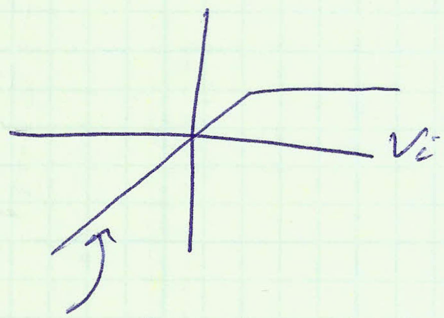
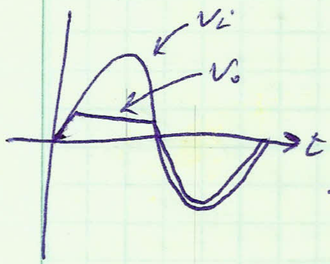
Small Signal Model

- Nonlinear devices behave as linear in small signal limit

Consider clipper circuit:





- circuit obviously behaves linearly here
- If we add 2 input voltages, we get sum at output

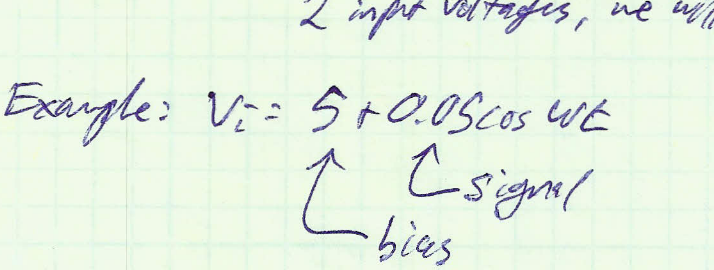
✲ Even in the clipped region if we add 2 input voltages, we will see <u>something</u> at the output

Example: $V_i = 5 + 0.05\cos\omega t$

5 ← bias, $0.05\cos\omega t$ ← signal

- With our piecewise linear model, $V_o = 0.7V$ always
- In reality, $V_o = 0.6921 + 0.0006\cos\omega t$ (simulate it!)
- If we double the input signal, $V_i = 5 + 0.1\cos\omega t$

we get double the output signal: $V_o = 0.6921 + 0.0012\cos\omega t$



The clipper circuit behaves <u>linearly</u> for small signals

- Sine wave in → sine wave out
- proportional to input (double input → double output)
- we could also add other frequency components
- this is why we can build linear amplifiers with nonlinear devices
- devices must always be in <u>one state</u>, e.g. ON, active, etc.
- note that any nonlinear function can be approximated by its <u>tangent line</u> for values close to a given point

We can see this from the Taylor series expansion:

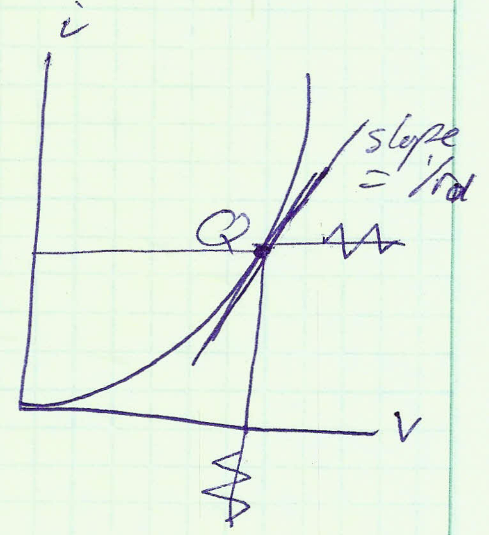
$$f(x_0+\Delta x) = \underline{f(x_0) + \Delta x \frac{df}{dx}\Big|_{x=x_0}} + \frac{(\Delta x)^2}{2!}\frac{d^2 f}{dx^2}\Big|_{x=x_0} + \cdots$$

If Δx is small, $(\Delta x)^n$ is small for $n=2,3,\ldots$

<u>Diode small signal model</u>

$$i_D(V_D+v_d) \simeq i_D(V_D) + v_d \frac{di_D}{dv_D}\Big|_{v_D=V_D}$$

$$= \underset{\substack{\uparrow \\ \text{bias current}}}{I_D} + \underbrace{v_d \frac{di_D}{dv_D}\Big|_{v_D=V_D}}_{\text{response to signal}}$$



Diode response to small signal is like a resistor

$i_d = v_d / r_d$

$r_d \equiv \left(\left. di_D / dv_D \right|_{v_D = V_D} \right)^{-1}$

r_d is inverse of slope of tangent line at bias point

find r_d from full expression for diode current:

$i_D = I_S e^{v_D / nV_T}$

$$\frac{di_D}{dv_D} = \frac{1}{nV_T} \times I_S e^{v_D / nV_T}$$

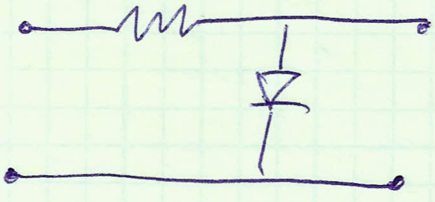
$$\left. \frac{di_D}{dv_D} \right|_{v_D = V_D} = \frac{1}{nV_T} I_S e^{V_D / nV_T} = \frac{I_D}{nV_T}$$

$$r_d = \frac{nV_T}{I_D}$$

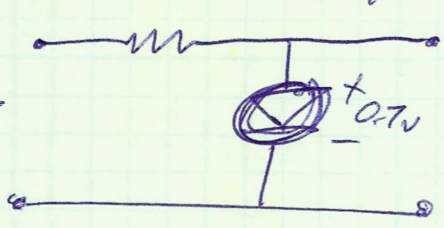
I_D is the bias current

r_d changes with temperature through V_T

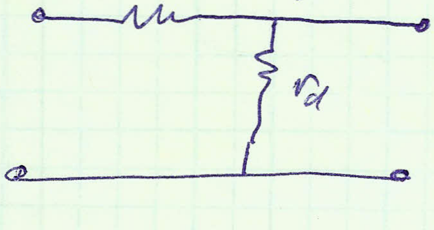
circuit



bias analysis (large sig.)



small signal analysis



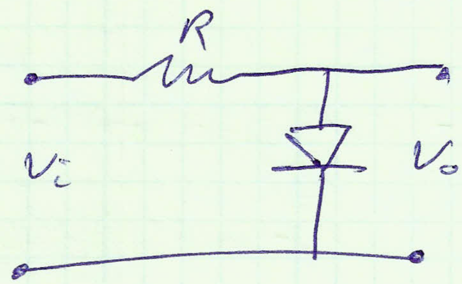
Bias analysis → set signal to zero

$R = 750\,\Omega$, $v_i = 5 + 0.05\cos\omega t \rightarrow v_i = 5$

$v_i = i_D R + v_D$

$5 = 750 I_D + 0.7$

$I_D = 5.73\,mA$



Small signal analysis

$$r_d = \frac{n V_T}{I_D}$$

$n = 2$ for discrete Si diodes

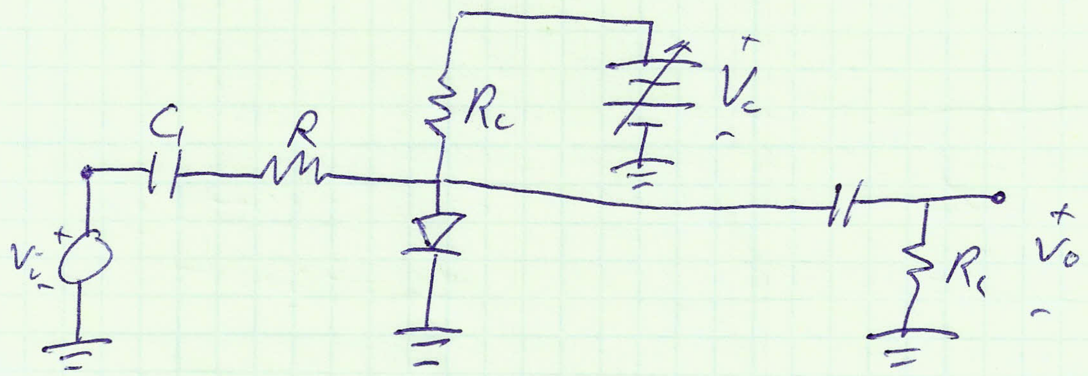
$V_T = 25\,mV$ at $300K$

$$r_d = \frac{2 \times 25 \times 10^{-3}}{5.73 \times 10^{-3}} = 8.73\,\Omega$$

$$v_d = \frac{r_d}{r_d + R} \times 0.05\cos\omega t = 0.00058\cos\omega t$$

$$V_o = 700 + 0.58\cos\omega t \quad (mV)$$

Voltage-controlled attenuator



Bias: $I_D = \dfrac{V_C - V_{D0}}{R_C}$

(note: capacitors are open circuits at DC)

Small signal: define $R_P \equiv R_C \| r_d \| R_L$

these all appear in parallel when $V_C = 0$

(note: capacitors are short circuits for the signal)

$$\frac{v_o}{v_i} = \frac{R_P}{R_P + R}$$

for $r_D \ll R_C$ and $r_D \ll R_L$, $R_P \simeq r_d$

$$\boxed{\frac{v_o}{v_i} = \frac{r_D}{r_D + R} \qquad r_D = \frac{nV_T}{I_D} = \frac{nV_T R_C}{V_C - V_{D0}}}$$

$V_C \uparrow \ldots \dfrac{v_o}{v_i} \downarrow$