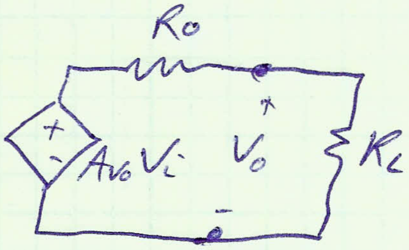
Low-Frequency Response



$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_o + R_L} A_{v_o}$$

capital Vs are <u>phasers</u>

$$\frac{V_i}{V_{sig}} = \frac{R_i}{R_i + R_{sig} + \frac{1}{sC_1}}$$

$\underline{s = j\omega}$

- this will have the form of constant-frequency voltage divider × frequency response

$$\frac{V_i}{V_{sig}} = \underbrace{\frac{R_i}{R_i + R_{sig}}} \times \frac{s}{s + \omega_{p1}}$$

could set $C \to \infty$ to find this part, then use that to find ω_{p1}

$$\frac{V_i}{V_{sig}} = \underbrace{\frac{R_i}{R_i + R_{sig} + \frac{1}{sC_1}} \cdot \frac{R_i + R_{sig}}{R_i}} \cdot \frac{R_i}{R_i + R_{sig}}$$

$$= \frac{R_i + R_{sig}}{R_i + R_{sig} + \frac{1}{sC_1}} = \frac{s}{s + \frac{1}{C_1(R_i + R_{sig})}} = \frac{s}{s + \omega_{p1}}$$

$$\omega_{p1} = 2\pi f_{p1} = \frac{1}{C_1(R_i + R_{sig})}$$

$$\frac{V_o}{V_{sig}} = \frac{R_i}{R_i + R_{sig}} \times A_v \times \frac{s}{s + \omega_{p1}}$$

More general approach for finding poles:

(have the students taken a linear systems or control class yet?)

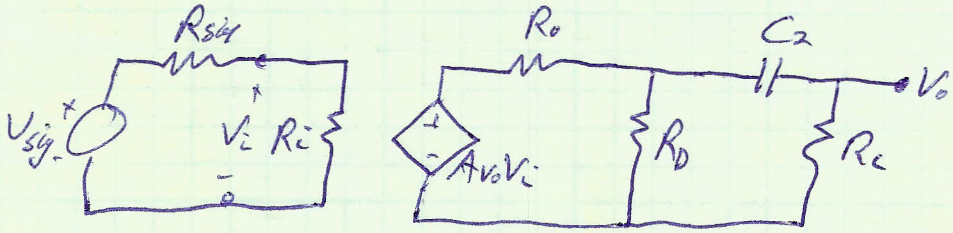
- For transfer function $\frac{V_o}{V_i} = \frac{\text{polynomial in } s}{\text{other polynomial in } s}$
- we can find poles and zeros

  zeros: let numerator = 0, solve for s

  poles: let denominator = 0, solve for s
- Our circuits will not have any zeros, but have <u>one pole</u> for each <u>capacitor</u>

Pole from capacitor at output

example: common drain amplifier



$$\frac{V_o}{V_{sig}} = \frac{R_i}{R_i + R_{sig}} \times A_v \times \frac{s}{s + \omega_{p2}}$$

$$\omega_{p2} = 2\pi f_{p2} = \frac{1}{C_2\left(R_L + R_D \,||\, R_o\right)}$$

for <u>unilateral</u> amplifiers, this is the correct value of f_{p2} from C_2

However if <u>not unilateral</u> (R_i depends on R_L) then R_i will include C_2

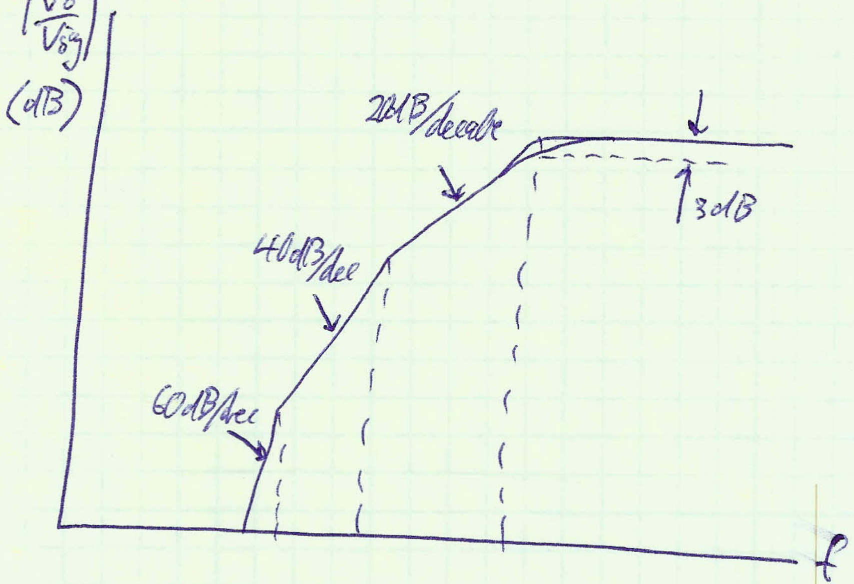
- note $R_L' = R_D \,||\, \left(R_L + \frac{1}{sC_2}\right)$

How to find poles by inspection:

1. Zero out V_{sig}
2. Consider each capacitor separately (others are shorts)
3. Compute the total effective resistance between capacitor terminals
   the pole is given by $f_P = \frac{1}{2\pi CR}$

- the total response is:

$$\frac{V_o}{V_{sig}} = A_V \times \frac{s}{s+\omega_{p1}} \times \frac{s}{s+\omega_{p2}} \times \frac{s}{s+\omega_{p3}} \times \cdots$$



- If the poles are very far apart, then low frequency cutoff is <u>highest</u> pole
- If they are nearby, $f_L \simeq f_{P1} + f_{P2} + f_{P3} + \ldots$

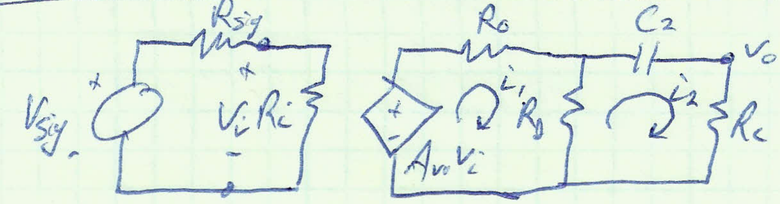
Note: Output resistance is R_o (output resistance with $V_i = 0$)

R_{out} is output resistance with $V_{sig} = 0$

pole for C_2 includes $\frac{R_i}{R_i + R_{sig}}$ term when <u>not unilateral</u>

same formulas can be used, but replace R_o with R_{out}

compare hard way and easy way to find f_p



$$A_{vo}V_i - R_o i_1 - R_D(i_1 - i_2) = 0$$

$$+R_D(i_2 - i_1) + \frac{1}{sC_2} i_2 + R_L i_2 = 0$$

$$i_2\left(R_D + \frac{1}{sC_2} + R_L\right) = i_1 R_D$$

$$i_1 = i_2 \frac{R_D + \frac{1}{sC_2} + R_L}{R_D}$$

$$A_{vo}V_i - i_1(R_o + R_D) + i_2 R_D = 0$$

$$A_{vo}V_i - i_2 \frac{(R_o+R_D)(R_D + \frac{1}{sC_2} + R_L)}{R_D} + i_2 R_D = 0$$

$$A_{vo}V_i = i_2\left[\frac{(R_o+R_D)(R_D + \frac{1}{sC_2} + R_L)}{R_D} - R_D\right]$$

note $V_o = i_2 R_L$

$\rightarrow i_2 = \frac{V_o}{R_L}$

$$A_{vo}V_i = \frac{V_o}{R_L}\left[\quad\right]$$

$$A_{vo}\frac{V_i}{V_o} = \frac{1}{R_L}\left[\quad\right] \rightarrow 0$$ to find pole. Solve for s

$$(R_o+R_D)\left(R_D + \frac{1}{sC_2} + R_L\right) = R_D^2$$

$$R_oR_D + \cancel{R_D^2} + (R_o+R_D)\left(\frac{1}{sC_2} + R_L\right) = \cancel{R_D^2}$$

$$(R_o+R_D)\left(\frac{1}{sC_2} + R_L\right) = -R_oR_D$$

$$\frac{1}{sC_2} + R_L = \frac{-R_oR_D}{R_o+R_D}$$

$$\frac{1}{sC_2} = \frac{-R_oR_D}{R_o+R_D} - R_L$$

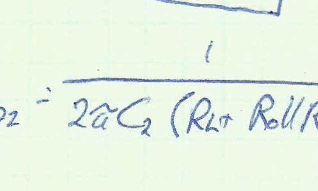
$$s = \frac{-1}{C_2(R_L + R_o \| R_D)}$$

$$f_{p2} = \frac{1}{2\pi C_2(R_L + R_o \| R_D)}$$

Now, by inspection:

Zero at $V_{sig} \rightarrow V_i = 0$ too

then $A_{vo}V_i = 0$



$$f_{p2} = \frac{1}{2\pi C_2(R_L + R_o \| R_D)}$$