Summary

Common Source, Common Emitter

- High gain
- CS has infinite R_i, CE has modest R_i (ignoring R_G, R_B)
- High output resistance r_o
- Limited high frequency response (will learn in 102)
- Main configuration for practical amplifier

Add source or emitter resistor

- gain less sensitive to temperature
- higher input resistance for CE → add $(1+\beta)R_E$
- lower gain

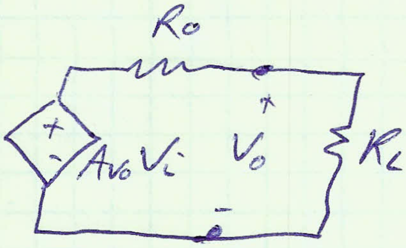
Common gate, common base

- high gain
- low input resistance → only for specialized applications
- typically used in combination with CS or CE (e.g. cascode amplifiers)

Source follower, emitter follower

- high input resistance
- low output resistance
- gain $\simeq 1$
- Used as buffer, or as output stage to increase current and power to load

Low-Frequency Response



$$A_v = \frac{V_o}{V_i} = \frac{R_L}{R_o + R_L} A_{vo}$$

capital Vs are <u>phasors</u>

$$\frac{V_i}{V_{sig}} = \frac{R_i}{R_i + R_{sig} + \frac{1}{sC_1}}$$

$s = j\omega$

- this will have the form of constant-frequency voltage divider × frequency response

$$\frac{V_i}{V_{sig}} = \underbrace{\frac{R_i}{R_i + R_{sig}}}_{\text{could set } C \to \infty \text{ to find this part, then use that to find } \omega_{p1}} \times \frac{s}{s + \omega_{p1}}$$

$$\frac{V_i}{V_{sig}} = \underbrace{\frac{R_i}{R_i + R_{sig} + \frac{1}{sC_1}} \cdot \frac{R_i + R_{sig}}{R_i}}_{} \cdot \frac{R_i}{R_i + R_{sig}}$$

$$= \frac{R_i + R_{sig}}{R_i + R_{sig} + \frac{1}{sC_1}} = \frac{s}{s + \frac{1}{C_1(R_i + R_{sig})}} = \frac{s}{s + \omega_{p1}}$$

$$\omega_{p1} = 2\pi f_{p1} = \frac{1}{C_1(R_i + R_{sig})}$$

$$\frac{V_o}{V_{sig}} = \frac{R_i}{R_i + R_{sig}} \times A_v \times \frac{s}{s + \omega_{p1}}$$

More general approach for finding poles:

(have the students taken a linear systems or control class yet?)

- For transfer function $\frac{V_o}{V_i} = \frac{\text{polynomial in } s}{\text{other polynomial in } s}$

- we can find poles and zeros

  zeros: let numerator = 0, solve for s

  poles: let denominator = 0, solve for s

- Our circuits will not have any zeros, but have one pole for each capacitor

Pole from capacitor at output

example: common drain amplifier

$$\frac{V_o}{V_{sig}} = \frac{R_i}{R_i + R_{sig}} \times A_v \times \frac{s}{s + \omega_{p2}}$$

$$\omega_{p2} = 2\pi f_{p2} = \frac{1}{C_2 (R_L + R_D \| R_o)}$$

for unilateral amplifiers, this is the correct value of f_{p2} from C_2

However if not unilateral (R_i depends on R_L) then R_i will include C_2

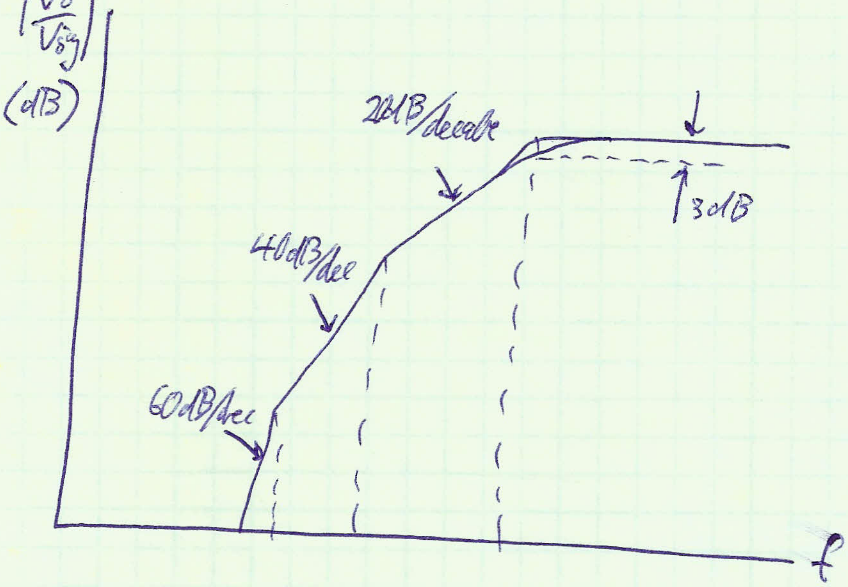
- note $R_L' = R_D \| (R_L + \frac{1}{sC_2})$

How to find poles by inspection:

1. Zero out V_{sig}
2. Consider each capacitor separately (others are shorts)
3. Compute the total effective resistance between capacitor terminals the pole is given by $f_p = \frac{1}{2\pi CR}$

- the total response is:

$$\frac{V_o}{V_{sig}} = A_V \times \frac{s}{s+\omega_{p1}} \times \frac{s}{s+\omega_{p2}} \times \frac{s}{s+\omega_{p3}} \times \cdots$$



- If the poles are very far apart, then low frequency cutoff is <u>highest</u> pole
- If they are nearby, $f_L \simeq f_{p1} + f_{p2} + f_{p3} + \ldots$

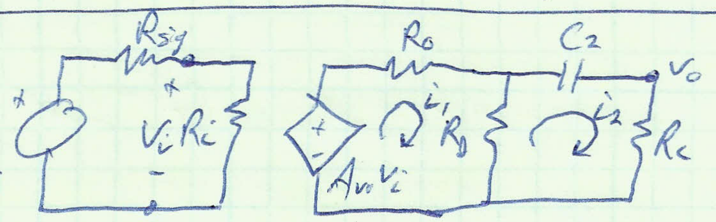
Note: Output resistance is R_o (output resistance with $V_i = 0$)

R_{out} is output resistance with $V_{sig} = 0$

pole for C_2 includes $\frac{R_i}{R_i + R_{sig}}$ term when <u>not unilateral</u>

same formulas can be used, but replace R_o with R_{out}

compare hard way and easy way to find f_p



$$A_{vo}V_i - R_o i_1 - R_D(i_1 - i_2) = 0$$

$$A_{vo}V_i - i_1(R_o + R_D) + i_2 R_D = 0$$

$$A_{vo}V_i - i_2\frac{(R_o+R_D)(R_D + \frac{1}{sC_2} + R_L)}{R_D} + i_2 R_D = 0$$

$$A_{vo}V_i = i_2\left[\frac{(R_o+R_D)(R_D+\frac{1}{sC_2}+R_L)}{R_D} - R_D\right]$$

$$+R_D(i_2 - i_1) + \frac{1}{sC_2}i_2 + R_L i_2 = 0$$

$$i_2\left(R_D + \frac{1}{sC_2} + R_L\right) = i_1 R_D$$

$$i_1 = i_2\,\frac{R_D + \frac{1}{sC_2} + R_L}{R_D}$$

note $V_o = i_2 R_L$

$\rightarrow i_2 = \frac{V_o}{R_L}$

$$A_{v_o}V_i = \frac{V_o}{R_L}[\quad]$$

$$A_{vo}\frac{V_i}{V_o} = \frac{1}{R_L}[\quad] \rightarrow 0$$ to find pole. Solve for S

$$(R_o+R_D)\left(R_D + \frac{1}{sC_2} + R_L\right) = R_D^{\,2}$$

$$R_oR_D + \cancel{R_D^2} + (R_o+R_D)\left(\frac{1}{sC_2} + R_L\right) = \cancel{R_D^2}$$

$$(R_o+R_D)\left(\frac{1}{sC_2}+R_L\right) = -R_oR_D$$

$$\frac{1}{sC_2} + R_L = \frac{-R_oR_D}{R_o+R_D}$$

$$\frac{1}{sC_2} = \frac{-R_oR_D}{R_o+R_D} - R_L$$

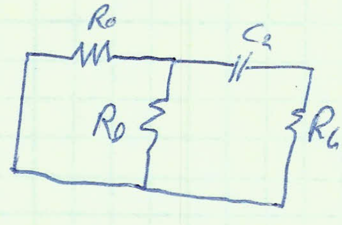
$$s = \frac{-1}{C_2(R_L + R_o \| R_D)}$$

$$f_{P2} = \frac{1}{2\pi C_2(R_L + R_o\|R_D)}$$

Now, by inspection:

Zero out $V_{sig} \rightarrow V_i = 0$ too

then $A_{vo}V_i = 0$



$$f_{p2} = \frac{1}{2\pi C_2(R_L + R_o\|R_D)}$$

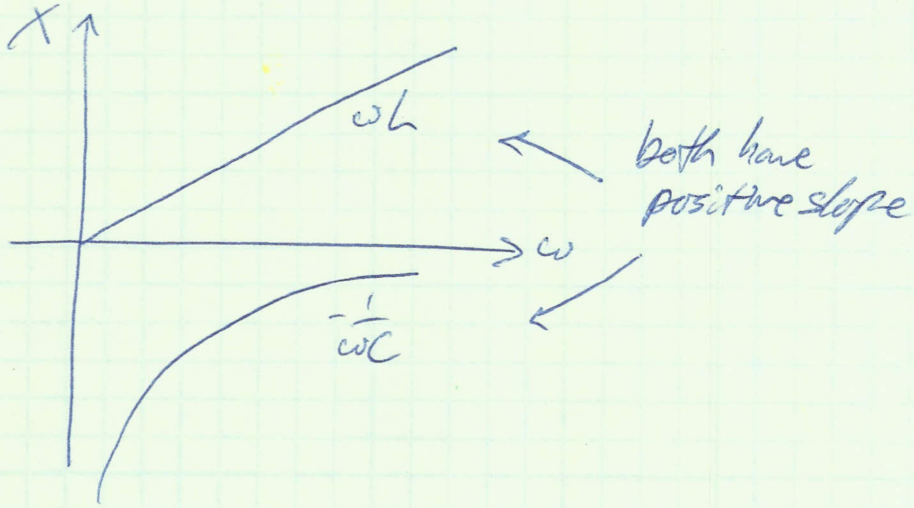
Special topics

Non-Foster circuits

Foster's theorem: reactance of any passive network is always <u>increasing</u> with frequency

Examples: capacitor or inductor

$$X_C = -\frac{1}{\omega C}, \quad X_L = \omega L$$

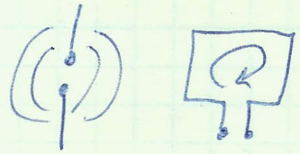


Why is this important?

1. Matching circuits.
2. propagating faster than the speed of light.

Matching small antennas

Small antennas behave as a capacitor or inductor with a small radiation resistance



oscillating currents cause radiation

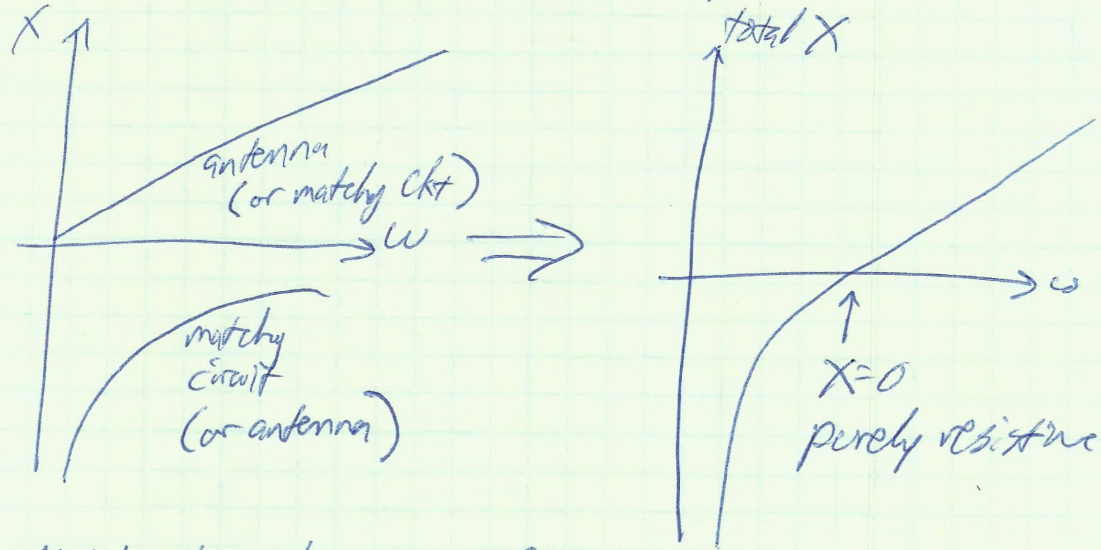
$P_{rad} = I^2 R_{rad}$

dipole   loop

C   L

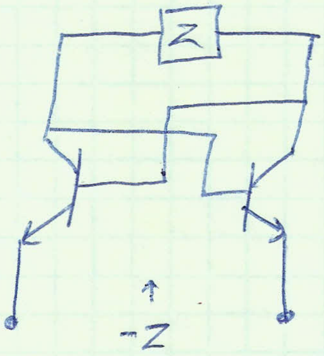
Can model as  or 

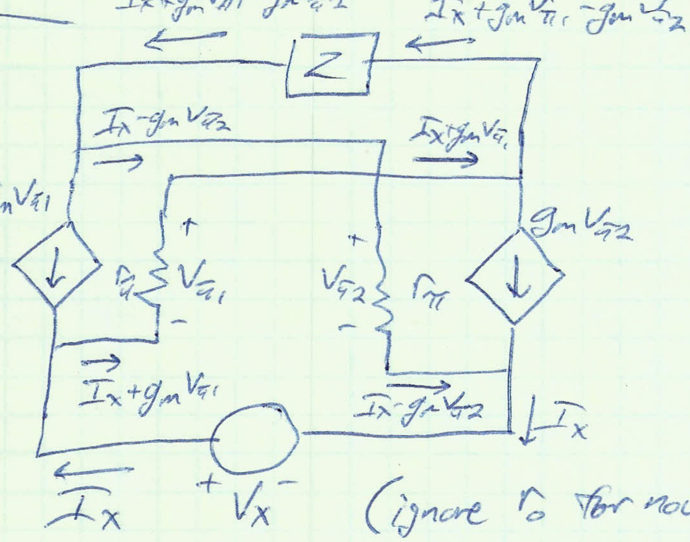
Inductive antenna - match with capacitor



- Match only works at one frequency → narrow band
- Could match over broadband if we have negative capacitors, or negative inductors

Negative impedance converter





(ignore r_o for now)

$$V_x - r_\pi(I_x + g_m V_{\pi1}) - Z(I_x + g_m V_{\pi1} - g_m V_{\pi2}) - r_\pi(I_x - g_m V_{\pi2}) = 0$$

$$V_x - r_\pi(2I_x + g_m V_{\pi1} - g_m V_{\pi2}) - Z(I_x + g_m V_{\pi1} - g_m V_{\pi2}) = 0$$

$$V_{\pi1} = -(I_x + g_m V_{\pi1}) r_\pi$$

$$-\frac{V_{\pi1}}{r_\pi} = I_x + g_m V_{\pi1}$$

$$V_{\pi1}\left(-\frac{1}{r_\pi} - g_m\right) = I_x$$

$$V_{\pi1} = \frac{-I_x}{\frac{1}{r_\pi} + g_m}$$

$$V_{\pi2} = (I_x - g_m V_{\pi2}) r_\pi$$

$$\frac{V_{\pi2}}{r_\pi} = I_x - g_m V_{\pi2}$$

$$V_{\pi2}\left(\frac{1}{r_\pi} + g_m\right) = I_x$$

$$V_{\pi2} = \frac{I_x}{\frac{1}{r_\pi} + g_m}$$

$$V_x - r_\pi\left(2I_x + g_m(V_{\pi1} - V_{\pi2})\right) - Z\left(I_x + g_m(V_{\pi1} - V_{\pi2})\right) = 0$$

$$V_{\pi1} - V_{\pi2} = \frac{-2I_x}{\frac{1}{r_\pi} + g_m}$$

$$V_x - r_\pi\left(2I_x - g_m\frac{2I_x}{\frac{1}{r_\pi}+g_m}\right) - Z\left(I_x - \underbrace{g_m\frac{2I_x}{\frac{1}{r_\pi}+g_m}}\right) = 0$$

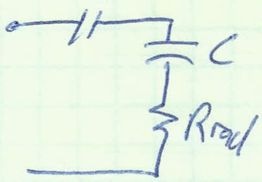
$$\hookrightarrow = 2I_x \cdot \frac{g_m r_\pi}{1 + g_m r_\pi} = 2I_x\frac{\beta}{\beta+1} \approx 2I_x$$

$$V_x - r_\pi(\cancel{2I_x - 2I_x}) - Z(I_x - 2I_x) = 0$$

$$V_x + Z I_x = 0$$

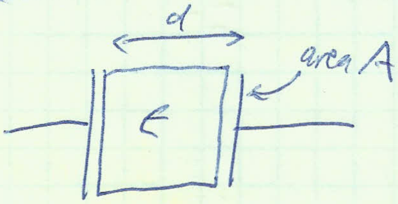
$$V_x / I_x = -Z$$

Antenna matched with non-Foster circuit



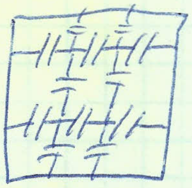
breadband match

Superluminal propagation



measure capacitance

$$C = \frac{\epsilon A}{d}$$



insert negative capacitors

overall $\epsilon < 1 \rightarrow n = \sqrt{\epsilon} < 1$

waves (but not information) can go faster than c