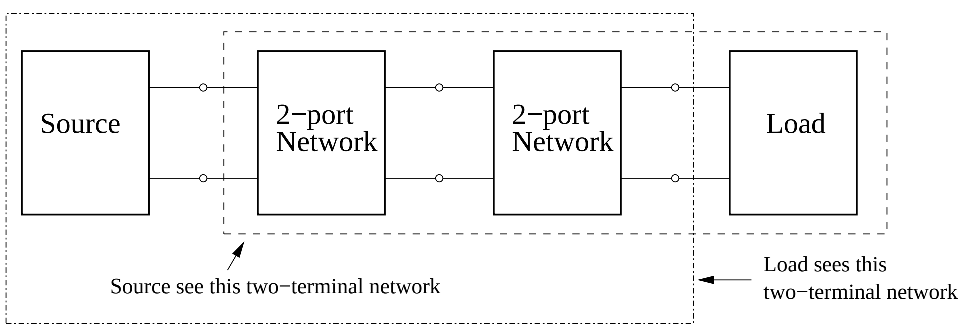


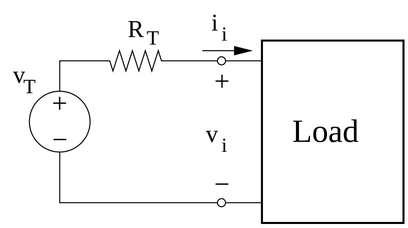
For two-terminal networks, the relationship between output voltage and current dictates how this network behave. If we derive this relationship once for a given two-terminal-network, we can solve any circuit which includes that two-terminal network without solving for internals of the two-terminal network. For two-terminal networks containing only linear elements, Thevenin theorem can be used to model such a network with two fundamental circuit elements. For two-terminal networks with non-linear elements, the relationship between output voltage and current is non-linear (see for example Zener diode power supply of Sec. 2).

In two-port networks the input signal (either input current or input voltage) is modified by the circuit and an output signal (either output current or output voltage) is generated. For most electronic circuits, we keep the currents low and modify voltages (as discussed in Appendix). As such the relationship between output voltage and input voltage dictates the response of the two-port network. This is called the transfer function. For two-port networks with non-linear elements, this is a non-linear relationship (*e.g.,* diode waveform shaping circuits of Sec. 2). If a two-port network includes only linear elements, the network can be modeled by four linear circuit elements (often 3) and its transfer functions is linear (*i.e.,* ratio of v_o/v_i does not depend on v_i). In Sec. 5 we will show that linear amplifiers can be characterized by three parameters and we will use this technique to divide the circuit into components and simplify the analysis considerably. A more general version of this approach is given in the Appendix.

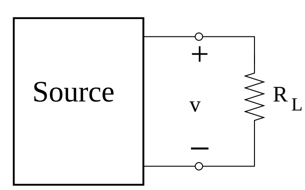
1.4.1 How each sub-circuit sees other elements

The strategy of dividing a "linear" circuit into individual components works because of the Thevenin Theorem. Recall that any two-terminal network can be replaced by its Thevenin equivalent. In addition, if a two-terminal network does not include an "independent source" it will be reduced to a single "impedance" (even if it includes dependent sources).



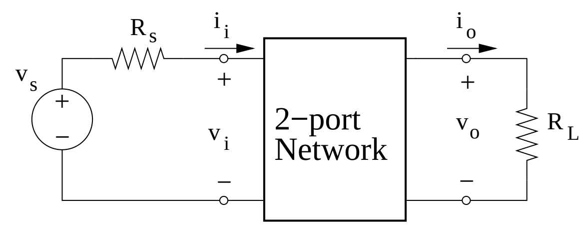


What Load sees: The load sees a two-terminal network. This two-terminal network contains an independent source. So it can be reduced to its Thevenin equivalent.



What Source sees: The source sees a two-terminal network. This two-terminal network does not contain an independent source. So it can be reduced to a single impedance.

What each two-port network sees: Following the logic above, its obvious that each two-port network sees a two-terminal network containing an independent source in the input side (can be reduced to a Thevenin form) and a two-terminal network that does not contain an independent source on the output side (so it can be reduced to a single impedance).



The above observations indicate that we do not need to solve a complete circuit. For two-terminal networks like the source, we only need to find their iv characteristics (or v_T and R_T for linear circuits) to be able to predict its response when it is attached to any circuit (here modeled as R_L). For a two-port network, we only need to solve the circuit above with v_s, R_s, and R_L as parameters. Then, wherever this two-port network appears in a circuit, we can use these results.

1.5 Mathematics versus Engineering

You should have learned by now that one cannot achieve "mathematical" accuracy in practical systems. Firstly, our instruments have a finite accuracy in measuring values. When a number (or measurement), A, has a tolerance of ϵ, it means that its value is between $A(1 \pm \epsilon) = A \pm \epsilon A$. This means that we cannot differentiate between any number in the range $A - \epsilon A$ to $A + \epsilon A$. We would say that all numbers in this range are "approximately equal" to each other:

$$B \approx A \qquad \Leftrightarrow \qquad A - \epsilon A \leq B \leq A + \epsilon A$$

and we can use B and A interchangeably as we cannot distinguish between them.

As an example, the scopes in ECE65 lab are accurate within 2%. So, if the scope ($\epsilon = 0.02$) reads a value of 1.352 V, the "real" value is anywhere between $1.352 \pm 0.02 \times 1.352$ or in the range of 1.325 to 1.379. In this context any number between 1.325 and 1.379 is approximately equal to 1.352 as we CANNOT differentiate among them by our measurements: $1.325 \approx 1.352$ and $1.379 \approx 1.325$.

Corollary: In the example above, the 4th significant digits in 1.352 is totally meaningless (see the range of numbers we cannot distinguish). It is a poor engineering practice to even report this 4th significant digit! (Still some ECE65 students report their calculations to 8th significant digits, directly writing the number from their calculators!). Similarly, it is poor engineering practice to report numbers in whole fractions (*e.g.,* 4/3). No measuring instrument measure any property in whole numbers!

Secondly, each element/component/system is manufactured to a certain tolerance – the smaller the tolerance, the more expensive is to build that component. For example, resistors we will use in the Lab have a tolerance of 5%. This means that a 1 kΩ has a value of $1,000 \pm 5\% = 1,000 \pm 50$ Ω or somewhere between 950 and 1,050 Ω.

A corollary of this concept is that if you designed a circuit and found that you need a 1,010 Ω resistor, you CANNOT put a 1 kΩ and a 10 Ω resistor (with 5% tolerance) in series. The resultant combination would have a value between 959.5 and 1,060.5 Ω which is no better than a 5% 1 kΩ resistor. If you need to have a 1,010 Ω resistor (*i.e.,* more precision), you should use 1% resistors (which are more expensive).

Concepts of infinity and zero are meaningless in abstract. They are used in the context of "much bigger" and "much smaller." For example, in the discussion of page 1-8 of measuring Thevenin parameters, we noted $v_2 = v_{oc} = v_T$ for $R_{L2} \to \infty$, since

$$\frac{v_2}{v_T} = \frac{R_{L2}}{R_T + R_{L2}}$$

However, from the above equation $v_2 \approx v_{oc} = v_T$ when $R_{L2} \gg R_T$. This means that we have defined "infinite" R_{L2} as $R_{L2} \gg R_T$, *i.e.,* with respect to another resistance. For example, if $R_T = 1$ Ω, even a $R_{L2} = 100$ Ω resistance would be infinite, while if $R_T = 1000$ Ω, a 100 Ω load resistance would actually be small.

Similarly, $v_2 = v_T$ for $R_T = 0$, However, from the above equation, $v_2 \approx v_T$ when $R_T \ll R_{L2}$. Again $R_T = 0$ is defined as $R_T \ll R_{L2}$. So, concepts of large and small (zero and infinite) require a frame of reference, *i.e.,* big or small compared to what, and should be stated as "much smaller" or "much bigger" than $\cdots$.

Notions of much smaller ($\ll$) and much greater ($\gg$) are meaningful only in term of a given or needed "relative" tolerance, ϵ. Consider quantity $B = A + a$. We use the concept of much smaller, $a \ll A$, to write $B \approx A$. From the above definition of approximate, we should have (assuming that a and A are positive):

$$B \approx A \qquad \to \qquad A - \epsilon A \leq B \leq A + \epsilon A$$

$$A - \epsilon A \leq A + a \leq A + \epsilon A \qquad \to \qquad a \leq \epsilon A$$

$$a \ll A \qquad \Leftrightarrow \qquad a \leq \epsilon A$$

Exercise: Show that with a tolerance of ϵ, $A \gg a$ means $A \geq (1/\epsilon)a$.

For most day-to-day use, a tolerance of 5% to 10% is more than sufficient. As a general rule, we will use a tolerance of 10% in the analysis in ECE65 unless otherwise stated.