Queueing Theory.

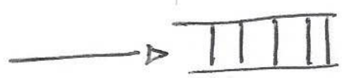
Queueing Theory is used to analize network performance in packet networks where there are

- RANDOM packet arrivals
- RANDOM packet lengths

We care about questions such as:

- How long does a packet spend in a buffer?
- How large is a buffer?

Packet arrival process is modeled as POISSON PROCESS that is defined by the following two properties:



$$P(n \text{ arrivals in interval size } T) = \frac{(\lambda T)^n e^{-\lambda T}}{n!}$$

arrivals n_1, n_2 in disjoint intervals are independent of each other

WHY POISSON?

It can be shown that this mathematical expression arises when the probability of having one arrival in a very small interval is proportional to the size of the interval. ~~[illegible]~~ If you want to generate these arrivals you could draw a random number n according to the expression and uniformly distribute arrivals in the interval. In this way you will have

- random (Poisson) # arrivals
- uniformly distributed interval
- distance between any two consecutive arrivals is exponential

$$P(t_1 - t_2 < t) = 1 - e^{-\lambda t} \qquad \boxed{\text{CDF}}$$

$$P_{|t_1-t_2|}(t) = \lambda e^{-\lambda t} \qquad \boxed{\text{PDF}}$$

$$\mathbb{E}(|t_1 - t_2|) = 1/\lambda$$

$$\text{Var}(|t_1 - t_2|) = \frac{1}{\lambda^2}$$

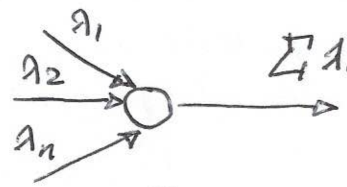
$$\mathbb{E}[n] = \lambda T$$

$$\text{Var}[n] = \mathbb{E}[n^2] - \mathbb{E}^2[n] = \lambda T$$

$$\mathbb{E}[n^2] = \lambda T + (\lambda T)^2$$

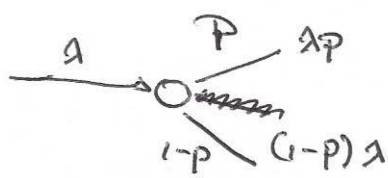
Two important basic properties of Poisson arrivals

① Merging Property



If $X_1, X_2 \ldots X_n$ are Poisson and independent of rates $\lambda_1, \lambda_2\ \lambda_n$ Then : $Y = \sum_i X_i$ is poisson of rate $\sum_i \lambda_i$

② Splitting Property



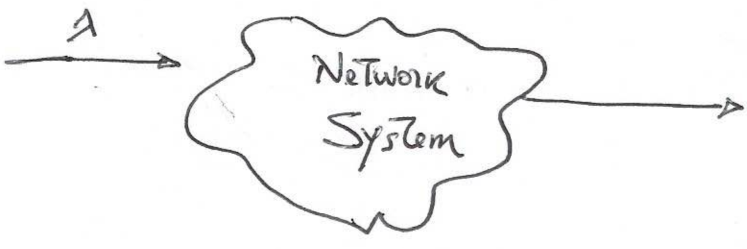
If X is Poisson and packet is switched on one path with prob. p and on other path with probability $(1-p)$

Then : we generate Two new streams

$Y_1 \sim$ Poisson λp

$Y_2 \sim$ Poisson $\lambda(1-p)$

LITTLE'S Theorem



L = $\mathbb{E}$(# packets in system) representing length of queue
W = $\mathbb{E}$(time one packet spends in system) representing waiting time

$$\boxed{L = \lambda W}$$

Consider now a queue where packets arrive according to Poisson λ and exit queue according to Poisson μ.
This means that the service time $|t_1 - t_2|$ to extract a packet from the queue is exponential of parameter μ

Clearly, the system is stable and the queue does not grow to ∞ only is $\boxed{\lambda < \mu}$ otherwise there is accumulation of packets

The utilization factor is then defined as:

$$\left[\rho = \frac{\lambda}{\mu}\right]$$

indicating that when $\lambda = \mu$ the queue is always utilized
We can show (see problems in the PDF addendum) that

$$W = \frac{1}{\mu - \lambda} \quad \text{ave waiting time [assuming system is stable]}$$

$$L = \frac{\lambda}{\mu - \lambda} \quad \text{ave queue length [assuming system is stable]}$$

<u>Question</u>: before we compute these formulas, do they make sense intuitively?