Regeneration of turbulent fluctuations in low Froude number flow over a sphere at Re = 3700.

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Direct numerical simulations (DNS) are performed to study the behavior of flow past a sphere in the regime of high stratification (low Froude number Fr). In contrast to previous results at lower Re that suggest monotone suppression of turbulence with increasing stratification in flow past a sphere, it is found that, below a critical Fr, increasing the stratification induces unsteady vortical motion and turbulent fluctuations in the near wake. The near wake is quantified by computing the energy spectra, the turbulence energy equation, partition of energy into horizontal and vertical components, and the buoyancy Reynolds number. These diagnostics show that the stabilizing effect of buoyancy changes flow over the sphere to flow around the sphere. This qualitative change in the flow leads to a new regime of unsteady vortex shedding in the horizontal planes that results in turbulence regeneration.

Key words:
covered a wide range of $Fr$ and $Re$, but the low $Fr$ cases had low $Re$ as well. The near wake was classified in four regimes (Chomaz et al. 1993a) depending on Froude number, including the quasi 2-D regime that occurred for the lowest examined values of $Fr \in \{0.125, 0.4\}$. A recent DNS (Orr et al. 2015) included $Fr < 1$ cases but at low $Re = 200$. None of these prior studies report turbulence in the low $Fr$ regime. It has been suggested (Chomaz et al. 1993a) that the effect of $Re$ is weak when $Fr < 0.35$ as long as $Re$ exceeds 100. On the other hand, quasi-2D motion in strongly stratified flow can be turbulent when the Reynolds number is large as found for Taylor-Green vortices (Riley & deBruynKops 2003), homogeneous turbulence (Lindborg 2006; Brethouwer et al. 2007) and a far wake (Diamessis et al. 2011). The nonequilibrium region of the far wake is also lengthened for large $Re$ (Brucker & Sarkar 2010).

2. Problem formulation, numerical details and validation

Motivated by the unanswered question regarding near-wake turbulence when $Fr$ is low but $Re$ is not, we use DNS to investigate the flow past a sphere at $Re = 3,700$ and $Fr \in \{0.025, 1\}$. The three-dimensional Navier Stokes equations are solved on a cylindrical coordinate system on a staggered grid using an immersed boundary method (IBM) (Balaras 2004; Yang & Balaras 2006) for representing the sphere.

The simulation parameters, domain size and grid distribution for the different cases are given in table 1. High resolution is used at the sphere surface (20 points across the boundary layer thickness at the point of maximum wall shear stress) and in the wake. The radial grid spacing is $\Delta r \approx 0.0016$ in the cylindrical region ($r < 0.65$) that encloses the sphere, the azimuthal direction has 128 points, and $\Delta x \approx 0.0016$ near the surface. The grid has mild stretching, radially and streamwise, away from the body. The IBM results and the grid resolution to resolve the flow have been successfully validated in the unstratified case against both previous simulations and laboratory experiments. Figures 1 (a)-(b) show that the variation of the surface pressure coefficient, $C_p$, and the surface shear stress ($\tau/\rho U^2 Re^{0.5}$), as a function of azimuthal angle, matches well with results in the available literature. Table 2 shows that key characteristics of the near-body flow such as Strouhal number of the dominant shedding frequency, separation angle, coefficient of drag ($C_d$) and pressure coefficient ($C_{pb}$) at the rearward stagnation point also match with previously reported values.

<table>
<thead>
<tr>
<th>Case</th>
<th>$Re$</th>
<th>$Fr$</th>
<th>$L_r$</th>
<th>$L_\theta$</th>
<th>$L_z$</th>
<th>$N_r$</th>
<th>$N_\theta$</th>
<th>$N_z$</th>
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<tr>
<td>1.</td>
<td>3700</td>
<td>0.025</td>
<td>58</td>
<td>$2\pi$</td>
<td>63 (40 upstream; 23 downstream)</td>
<td>690</td>
<td>128</td>
<td>3072</td>
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<tr>
<td>2.</td>
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<td>0.05</td>
<td>58</td>
<td>$2\pi$</td>
<td>63 (40 upstream; 23 downstream)</td>
<td>690</td>
<td>128</td>
<td>3072</td>
</tr>
<tr>
<td>3.</td>
<td>3700</td>
<td>0.125</td>
<td>58</td>
<td>$2\pi$</td>
<td>120 (40 upstream; 80 downstream)</td>
<td>690</td>
<td>128</td>
<td>4608</td>
</tr>
<tr>
<td>4.</td>
<td>3700</td>
<td>0.17</td>
<td>58</td>
<td>$2\pi$</td>
<td>56 (40 upstream; 16 downstream)</td>
<td>690</td>
<td>128</td>
<td>2560</td>
</tr>
<tr>
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<td>$2\pi$</td>
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<td>690</td>
<td>128</td>
<td>2560</td>
</tr>
<tr>
<td>6.</td>
<td>3700</td>
<td>0.25</td>
<td>58</td>
<td>$2\pi$</td>
<td>120 (40 upstream; 80 downstream)</td>
<td>690</td>
<td>128</td>
<td>4608</td>
</tr>
<tr>
<td>7.</td>
<td>3700</td>
<td>0.5</td>
<td>58</td>
<td>$2\pi$</td>
<td>120 (40 upstream; 16 downstream)</td>
<td>690</td>
<td>128</td>
<td>4608</td>
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<tr>
<td>8.</td>
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<td>0.8</td>
<td>58</td>
<td>$2\pi$</td>
<td>103 (25 upstream; 80 downstream)</td>
<td>690</td>
<td>128</td>
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</tr>
<tr>
<td>9.</td>
<td>3700</td>
<td>1.0</td>
<td>58</td>
<td>$2\pi$</td>
<td>95(13 upstream; 80 downstream)</td>
<td>630</td>
<td>128</td>
<td>4608</td>
</tr>
<tr>
<td>10.</td>
<td>3700</td>
<td>$\infty$</td>
<td>16</td>
<td>$2\pi$</td>
<td>95(13 upstream; 80 downstream)</td>
<td>630</td>
<td>128</td>
<td>4608</td>
</tr>
</tbody>
</table>

Table 1. Simulation parameters. The sphere is located at $(0, 0, 0)$. The substantial domain size in the radial and upstream direction, along with the sponge region, eliminates the spurious reflection of internal waves.
Regeneration of turbulent fluctuations in low Froude number flow over a sphere at Re = 3700.

3. Results and discussion

Figure 2 shows the downstream evolution of turbulent kinetic energy (TKE) integrated over cross-stream \((x_2-x_3)\) planes for cases with different \(Fr\). Note that \(x_3\) denotes the vertical coordinate, the horizontal directions are \(x_1\) (streamwise) and \(x_2\) (lateral), and the sphere center is at the origin. All statistics are computed after the initial transient by time averaging over an interval of 1.5 \(L_x/U\) which is sufficient to obtain converged statistics. Buoyancy in a stratified wake is found to suppress turbulence in previous studies and, accordingly, TKE decreases when \(Fr\) decreases from 1 to 0.8 to 0.5. However, the trend reverses when \(Fr\) decreases to 0.25 and beyond: TKE increases with decreasing \(Fr\). The value of TKE in the \(Fr = 0.25\) case increases to a level comparable to the \(Fr = 0.8\) case and a further decrease of \(Fr\) to 0.21 lead to values of TKE larger than in the unstratified case. Subsequent reduction in \(Fr\) beyond 0.21 leads to progressive augmentation of TKE.

To understand the remarkable regeneration of fluctuations in the near wake at low \(Fr\), contour plots of azimuthal vorticity magnitude in the horizontal \((x_1 - x_2)\) and vertical \((x_1 - x_3)\) planes (figure 3) are examined. The near-wake dynamics changes qualitatively for cases with \(Fr \leq 0.25\), as elaborated below. The \(Fr = 1\) wake displays the anisotropy
Figure 2. Evolution of integrated turbulent kinetic energy in streamwise direction.

Figure 3. Instantaneous azimuthal vorticity magnitude on the horizontal $x_1 - x_2$ plane ($x_3 = 0$) and the vertical $x_1 - x_3$ plane ($x_2 = 0$). Snapshots compared among cases with different $Fr$. 
Regeneration of turbulent fluctuations in low Froude number flow over a sphere at Re = 37005

of a moderate-$Fr$ wake: a large spread in the horizontal plane (figure 3a) and small scale structures associated with the shear layer instability while, in the vertical plane, the separated boundary layers (figure 3b) contract, followed by an undulation of the wake. At $Fr = 0.5$ (not shown here), the recirculation bubble is steady, the disintegration of the shear layer is suppressed in the horizontal plane, and the separating shear layers dip to the centerline in the vertical plane. The shear layer formed by the separating boundary layer exhibits large steady waviness in the vertical plane, there is little unsteadiness in the near wake and, therefore, the TKE for $Fr = 0.5$ is insignificant as was shown in figure 3. A quasi-steady recirculation bubble attached to the sphere is found in the horizontal plane (figure 3c) for a larger stratification, $Fr = 0.25$. At the end of the recirculation zone, the wake undergoes an unsteady undulation with the shedding of vortices further downstream. The shear layer in the vertical direction (figure 3d) manifests waviness (induced by lee waves), but the instability does not break down into turbulence. The flow between the upper and lower shear layers displays thin strips of enhanced vorticity symptomatic of vorticity layering.

The flow organization changes significantly with further decrease in $Fr$ to 0.125 and beyond. There is unsteady motion of the shear layers in the horizontal plane accompanied by patches of small-scale turbulence (figure 3e) as compared to the steady recirculation bubble in the $Fr = 0.25$ wake. This reappearance of small scale fluctuations at $Fr = 0.125$ occurs due to unsteady vortex shedding in the horizontal plane, which results in both flapping and destabilization of the shear layer. A similar vertical layering of vorticity as $Fr = 0.25$ is also seen at $Fr = 0.125$ but, in this case, the layers roll up intermittently to form Kelvin-Helmholtz (KH) billows (figure 3f) which then break down into finer-scale fluctuations. A secondary instability of pancake vortices in the far wake to form KH rolls was noted in previous temporal simulations (Diamessis et al. 2011) for sufficiently high $Re$. In the present near wake, the perturbations provided by the horizontal flapping motion and the value of the local $Re$ are sufficient to destabilize the vertically layered vorticity into KH billows. As $Fr$ approaches 0.025, the unsteady vortex shedding from the sphere in the horizontal plane becomes more noticeable. The $TKE$ in the region $x/D < 1$ that belongs to the very near wake is also the largest among all simulated cases as shown in figure 2. In the horizontal plane (figure 3g), there are coherent vortices with interspersed threads of rolled-up vorticity. In the vertical plane (figure 3h), layered vortical structures are seen but do not manifest KH billows. The fact that KH billows are absent in the $Fr = 0.025$ case will be explained, based on the value of buoyancy Reynolds number and the scaling analysis of Riley & deBruynKops (2003) and Brethouwer et al. (2007), later in the paper. The vorticity pattern at $Fr = 0.025$ appears to have less fine-scale activity relative to $Fr = 0.125$. Internal gravity waves at the body can be seen in the vertical plane (figure 3d, f, h) but their discussion is deferred to future work.

Both mean and turbulent kinetic energy are increasingly dominated by horizontal motions as $Fr$ decreases to 0.25 and below. The evolution of the ratio of area-integrated mean kinetic energy of the horizontal component ($MKE_{11} + MKE_{22}$) and vertical component ($MKE_{33}$) is shown in figure 4(a). For $Fr = 1$, horizontal $MKE$ is larger near the sphere but, beyond $x_1/D \approx 5$, $MKE$ becomes similarly distributed among the horizontal and vertical components. The undulations after $x_1/D \approx 5$ signify the exchange of $MKE$ between horizontal and vertical components. The ratio ($MKE_{11} + MKE_{22}$)/$MKE_{33}$ for $Fr = 0.25$ and 0.125 characterizes the transition of the near wake into quasi-horizontal motion. The case with $Fr = 0.025$ exhibits the complete dominance of horizontal motion wake unsteadiness, present primarily in the form of layered coherent vortices that span a
Figure 4. Evolution of (a) the ratio of area-integrated horizontal and vertical mean kinetic energy, (b) components of integrated turbulent kinetic energy, in streamwise direction. The area integration is over the $x_2, x_3$ plane normal to the streamwise direction.

Figure 5. Energy spectra of (a) lateral $v$ fluctuations and (b) vertical $w$ fluctuations at a downstream point ($x_1 = 1.6, x_2 = 0.51, x_3 = 0$) in the horizontal center plane at various Froude numbers. $E_{vv}$, $E_{ww}$ and Strouhal number, $St$ are nondimensional values based on $U$ and $D$.

Wide lateral ($x_2$) extent. The streamwise variation of the components of $TKE$ for $Fr = 1$ and 0.125 is presented in figure 4(b). The components of $TKE$ for $Fr = 1$ evolve in a similar manner, whereas for $Fr = 0.25$ (not shown here) the streamwise ($TKE_{11}$) and spanwise ($TKE_{22}$) components are larger relative to the vertical ($TKE_{33}$) component. A significant difference between the horizontal ($TKE_{11}, TKE_{22}$) and vertical components is observed as $Fr$ is further decreased to 0.125 (shown here) and 0.025 (not shown here).

Temporal spectra are examined to quantify buoyancy effects on the frequency content of lateral velocity, $v$. Figure 5(a) shows that there is a significant decrease of energy at all frequencies when stratification increases to change $Fr$ from 1 to 0.25. However, a further decrease of $Fr$ to 0.125 and 0.025 shows a re-energization of fluctuations at all frequencies. There is a strong low-frequency peak in these cases: (i) $St = \omega D/U = 0.163$ for $Fr = 0.125$, (ii) $St = 0.200$ for $Fr = 0.025$. Secondary peaks of $E_{vv}$ at harmonics of the low-frequency mode are also evident. There is substantial energy, much larger than at $Fr = 0.25$, at the intermediate frequencies as well. Notice that for flow over a circular cylinder in an unstratified environment at $Re = 3900$, the shedding frequency is found to be $\approx 0.2$ (Parnaudeau et al. 2008). Therefore, with increasing stratification, the vortex shedding of a sphere shifts towards that of a circular cylinder. This is because the flow at depths larger than $O(U/N)$ with respect to the top of the sphere tends to divert around the sphere rather than over the sphere because of the potential energy barrier.
Regeneration of turbulent fluctuations in low Froude number flow over a sphere at \( Re = 3700 \).

Figure 6. Streamwise \((U_{1,\text{mean}})\) and lateral \((U_{2,\text{mean}})\) mean velocity profiles are plotted as a function of lateral coordinate \( x_2 \) at two streamwise locations \((x_1/D = 0, 1)\) in the horizontal central plane, \( x_3 = 0 \).

We emphasize that the low-\( Fr \) near wake, apart from the similarity of vortex shedding, is quite different from the unstratified cylinder wake where the strong inhibition of vertical fluctuations by buoyancy is absent. For example, the vertical velocity spectra \( E_{ww} \) (figure 5(b)) at \( Fr = 0 \) and \( Fr = 0.25 \) have much smaller amplitude relative to their corresponding horizontal counterpart, \( E_{vv} \), and also have smaller amplitude with respect to \( E_{ww} \) for the \( Fr = 1 \) case.

The mean velocity profiles change significantly with decreasing \( Fr \) because of the preferential flow around the sphere rather than over it. Thus, the profile of the mean streamwise velocity (figure 6 a) along the lateral line \((x_1 = x_3 = 0, x_2 > 0.5)\) shows enhanced horizontal shear in the vicinity of the sphere boundary at \( x_2 = 0.5 \), for the lower-\( Fr \) cases in comparison to \( Fr = 1 \). At \( x_1 = 1 \) (figure 6 b), the shear is confined within a narrow band of \( 0.5 < x_2 < 0.8 \) for \( Fr = 1 \), whereas \( Fr = 0.25, 0.125, 0.025 \) show progressively broader regions of shear. The lateral, horizontal motion of the fluid near the sphere is also enhanced as shown by the profile of the lateral velocity \( U_{2,\text{mean}}(x_2) \) on the line \((x_1 = x_3 = 0, x_2 > 0.5)\) as shown by figure 6 (c). At \( x_1 = 1 \), the variation of \( U_{2,\text{mean}} \) as a function of \( x_2 \) (figure 6 d) is substantial for \( Fr = 0.25, 0.125, 0.025 \) and has a complex shape because of the three-dimensional mean flow near the body.

The production of \( TKE \) is given by \( P = -u'_i u'_j \partial \bar{U}_i / \partial x_j \) with the overbar denoting mean value. The various components, \( P_{\alpha,\beta} \) that comprise \( P \) change in the near wake \((x/D < 5)\) because of the buoyancy effect. Figure 7 shows the downstream evolution of the components, \( P_{\alpha,\beta} \), integrated over the cross-stream \( x_2 - x_3 \) plane. The integrated production for the \( Fr = 1 \) wake is primarily dominated by the components \((P_{13}, P_{31})\) involving vertical fluctuations \( u'_3 \) with some contributions from the components \((P_{12}, P_{22})\) involving horizontal fluctuations \( u'_2 \) as shown in figure 7 (a). This scenario changes when stratification increases. As illustrated in figure 7 (b) for \( Fr = 0.25 \), the components \( P_{13} \) and especially \( P_{31} \) are suppressed with respect to \( Fr = 1 \) and by \( Fr = 0.025 \) (figure 7 d), both become negligible as the buoyancy effect strengthens to make \( u'_3 \) negligible. However, \( P_{12} \) and \( P_{22} \) associated with horizontal fluctuations increase when \( Fr \) is reduced
Figure 7. Shear production components for different Fr cases, integrated over $x_2 - x_3$ planes.

Figure 8. Variation of buoyancy Reynolds number $Re_b = \varepsilon / (\nu N^2)$ for different Fr at the center line $x_2 = 0, x_3 = 0$ in the streamwise direction $x_1$.

to 0.125 from 0.25. The large lateral ($x_2$) gradients of mean $U_1$ (figure 6 b) and mean $U_2$ (figure 6 d) enhance $P_{12}$ and $P_{22}$, respectively, making them the leading production terms for $Fr = 0.125$ and 0.025.

The buoyancy Reynolds number, $Re_b = \varepsilon / (\nu N^2)$, where $\varepsilon$ is the turbulent dissipation
Regeneration of turbulent fluctuations in low Froude number flow over a sphere at Re = 3700.

rate and N is the background buoyancy frequency, is an often-used parameter to distinguish the turbulent nature of fluctuations in stratified flow. A similar parameter that distinguishes turbulence is $R = ReFr_h^2$, where $Fr_h = u/l_hN$ ($l_h$ is the length scale and $u$ is the velocity scale of horizontal fluctuations) is the horizontal Froude number, and $Re = ul_h/\nu$. The choice of $l_h = u^3/\varepsilon$ makes $R$ identical to $Re_b$. Riley & deBruynKops (2003) estimated the Richardson number of layered motions in strongly stratified flow by $Ri \approx 1/R$, and proposed that layer instability was possible if $Ri \lesssim 1$ or, equivalently, $R \gtrsim 1$. Brethouwer et al. (2007) concluded that if $R > 4$ an energy cascade from large to small scales is possible allowing an inertial range in horizontal energy spectra. In contrast for $R \ll 1$, the dissipation $\varepsilon$ is associated with quasi-two-dimensional scales.

Arobone & Sarkar (2010), in their DNS of a stratified fluid with horizontal shear found a network of quasi-2D vortices with interspersed dislocations that were laminar for small $Re_b$ but exhibited secondary instability for larger $Re_b$.

We find that the values of $Re_b$ (figure 8) provide guidance to the observed differences in the state of fluctuating motion at different $Fr$. The $Fr = 1$ case has $Re_b$ values between 10 – 100 at 0.54 < $x_1/D$ < 5.5, signifying broadband turbulence as observed from the energy content at high frequencies in the horizontal and vertical energy spectra (figure 5a and b). For the lower $Fr$ of 0.25, the streamwise locations 0.5 < $x_1/D$ < 3 have 0.1 < $Re_b$ < 1. At these streamwise locations, the vortices are still attached as shown in figure 3(c) and no small scale features are present. Some of the small scales observed in the $Fr = 0.25$ case (figure 3c) at $x_1/D = 4 - 5$ are consistent with $Re_b \gtrsim 1$ in this region. Small scales observed in figure 3(e) are consistent with the O(1) values of $Re_b$ for $Fr = 0.125$ at locations 1.14 < $x_1/D$ < 2.75 where $Re_b < 1$ and the flow transitions towards quasi-2D dissipation. For $Fr = 0.025$, $Re_b << 1$ at all $x_1/D$ locations. There is vertical shear between pancake eddies as shown in figure 3(f) and (h) that is quasi laminar for small $Re_b$ consistent with Brethouwer et al. (2007). Nevertheless the flow is far from laminar. The horizontal motion is unsteady owing to vortex shedding, there is broadband turbulence in the near wake as shown by velocity spectra, and there are small scales, e.g. thin braid vortices between the vortices being shed from the sphere (figure 3g) in the vorticity field.

From figure 8, it can be seen that for $Fr = 0.25$ and 0.125, the value of $Ri \approx 1/Re_b$ is $\lesssim 1$ and, therefore, secondary KH instabilities are present in the vertical layers (figure 3d and f). However, for $Fr = 0.125$ at $x_1/D > 5$, the value of $Ri > 1$ and for $Fr = 0.025$ the value of $Ri >> 1$ at all $x_1/D$ locations. Hence, secondary instability is absent in the vertical layers at $x_1/D \approx 5$ location in figure 3(f) and at all locations in figure 3(h).

4. Conclusions

To summarize, although turbulence decreases and is almost extinguished when stratification increases and $Fr$ decreases to 0.5, it is regenerated when $Fr$ decreases further to 0.25 and beyond at $Re = 3,700$. This new finding is contrary to the belief that turbulence suppression is monotone with increasing stratification for flow past a sphere that was based on experiments at low $Re$. Owing to the suppression of vertical motion, the fluid moves horizontally around the sphere. This leads to a new regime of unsteady vortex shedding with frequency similar to that for a circular cylinder, there is transition to broadband turbulence if $Re$ is sufficiently large, and the enhanced shear of the horizontal motion feeds energy into the fluctuation energy. The buoyancy Reynolds number is $Re_b = O(1)$ at locations in the low-$Fr$ wake where quasi-2D vortices are accompanied
with small-scale features in vertical layers between these vortices. Future simulations of flow past a sphere at higher $Re$ are desirable to explore the low-$Fr$ dynamics of the near wake at higher $Re_b$.

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Regeneration of turbulent fluctuations in low Froude number flow over a sphere at \( Re = 3700 \).
