MOSFET Small Signal model

- assume always in saturation
- 4 independent params i_G, i_D, v_{GS}, v_{DS} (NMOS)

$i_G(v_{GS}, v_{DS}) = 0$

$i_D(v_{GS}, v_{DS}) = \frac{1}{2} k_n' \frac{W}{L} (v_{GS} - V_T)^2 (1 + \lambda v_{DS})$

Bias point parameters are I_D, V_{GS}, V_{DS} $(I_G = 0)$

$i_D = I_D + i_d$, $v_{GS} = V_{GS} + v_{gs}$, $v_{DS} = V_{DS} + v_{ds}$

Use Taylor series expansion in terms of $\underline{2}$ variables

$$f(x_0 + \Delta x, y_0 + \Delta y) \simeq f(x_0, y_0) + \Delta x \frac{\partial f}{\partial x}\Big|_{x_0, y_0} + \Delta y \frac{\partial f}{\partial y}\Big|_{x_0, y_0}$$

$$i_D(V_{GS} + v_{gs}, V_{DS} + v_{ds}) = i_D(V_{GS}, V_{DS}) + \underbrace{v_{gs} \frac{\partial i_D}{\partial v_{GS}}\Big|_Q + v_{ds} \frac{\partial i_D}{\partial v_{DS}}\Big|_Q}_{\text{signal component}}$$

Define $g_m \equiv \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = 2 \times \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_T)(1 + \lambda V_{DS})\Big|_Q$

$$g_m = \frac{2 I_D}{V_{GS} - V_T}$$

Define $\frac{1}{r_o} \equiv \frac{\partial i_D}{\partial v_{DS}}\Big|_Q = \lambda \frac{1}{2} k_n' \frac{W}{L} (V_{GS} - V_T)^2 \Big|_Q$

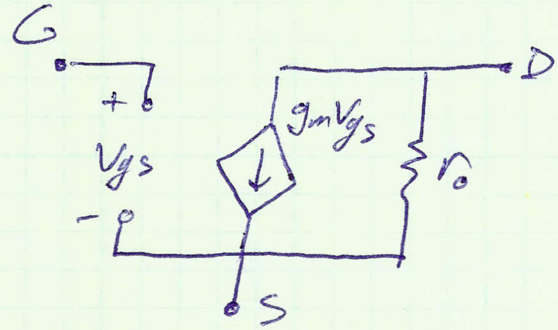
$$\frac{1}{r_o} = \frac{\lambda I_D}{1 + \lambda V_{DS}} \longrightarrow r_o = \frac{V_A + V_{DS}}{I_D} \simeq \frac{V_A}{I_D} \quad \text{where } V_A = \frac{1}{\lambda}$$

MOSFET small signal equations

$i_g = 0$

$i_d = g_m v_{gs} + \dfrac{v_{ds}}{r_o}$

small signal circuit



$$g_m = \frac{2 I_D}{V_{GS} - V_T}$$

$$r_o = \frac{V_A + V_{DS}}{I_D} \simeq \frac{V_A}{I_D}$$

for PMOS

$$g_m = \frac{2 I_D}{V_{SG} - |V_T|}$$

$$r_o = \frac{V_A + V_{SD}}{I_D} \simeq \frac{V_A}{I_D}$$

the circuit looks the same

we do <u>not</u> need to replace v_{gs} with v_{sg}

BJT small signal model

$$i_B = \frac{I_S}{\beta} e^{V_{BE}/nV_T}$$

$$i_C = I_S e^{V_{BE}/nV_T}\left(1 + \frac{V_{CE}}{V_A}\right)$$

- assume active state, bias point parameters I_B, I_C, V_{CE}, V_{BE}

$i_B = I_B + i_b$ $i_C = I_C + i_c$, $v_{BE} = V_{BE} + v_{be}$, $v_{CE} = V_{CE} + v_{ce}$

Use Taylor series expansion

- $i_B(V_{BE} + v_{be}) = i_B(V_{BE}) + v_{be} \frac{\partial i_B}{\partial v_{BE}}\Big|_Q$

$$i_b = \frac{\partial i_B}{\partial v_{BE}}\Big|_Q \cdot v_{be} \equiv \frac{v_{be}}{r_\pi}$$

$$\frac{1}{r_\pi} \equiv \frac{\partial i_B}{\partial v_{BE}}\Big|_Q = \frac{1}{nV_T} \cdot \frac{I_S}{\beta} e^{V_{BE}/nV_T} = \frac{I_B}{nV_T}$$

$$\longrightarrow r_\pi = \frac{nV_T}{I_B}$$

- $i_C(V_{BE} + v_{be}, V_{CE} + v_{ce}) = i_C(V_{BE}, V_{CE}) + v_{be} \frac{\partial i_C}{\partial v_{BE}}\Big|_Q + v_{ce} \frac{\partial i_C}{\partial v_{CE}}\Big|_Q$

$$i_c = v_{be} \frac{\partial i_C}{\partial v_{BE}}\Big|_Q + v_{ce} \frac{\partial i_C}{\partial v_{CE}}\Big|_Q \equiv g_m v_{be} + \frac{v_{ce}}{r_o}$$

$$g_m \equiv \frac{\partial i_C}{\partial v_{be}}\Big|_Q = \frac{1}{nV_T} I_S e^{V_{BE}/nV_T}\left(1 + \frac{V_{CE}}{V_A}\right) = \frac{I_C}{nV_T}$$

$$\frac{1}{r_o} \equiv \frac{\partial i_C}{\partial v_{CE}}\Big|_Q = \frac{1}{V_A} I_S e^{V_{BE}/nV_T} = \frac{I_C}{V_A(1 + V_{CE}/V_A)} = \frac{I_C}{V_A + V_{CE}}$$

BJT small signal equations

$$i_b = \frac{v_{be}}{r_\pi}$$

$$i_c = g_m v_{be} + v_{ce}/r_o$$

small signal circuit

$$r_\pi = \frac{nV_T}{I_B}$$

$$g_m = \frac{I_C}{nV_T} = \frac{I_C}{I_B} \cdot \frac{I_B}{nV_T} = \frac{\beta}{r_\pi}$$

$$r_o = \frac{V_A + V_{CE}}{I_C} \simeq \frac{V_A}{I_C}$$

PNP is the same, but with V_{EC} instead of V_{CE}