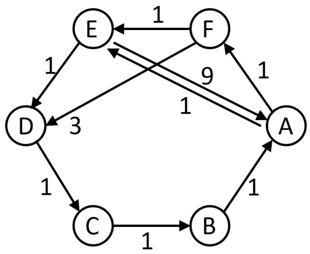
ECE 158A 2019 Fall Midterm Practice Questions

Problem 1 (routing)



(a) Find the shortest path from each node to node A using the Dijkstra's algorithm.

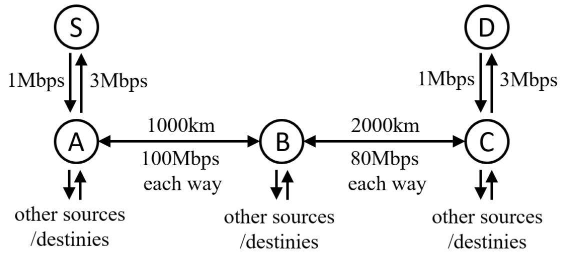
Step	Finalized Set	*A*	*B*	*C*	*D*	*E*	*F*
0		0	∞	∞	∞	∞	∞
1	A		1,A	∞	∞	9,A	∞
2	A,B			2,B	∞	9,A	∞
3	A,B,C				3,C	9,A	∞
4	A,B,C,D					4,D	6,D
5	A,B,C,D,E						5,E

(b) Find the shortest path from each node to node A using the Bellman-Ford algorithm.

Iteration	**B**	**C**	**D**	**E**		**F**		*shortest distance*
number	A	B	C	A	D	D	E	*changes*
1	1	∞	∞	9	∞	∞	∞	**B**($\infty \to 1$), **E**($\infty \to 9$)
2	1	2	∞	9	∞	∞	10	**C**($\infty \to 2$), **F**($\infty \to 10$)
3	1	2	3	9	∞	∞	10	**D**($\infty \to 3$)
4	1	2	3	9	4	6	10	**E**($9 \to 4$), **F**($10 \to 6$)
5	1	2	3	9	4	6	5	**F**($6 \to 5$)

Problem 2 (packet delay, M/M/1 queue)

Consider the network shown in the figure below. Node S sends a flow of packets to node D via node A, B, and C. Each packet has size 4kB (3.2×10^4 bit). At S, a packet is transmitted once the ACK message for the previous packet is received. The size of the ACK message is so small that its transmission delay on any link can be assumed negligible. The links between S,A and D,C are dedicated to the communication between S and D. Their physical length is very small and the propagation time over them is negligible. The links between A,B and B,C are shared by public traffics. These two links are full-duplex, with equal rates on both directions. A M/M/1 queue is maintained at each entrance of each of the two links. The traffic arriving at A directed towards B has rate 80Mbps, that arriving at B directed towards C has rate 60Mbps; the traffic arriving at C directed to B and the traffic arriving at B directed to A both have rate 70Mbps. The average packet size in the public traffic is 5kB (4×10^4 bit). Assume that the packet flow from S to D has negligible influence on the queues' status. The propagation speed is 3×10^8 m/s. Consider the following questions:



(a) How long does it take a packet to travel from S to D?

Ans: A packet sent from S to D experiences transmission delays on all 5 links, queueing delays at node A and B, propogation delays on links between A,B and B,C:

$$d_{trans}^{(SA)} = \frac{3.2 \times 10^4}{1 \times 10^6} = 32 \times 10^{-3} s$$

$$d_{queue}^{(AB)} = \frac{1}{\mu^{(AB)} - \lambda^{(AB)}} - \frac{1}{\mu^{(AB)}} = \frac{4 \times 10^4}{100 \times 10^6 - 80 \times 10^6} - \frac{4 \times 10^4}{100 \times 10^6} = 2 \times 10^{-3} - 0.4 \times 10^{-3}$$
$$= 1.6 \times 10^{-3} s$$

$$d_{trans}^{(AB)} = \frac{3.2 \times 10^4}{100 \times 10^6} = 0.32 \times 10^{-3} s$$

$$d_{queue}^{(BC)} = \frac{1}{\mu^{(BC)} - \lambda^{(BC)}} - \frac{1}{\mu^{(BC)}} = \frac{4 \times 10^4}{80 \times 10^6 - 60 \times 10^6} - \frac{4 \times 10^4}{80 \times 10^6} = 2 \times 10^{-3} - 0.5 \times 10^{-3}$$
$$= 1.5 \times 10^{-3} s$$

$$d_{trans}^{(BC)} = \frac{3.2 \times 10^4}{80 \times 10^6} = 0.4 \times 10^{-3} s$$

$$d_{trans}^{(CD)} = \frac{3.2 \times 10^4}{3 \times 10^6} = 10.67 \times 10^{-3} s$$

$$d_{prop}^{(AC)} = \frac{3000 \times 10^3}{3 \times 10^8} = 10 \times 10^{-3}$$

$$d_{total}^{(SD)} = d_{trans}^{(SA)} + d_{queue}^{(AB)} + d_{trans}^{(AB)} + d_{queue}^{(BC)} + d_{trans}^{(BC)} + d_{trans}^{(CD)} + d_{prop}^{(AC)} = 56.49 \times 10^{-3} s$$

(b) How long does it take a ACK message to travel from D to S?

Ans: A ACK sent from D to S experiences queueing delays at node C and B, propogation delays on links between C,B and B,A:

$$d_{queue}^{(CB)} = \frac{1}{\mu^{(CB)} - \lambda^{(CB)}} - \frac{1}{\mu^{(CB)}} = \frac{4 \times 10^4}{80 \times 10^6 - 70 \times 10^6} - \frac{4 \times 10^4}{80 \times 10^6} = 4 \times 10^{-3} - 0.5 \times 10^{-3}$$
$$= 3.5 \times 10^{-3} s$$

$$d_{queue}^{(BA)} = \frac{1}{\mu^{(BA)} - \lambda^{(BA)}} - \frac{1}{\mu^{(BA)}} = \frac{4 \times 10^4}{100 \times 10^6 - 70 \times 10^6} - \frac{4 \times 10^4}{100 \times 10^6} = 1.33 \times 10^{-3} - 0.4 \times 10^{-3}$$
$$= 0.93 \times 10^{-3} s$$

$$d_{prop}^{(CA)} = \frac{3000 \times 10^3}{3 \times 10^8} = 10 \times 10^{-3} s$$

$$d_{total}^{(DS)} = d_{queue}^{(CB)} + d_{queue}^{(BA)} + d_{prop}^{(CA)} = 14.43 \times 10^{-3} s$$

(c) Determine the average flow rate of the data flow from S to D.

Ans: According to the results in part (a) and (b), the round trip time between S and D is

$$RTT = 56.49 \times 10^{-3} + 14.43 \times 10^{-3} = 70.92 \times 10^{-3} s$$

Since a packet of data (3.2×10^4 bit) is delievered in each round trip time, the throuput is

$$\frac{packet\ size}{RTT} = \frac{3.2 \times 10^4}{70.92 \times 10^{-3}} \approx 0.45 Mbps$$

Problem 3 (flow control: AIMD)

Consider that three flows with rates x_1, x_2, x_3 share the same bottleneck link, whose capacity is C. The flow rates are managed by AIMD for congestion control:

- After each RTT, if no packet loss occurred, the source of a flow increases its congestion window size by one packet.
- The source of a flow halves its flow rate once a packet loss occurs.
- All flows experience packet losses immediately after $x_1 + x_2 + x_3$ exceeds C.

Assume all flows have the same RTT, and the initial flow rates are $x_1 = C/4$, $x_2 = C/8$, $x_3 = C/8$. Consider the following questions:

(a) Fill in the flow rates at the specified moments in the table below:

	x_1	x_2	x_3
before the 1st decrease	$10/24 \times C$	$7/24 \times C$	$7/24 \times C$
after the 1st decrease	$10/48 \times C$	$7/48 \times C$	$7/48 \times C$
before the 2nd decrease	$6/16 \times C$	$5/16 \times C$	$5/16 \times C$
after the 2nd decrease	$6/32 \times C$	$5/32 \times C$	$5/32 \times C$
before the 3rd decrease	$34/96 \times C$	$31/96 \times C$	$31/96 \times C$
after the 3rd decrease	$34/192 \times C$	$31/192 \times C$	$31/192 \times C$
before the 4th decrease	$22/64 \times C$	$21/64 \times C$	$21/64 \times C$

Ans: Consider an increasing phase in which the initial flow rates are $x_1 = x_1^{(0)}$, $x_2 = x_2^{(0)}$, $x_3 = x_3^{(0)}$. Because the RTT is the same for all three flows, x_1, x_2, x_3 increase at the same rate, for which they satisfy

$$(x_1 - x_1^{(0)}) = (x_2 - x_2^{(0)})$$
$$(x_1 - x_1^{(0)}) = (x_3 - x_3^{(0)})$$

Congestion happens when $x_1 + x_2 + x_3 = C$. Solving the system formed by these three equations gives the flow rates when congestion occurs, that is, just before the decrease happens:

$$\begin{aligned} x_1 &= \frac{1}{3} \cdot (C + (x_1^{(0)} - x_2^{(0)}) + (x_1^{(0)} - x_3^{(0)})) \\ x_2 &= \frac{1}{3} \cdot (C + (x_2^{(0)} - x_1^{(0)}) + (x_2^{(0)} - x_3^{(0)})) \\ x_3 &= \frac{1}{3} \cdot (C + (x_3^{(0)} - x_1^{(0)}) + (x_3^{(0)} - x_2^{(0)})) \end{aligned} \tag{1}$$

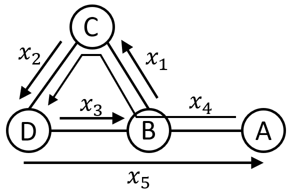
Equations in (1) can be used to compute the flow rates at the end of each increasing phase given those at the beginning of the phase. The flow rates at the beginning of the next increasing phase are just the halves of flow rates at the end of the previous increasing phase.

(b) After how many increase-decrease cycles does the difference $|x_1 - x_2|$ fall below $C \times 1\%$?

Ans: The difference $|x_1 - x_2|$ doesn't change in each increasing phase and is halved in each flow rate decrease. Denote $|x_1 - x_2|$ after the i-th decrease as $\Delta x^{(i)}$. Then $\Delta x^{(i)} = \Delta x^{(0)} \cdot \left(\frac{1}{2}\right)^i$, where $\Delta x^{(0)} = C/8$ is the initial difference between x_1 and x_2. For $\Delta x^{(i)}$ to be smaller than $C/100$, we see that i needs to be at least 4.

Problem 4 (flow control: utility maximization)

Consider the network shown in the figure above. Each link in the network has capacity C. The link between node A and node B is half-duplex, so the flows x_4 and x_5 are jointly constrained by the link's capacity. Assume the utility of the i-th flow is evaluated as $U(x_i) = \frac{1}{1-\alpha} x^{1-\alpha}$. Determine the flow rates which maximize the total utility without exceeding the links' capacities.



Ans: The constraints are

$$x_1 + x_4 = C,\ x_2 + x_4 = C,\ x_3 + x_5 = C,\ x_4 + x_5 = C$$

Given 4 equations with 5 variables, there are exactly one degree of freedom. Write x_2, x_3, x_4, x_5 as functions of x_1:

$$x_2 = x_1,\ x_3 = C - x_1,\ x_4 = C - x_1,\ x_5 = x_1$$

Note that $\frac{d}{dx}U(x) = x^{-\alpha}$. Taking derivative of the total utility in respect to x_1, and let it be 0 gives

$$3x_1^{-\alpha} - 2(C - x_1)^{-\alpha} = 0$$

Solving the equation gives

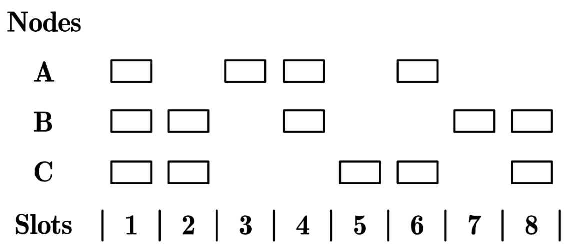
$$x_1 = \frac{1}{(2/3)^{1/\alpha} + 1} \cdot C$$

Then

$$x_2 = x_5 = x_1 = \frac{1}{(2/3)^{1/\alpha} + 1} \cdot C$$
$$x_3 = x_4 = \frac{1}{1 + (3/2)^{1/\alpha}} \cdot C$$

Problem 5 (random access: exponential backoff)

Suppose three nodes are communicating on a network that uses a random access protocol to mitigate collisions. Let the nodes transmit packets in slots as shown in the following diagram, where slot 1 begins at time t = 0:



In the protocol, each node transmits a packet, and if there is a collision then the node waits for X time slots before transmitting again, where X is a random variable following the distribution $Uniform\{0, 1, 2, ..., 2^N - 1\}$, $N = min\{m, 10\}$, and m is the number of previous collisions **for that packet**.

(a) How many packets has each node successfully transmitted after time slot 8 has completed?

Ans: Transmission is successful if exactly one node transmits in a timeslot. According to the figure, successful transmissions happened in timeslot 3, 5, and 7, so 3 packets have been transmitted successfully.

(b) After slot 1, what was the probability that there was a collision in slot 2?

Ans: After slot 1, $N = 1$ for all the three node. Thus at the beginning of timeslot 2, the backoff duration is chosen randomly from $\{0, 1\}$. Each node has probability $1/2$ to transmit in slot 2.

A collision happens in 4 cases: A,B transmit, A,C transmit, B,C transmit, A,B,C transmit. Its probability is

$$\left(\frac{1}{2}\right)^2 \times \left(1-\frac{1}{2}\right) \times 3 + \left(\frac{1}{2}\right)^3 = \frac{1}{8} \times 4 = \frac{1}{2}$$

(c) What is the probability no node transmits in slot 9?

Ans: For node A, the last attempt to transmit was in slot 6. This was the second attempt A made to transmit the packet (after it failed once in slot 4), so after that $N = 2$ for A. The backoff duration was then chosen randomly from $\{0, 1, 2, 3\}$, which means A has equal probability to transmit in slots 7,8,9,10. Now that slots 7 and 8 have passed and no transmission happend, so A has probability 1/2 to transmit in either slot 9 or 10. Similarly, for node B, $N = 1$. It has probability 1/2 to transmit in slot 9. For node C, $N = 2$. It has probability 1/4 to transmit in slot 9. Then the probability that no node transmits in slot 9 is

$$\left(1-\frac{1}{2}\right) \times \left(1-\frac{1}{2}\right) \times \left(1-\frac{1}{4}\right) = \frac{3}{16}$$

Problem 6 (random access: throughput)

Consider a scenario where N users randomly access a shared physical medium in the following manner: In each time slot, each user (if it has data to send) makes a random decision on whether to transmit a packet or not in the current slot. The transmission succeeds if no other user transmits in the same slot (otherwise a collision happens). The random decisions are independent across users and timeslots. Assume (packet size)/(length of time slot)=C, the transmission probability of the i-th station is p_i, and each user always has data to send, consider the following questions:

(a) What is the throughput (the amount of data successfully transmitted in a unit of time) of the i-th user?

Ans: The probability that the i-th user makes a successful transmission in a time slot is

$$p_i \cdot \prod_{j=1, j\neq i}^{N} (1-p_j)$$

Its throughput is the probability multiplies C.

(b) Assume three of the users have transmission probabilities $p_1 = 0.1$, $p_2 = 0.2$, $p_3 = 0.3$. What is the ratio between their throughputs? Ans: The expression of the throughput can be rewritten as

$$x_i = C \cdot \frac{p_i}{1-p_i} \cdot \prod_{j=1}^{N} (1-p_j)$$

so the per-user throughput is proportional to $\frac{p_i}{1-p_i}$.

$$x_1 : x_2 : x_3 = \frac{p_1}{1-p_1} : \frac{p_2}{1-p_2} : \frac{p_3}{1-p_3} = \frac{1}{9} : \frac{1}{4} : \frac{3}{7}$$

(c) What is the total throughput of all N users (in term of $\{p_i\}$ and C)?

Ans:

$$C \cdot \left(\sum_{i=1}^{N} \frac{p_i}{1-p_i}\right) \cdot \left(\prod_{j=1}^{N} (1-p_j)\right)$$