NETWORK GRAPH MODELS II

ADD. READING
CHIANG ch 9-10

In the previous lecture we asked: how does the internet graph look like?

We looked for a reasonable model having
- ⓐ small diameter
- ⓑ ~~[illegible]~~ most nodes having small node degree (i.e. #connections)

We expressed the small diameter requirement as a logarithmic relationship

$$\boxed{d \approx \log N}$$

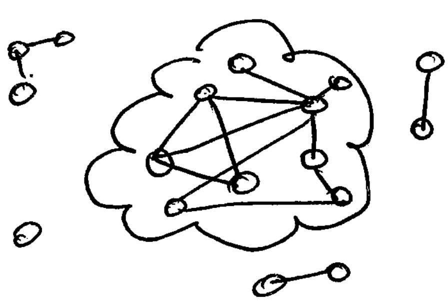
In practice, I can reach a billion nodes in 9 hops. Currently, there are about 3 billion internet nodes and studies through measurements give about 10 hops median. So it is reasonable. See link to internet users count & handout on hop-count on our web page.

A first model we examined was a random graph. This has the two properties we want, but it also has one important drawback:

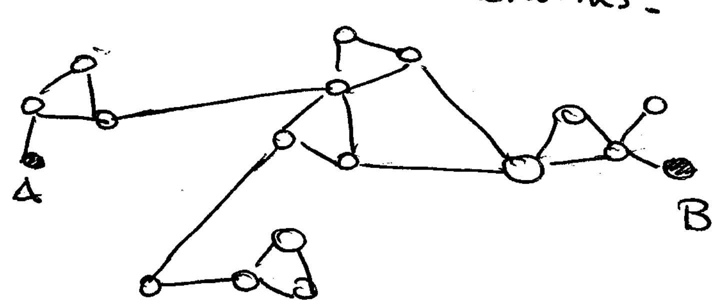
NO STRUCTURE

Any node can connect to any other node with the same probability when $\lambda = Np > 1 \Rightarrow$ we get a giant connected component with a small diameter, but it looks like a giant "BLOB"

The "Spaghetti Internet"

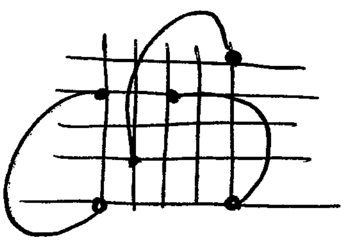


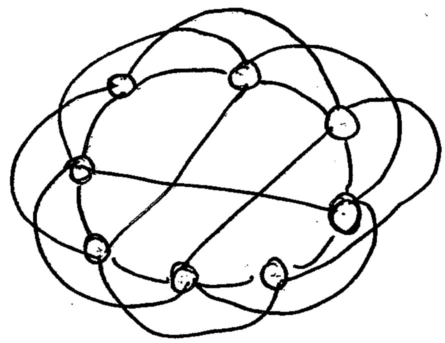
The "real internet" has structure, it is a collection of interconnected networks.



So we introduced a notion of clustering coeff.
We want a network that has high clustering & small diameter.

One possibility is to start with a regular graph, like a grid, or a ring and then add random connections between the nodes to effectively reduce the diameter





WATTS - STROGATZ model (1998)

- start with ring lattice with N nodes, each connected to K neighbors $K/2$ on each side
- For every node "rewire" every edge of that node with probability β and selecting the destination node uniformly at random
Avoiding self-loops & duplication.

QUESTION : At the end, how many "long distance" edges you will have?

There will be on average $\beta \frac{NK}{2}$ "long distance" edges

$\beta = 0 \Rightarrow$ Regular graph

$\beta = 1 \Rightarrow$ Random graph with node degree $\approx K$ and every edge occurring with probability

$$\boxed{P_{edge} = \frac{NK}{2} \frac{1}{\binom{N}{2}}}$$

This requires a bit of math to explain.

At the end we have $M = N\frac{K}{2}$ edges, and a graph $G(N, M)$ of N nodes and M edges chosen at random among all graphs of this type.

In random graph model $G(N, P_{edge})$ $M \approx \binom{n}{2} P_{edge}$

So it follows that $\binom{n}{2} P_{edge} = \frac{NK}{2} \Rightarrow P_{edge} = \frac{NK/2}{}$

Average path length in WS model is in between

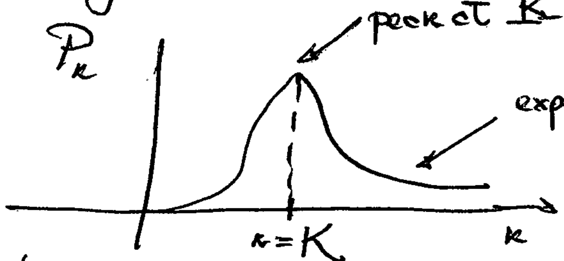
$$\boxed{\begin{array}{ll} \beta = 0 & \mathbb{E}(d) = \dfrac{N}{2K} \\ \beta = 1 & \mathbb{E}(d) = \dfrac{\log N}{\log K} \end{array}}$$

(regular ring)

(random graph) of average degree K

As $\beta \to 1$ it approaches the second value very rapidly

The model also has large clustering for moderate values of β and a degree distribution similar to that of a random graph, namely



This means that all nodes have roughly the same degree $\approx K$
In fact the only way to change degree is if a node ~~the~~ rewires randomly to the same node more than once and for large N this is very unlikely.

Similar models can be obtained for Grid or connecting other structured networks. But is the model of having almost constant node degree a good one?

We would like to have more variability in node degree distribution