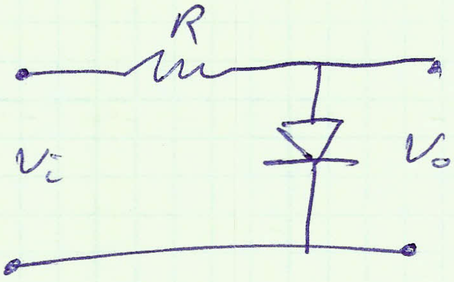
<u>Bias analysis</u> $\rightarrow$ set signal to zero

$R = 750\,\Omega$, $v_i = 5 + 0.05\cos\omega t \rightarrow v_i = 5$

$v_i = i_D R + V_D$



$5 = 750\,I_D + 0.7$

$I_D = 5.73\,mA$

<u>Small signal analysis</u>

$$r_d = \frac{nV_T}{I_D}$$

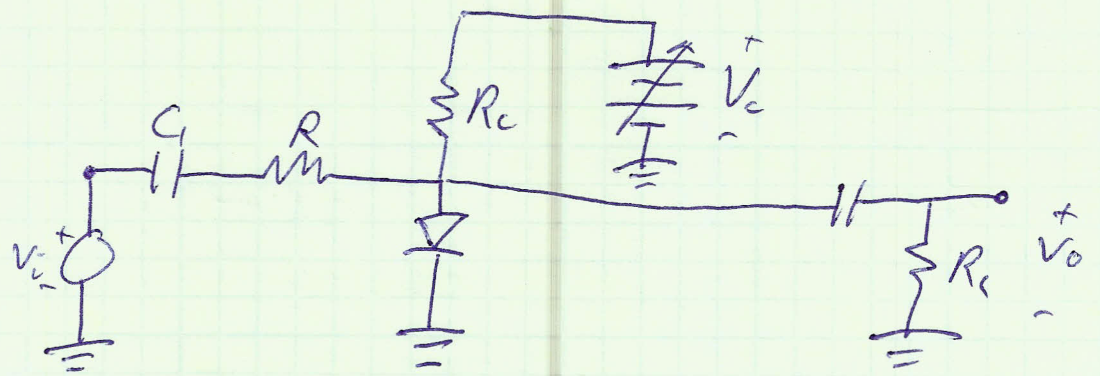
$n = 2$ for discrete Si diodes
$V_T = 25\,mV$ at $300K$

$$r_d = \frac{2\times 25\times 10^{-3}}{5.73\times 10^{-3}} = 8.73\,\Omega$$

$$v_d = \frac{r_d}{r_d + R}\times 0.05\cos\omega t = 0.00058\cos\omega t$$

$$V_o = 700 + 0.58\cos\omega t \quad (mV)$$

Voltage-controlled attenuator



Bias: $I_D = \dfrac{V_C - V_{D0}}{R_C}$

(note: capacitors are <u>open</u> circuits at DC)

Small signal: define $R_P \equiv R_C \| r_d \| R_L$

these all appear in parallel when $V_C = 0$

(note: capacitors are <u>short</u> circuits for the signal)

$$\frac{v_o}{v_i} = \frac{R_P}{R_P + R}$$

for $r_D \ll R_C$ and $r_D \ll R_L$, $R_P \simeq r_d$

$$\boxed{\begin{aligned} \frac{v_o}{v_i} &= \frac{r_D}{r_D + R} \\ r_D &= \frac{n V_T}{I_D} = \frac{n V_T R_C}{V_C - V_{D0}} \end{aligned}}$$

$V_C \uparrow \ldots \dfrac{v_o}{v_i} \downarrow$

MOSFET Small Signal model

- assume always in saturation
- 4 independent parameters i_G, i_D, v_{GS}, v_{DS} (NMOS)

$i_G(v_{GS}, v_{DS}) = 0$

$i_D(v_{GS}, v_{DS}) = \frac{1}{2} k_n' \frac{W}{L}(v_{GS} - V_T)^2 (1 + \lambda v_{DS})$

Bias point parameters are I_D, V_{GS}, V_{DS} $(I_G = 0)$

$i_D = I_D + i_d$, $v_{GS} = V_{GS} + v_{gs}$, $v_{DS} = V_{DS} + v_{ds}$

Use Taylor series expansion in terms of $\underline{2}$ variables

$$f(x_0 + \Delta x, y_0 + \Delta y) \simeq f(x_0, y_0) + \Delta x \frac{\partial f}{\partial x}\Big|_{x_0, y_0} + \Delta y \frac{\partial f}{\partial y}\Big|_{x_0, y_0}$$

$$i_D(V_{GS} + v_{gs}, V_{DS} + v_{ds}) = i_D(V_{GS}, V_{DS}) + \underbrace{v_{gs} \frac{\partial i_D}{\partial v_{GS}}\Big|_Q + v_{ds} \frac{\partial i_D}{\partial v_{DS}}\Big|_Q}_{\text{signal component}}$$

Define $g_m \equiv \frac{\partial i_D}{\partial v_{GS}}\Big|_Q = 2 \times \frac{1}{2} k_n' \frac{W}{L}(v_{GS} - V_T)(1 + \lambda v_{DS})\Big|_Q$

$$g_m = \frac{2 I_D}{V_{GS} - V_T}$$

Define $\frac{1}{r_o} \equiv \frac{\partial i_D}{\partial V_{DS}}\Big|_Q = \lambda \frac{1}{2} k_n' \frac{W}{L}(v_{GS} - V_T)^2\Big|_Q$

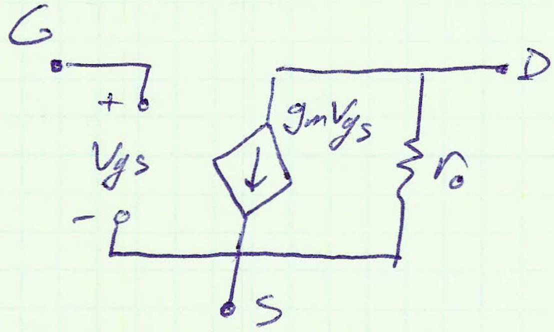
$$\frac{1}{r_o} = \frac{\lambda I_D}{1 + \lambda V_{DS}} \longrightarrow r_o = \frac{V_A + V_{DS}}{I_D} \simeq \frac{V_A}{I_D} \quad \text{where } V_A = \frac{1}{\lambda}$$

MOSFET small signal equations

$i_g = 0$

$i_d = g_m v_{gs} + \frac{v_{ds}}{r_o}$

small signal circuit



$$g_m = \frac{2 I_D}{V_{GS} - V_T}$$

$$r_o = \frac{V_A + V_{DS}}{I_D} \simeq \frac{V_A}{I_D}$$

for PMOS

$$g_m = \frac{2 I_D}{V_{SG} - |V_T|}$$

$$r_o = \frac{V_A + V_{SD}}{I_D} \simeq \frac{V_A}{I_D}$$

the circuit looks the same

we do <u>not</u> need to replace v_{gs} with v_{sg}

BJT small signal model

$$i_B = \frac{I_s}{\beta} e^{v_{BE}/nV_T}$$

$$i_C = I_s e^{v_{BE}/nV_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

- assume active state, bias point parameters I_B, I_C, V_{CE}, V_{BE}

$i_B = I_B + i_b \quad i_C = I_c + i_c \quad , \quad v_{BE} = V_{BE} + v_{be} \quad , \quad v_{CE} = V_{CE} + v_{ce}$

Use Taylor series expansion

- $$i_B(V_{BE} + v_{be}) = i_B(V_{BE}) + v_{be} \frac{\partial i_B}{\partial v_{BE}}\Big|_Q$$

$$i_b = \frac{\partial i_B}{\partial v_{BE}}\Big|_Q \cdot v_{be} \equiv \frac{v_{be}}{r_\pi}$$

$$\frac{1}{r_\pi} \equiv \frac{\partial i_B}{\partial v_{BE}}\Big|_Q = \frac{1}{nV_T} \cdot \frac{I_s}{\beta} e^{V_{BE}/nV_T} = \frac{I_B}{nV_T}$$

$$\longrightarrow r_\pi = \frac{nV_T}{I_B}$$

- $$i_C(V_{BE} + v_{be}, V_{CE} + v_{ce}) = i_C(V_{BE}, V_{CE}) + v_{be} \frac{\partial i_C}{\partial v_{BE}}\Big|_Q + v_{ce} \frac{\partial i_C}{\partial v_{CE}}\Big|_Q$$

$$i_c = v_{be} \frac{\partial i_C}{\partial v_{BE}}\Big|_Q + v_{ce} \frac{\partial i_C}{\partial v_{CE}}\Big|_Q \equiv g_m v_{be} + \frac{v_{ce}}{r_o}$$

$$g_m \equiv \frac{\partial i_C}{\partial v_{be}}\Big|_Q = \frac{1}{nV_T} I_s e^{V_{BE}/nV_T}\left(1 + \frac{V_{CE}}{V_A}\right) = \frac{I_C}{nV_T}$$

$$\frac{1}{r_o} \equiv \frac{\partial i_C}{\partial v_{CE}}\Big|_Q = \frac{1}{V_A} I_s e^{V_{BE}/nV_T} = \frac{I_C}{V_A(1 + V_{CE}/V_A)} = \frac{I_C}{V_A + V_{CE}}$$

BJT small signal equations

$$i_b = \frac{v_{be}}{r_\pi}$$

$$i_c = g_m v_{be} + v_{ce}/r_o$$

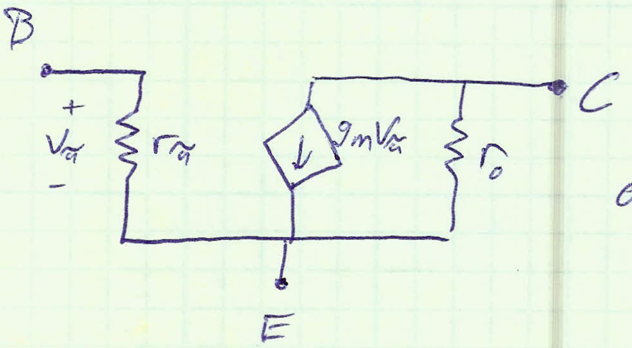
small signal circuit

$$r_\pi = \frac{nV_T}{I_B}$$

$$g_m = \frac{I_c}{nV_T} = \frac{I_c}{I_B} \cdot \frac{I_B}{nV_T} = \frac{\beta}{r_\pi}$$

$$r_o = \frac{V_A + V_{CE}}{I_c} \simeq \frac{V_A}{I_c}$$

PNP is the same, but with V_{EC} instead of V_{CE}



or