# ALOHA

Analysis of Aloha protocol

- Transmit
- wait for ack
- if not ack retransmit after random delay

Aloha exploits statistical multiplexing.
Assume time is divided into slots

|1|2|3|4|... $\longrightarrow t$

1 pk can be transmitted per slot
$N$ users
each transmit indep. w. prob $p$

$$P(\text{success}) = P(\text{exactly one transmits}) = P(\text{only } 1 \text{ or only } 2 \text{ or only } 3 \ldots)$$
$$= \sum_{i=1}^{N} P(\text{only } i) = \sum_{i=1}^{N} p(1-p)^{N-1} = \boxed{Np(1-p)^{N-1}}$$

Notice that as $p \to 0$ $\quad P(\text{success}) = 0$ [nobody transmits]

$p \to 1$ $\quad P(\text{success}) = 0$ [everybody transmits] $\Rightarrow$ congestion

**QUESTION**: What is the optimal value of $p$ and the corresponding % of packets that are transmitted successfully?

Let's take the derivative:

$$\frac{d\,P(\text{success})}{dp} = -N(N-1)p(1-p)^{N-2} + N(1-p)^{N-1} = 0$$

$$N(1-p)^{N-2}\left[(1-p)-(N-1)p\right] = 0$$

$$N(1-p)^{N-2}\left[1-\cancel{p}-Np+\cancel{p}\right] = 0$$

$$N(1-p)^{N-2}\left[1-Np\right] = 0$$

$$\boxed{p=\frac{1}{N}}$$

It follows that the optimal transmission probability is $\frac{1}{N}$
namely decrease the probability when the number of users increases,
it makes sense !

$$P_{opt}(success) = \left(1-\frac{1}{N}\right)^{N-1}$$

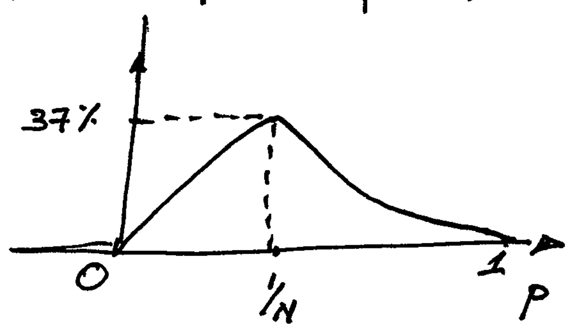
now, remember that

$$\left(1+\frac{a}{N}\right)^N \longrightarrow e^a \text{ when } N \to \infty$$

in our case let $a = -1$, we have

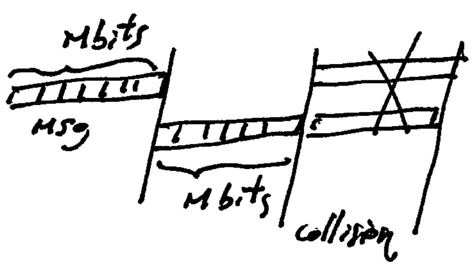
$$P_{opt}(success) \longrightarrow \boxed{\frac{1}{e} \approx 0.37}$$

only 37% of packets go through. This is not very good and there is a lot of room for improvement. The channel is not well utilized by random access.

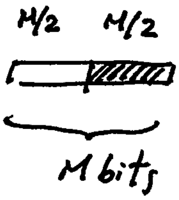



One way Aloha can be improved is by using coding (see handout)
This idea is at the basis of cellular systems: do not transmit uncoded packets but use coding to improve channel utilization.
With uncoded packets of M bits, we send

$$\frac{1 \text{ pk}}{\text{slot}} = \frac{M \text{ info bits}}{\text{slot}} = \text{RATE}$$

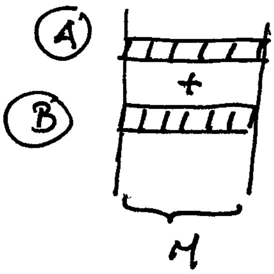


Suppose instead we send <u>coded</u> packets at RATE 1/2




M/2 are "info bits"
M/2 are "redundant" bits used for coding

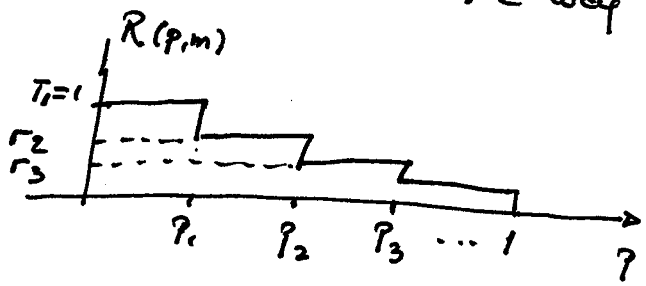
When a collision occurs receive superposition of 2M bits in a codeword of M bits. By choosing redundant bits in a "smart way"



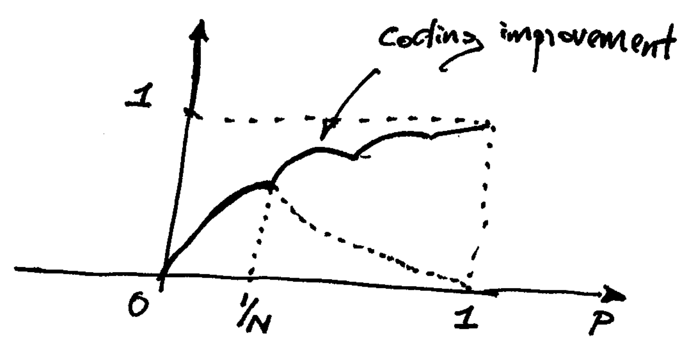

we can recover the M/2 bits of user 1 and the M/2 bits of user 2.

Notice that the bandwidth is always the same (we are transmitting M total # of bits in one slot) but we are using channel more efficiently by using coding.

The handout shows there is a way to choose R optimally depending on p and N




$p_i$ are solutions to equation depend on N and p




Achieve full utilization of channel by transmitting at lower rates.

<u>There is a catch!</u> When $p \to 1$ we have full utilization of channel, however each user sends at rate ~~same~~ decreases, very few info bits per time slot $\Rightarrow$ channel is crowded delay increases.