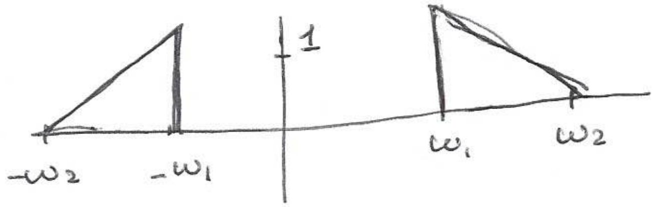
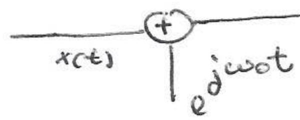
PROBLEM SETS



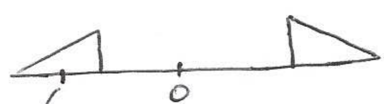
min sampling rate is $\geq 2\omega_2$
however, there is a lot of "white space"
what can we do?

① <u>Modulate</u>

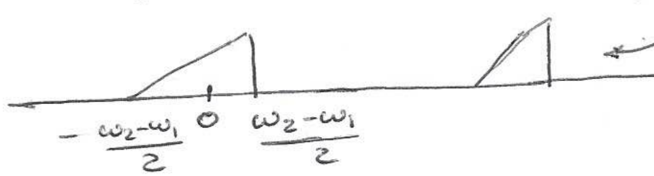


$x(t)\, e^{j\omega_0 t} \xleftrightarrow{FT} X(j(\omega-\omega_0))$

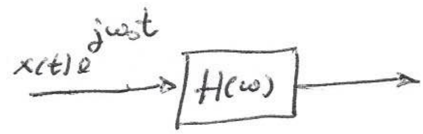
The effect is to shift spectrum on <u>right</u> by ω_0 in freq.



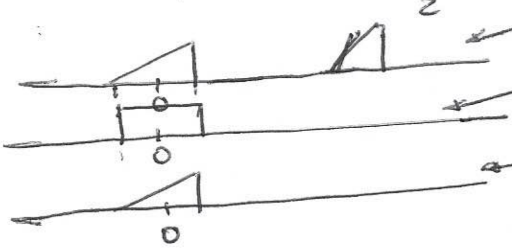
$-\left[\omega_1 + \frac{\omega_2-\omega_1}{2}\right] = -\frac{\omega_1+\omega_2}{2} \Rightarrow$ choose $\boxed{\omega_0 = \frac{\omega_1+\omega_2}{2}}$



modulated signal. in freq. domain



② <u>Low Pass</u> with cut-off $\omega_0' = \frac{\omega_2-\omega_1}{2}$



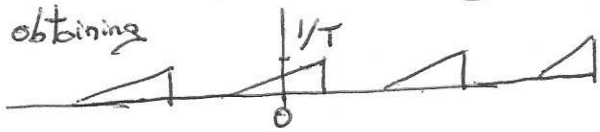
modulated sig

Freq. Response Low-Pass

new signal

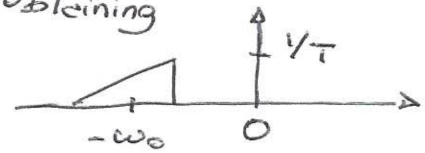
③ <u>Sample</u> new signal with freq $\geq \frac{\omega_2-\omega_1}{2}$

obtaining



spectrum of sampled signal

(4) To recover original signal first LOW-PASS again with cut-off $\frac{\omega_2-\omega_1}{2}$
then modulate by multiply by $e^{-j\omega_0 t}$ $\omega_0 = \frac{\omega_1+\omega_2}{2}$
obtaining



(5) Then to obtain original signal either multiply by 2 and take real part:

$$Re[x(t)] = \frac{1}{2}[x(t) + x^*(t)] \xrightarrow[FT]{} \frac{1}{2}X(j\omega) + \frac{1}{2}X^*(j\omega)$$

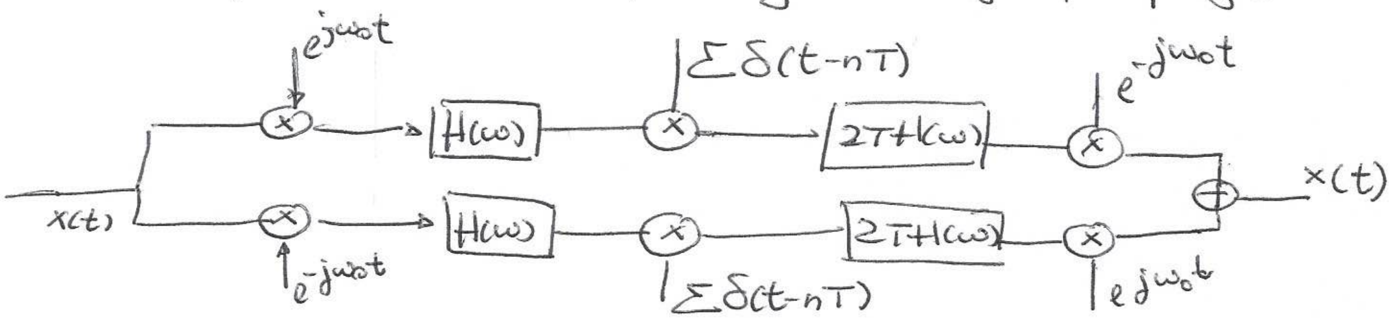
The absolute value is

$$\frac{1}{2}|X(j\omega) + X^*(j\omega)|$$

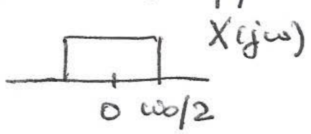
if support of $X(j\omega)$ does not contain origin we have

$$\frac{1}{2}|X(j\omega)| + \frac{1}{2}|X^*(j\omega)|$$
$$= \frac{1}{2}|X(j\omega)| + \frac{1}{2}|X(-j\omega)|$$

Alternatively redo the same steps to get the right part of signal



Determine Nyquist sampling rate of the following signals

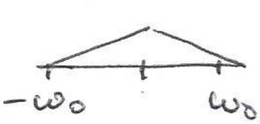


Nyquist rate $= \omega_0$

$x(t) + x(t-1) \longleftrightarrow X(j\omega) + X(j\omega)e^{-j\omega} \Rightarrow$ Nyquist rate $= \omega_0$

$\frac{dx(t)}{dt} \longleftrightarrow j\omega X(j\omega) \Rightarrow$ Nyquist rate $= \omega_0$

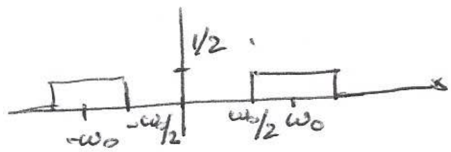
$x^2(t) \longleftrightarrow \frac{1}{2\pi} X \circledast X(j\omega)$



support in freq is $2\omega_0$ $\Rightarrow$ Nyquist rate $= 2\omega_0$

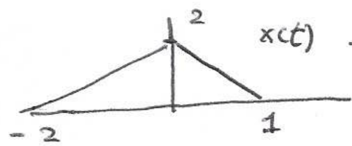
consider $x(t)\cos(\omega_0 t) \longleftrightarrow \frac{1}{2}X(\omega-\omega_0)+\frac{1}{2}X(\omega+\omega_0)$

$$= \frac{x(t)\,e^{j\omega_0 t}}{2} + \frac{x(t)\,e^{-j\omega_0 t}}{2}$$

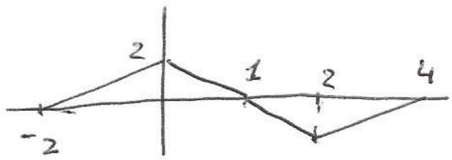


New Nyquist rate $\boxed{3\omega_0}$

MT2 P3

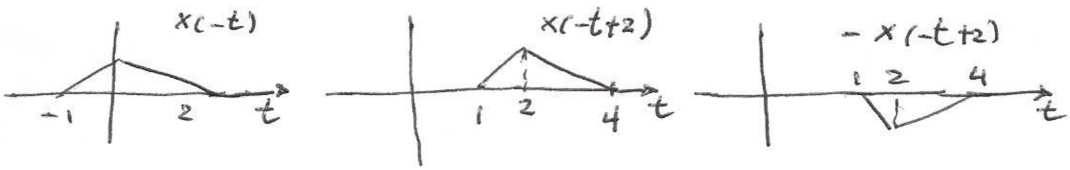


$x(t) \longrightarrow X(\omega) = \dfrac{3 - e^{j2\omega} - 2e^{-j\omega}}{\omega^2}$



$y(t) = x(t) - x(-(t-2))$

since



Now There was a mistake in the solution

$x(t-2) \longleftrightarrow X(j\omega)\, e^{-2j\omega}$ (1)

$x(-t) \longrightarrow X(-j\omega)$ (2)

$x(-t+2) \longrightarrow X(-j\omega)\, e^{2j\omega}$ (3)

However the above is wrong. To see this, let's apply the definition directly

$$\int_{-\infty}^{+\infty} x(-t+2)\, e^{-j\omega t}\, dt = \int_{-\infty}^{+\infty} x(-z)\, e^{-j\omega(z+2)}\, dz = e^{-2j\omega} \int_{-\infty}^{+\infty} e^{-j\omega z}\, x(-z)\, dz = \boxed{X(-j\omega)\, e^{-2j\omega}}$$

call
$t-2=z$

so the correct result is $X(-j\omega)\, e^{-2j\omega}$

The problem occurs in the third line (3) $x(-(t-2)) \longrightarrow X(-j\omega)\, e^{2j\omega}$ this is incorrect because it is incorrectly applying the shifting property (1) which could be valid to $x(+t+2) \longrightarrow X(j\omega)\, e^{+2j\omega}$ namely when the variable is $+t$.

In our case, however, the variable is $-t$ and we have

$$x(-(t-2)) \longrightarrow e^{-2j\omega} X(-j\omega)$$

where we are correctly applying the scaling property

$$x(at) \longrightarrow \frac{1}{|a|} X(aj\omega)$$ where $a=-1$ and $t = t-2$
ω is the transformed var of $t-2$

$$x(-(t-2)) \longrightarrow X(-j\omega)\, e^{-2j\omega}.$$

now we can continue with the solution and we have

$$Y(j\omega) = X(j\omega) - X(-j\omega)\, e^{-2j\omega}$$

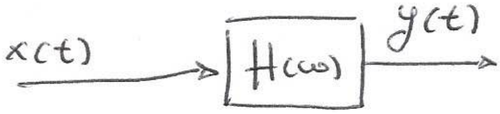
$$= \frac{3 - e^{j2\omega} - 2e^{-j\omega}}{\omega^2} - \frac{3 - e^{-2j\omega} - 2e^{j\omega}}{\omega^2}\, e^{-2j\omega}$$

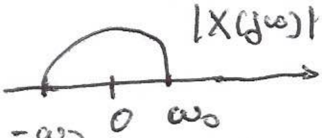
$$= \frac{3 - e^{j2\omega} \cancel{-2e^{-j\omega}} - 3e^{-2j\omega} + e^{-4j\omega} \cancel{+2e^{-j\omega}}}{\omega^2}$$

$$= \frac{3 - e^{2j\omega} - 3e^{-2j\omega} + e^{-4j\omega}}{\omega^2}$$

MODULATION PROBLEM

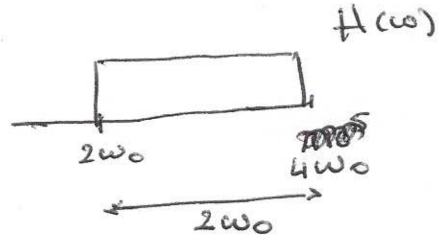
Assume you want to send a signal $x_o(t)$ through a LTI system and recover it at the receiver





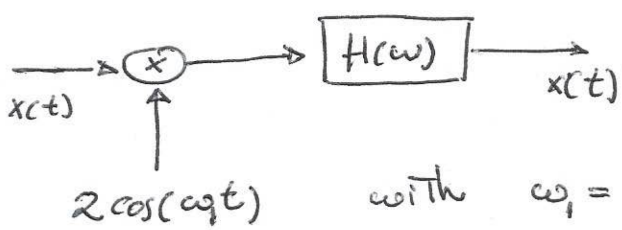
$\omega_0 = 2\pi 10^4$

$$H(\omega) = \begin{cases} 1 & \text{for } 2\omega_0 \leq \omega \leq 4\omega_0 \\ 0 & \text{otherwise} \end{cases} \Rightarrow$$

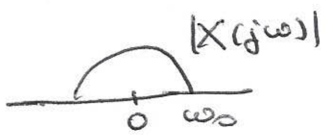


① check if additional processing is needed
YES because band-pass filter and signal is out-of band $\omega_0 < 2\omega_0$

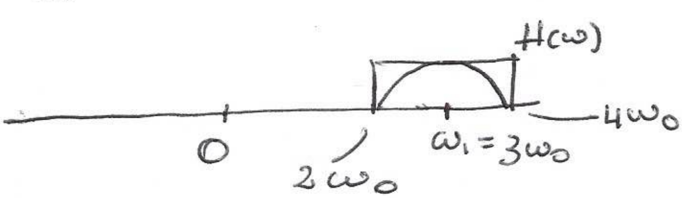
② If needed what processing can you design?



with $\omega_1 = 3\omega_0$



$$|X(j\omega)\cos(\omega_1 t)| = \frac{1}{2} X(j(\omega+\omega_0)) + \frac{1}{2} X(j(\omega-\omega_0))$$



Consider an LTI system

$$x(t) \longrightarrow \boxed{H(\omega)} \longrightarrow y(t)$$

Let $h(t) = u(t)$ step function at the origin

determine $y(t)$ if $x(t) = \frac{d}{dt} f(t)$ for some known signal $f(t)$

We proceed in frequency domain $x(t) \longleftrightarrow X(j\omega)$ $f(t) \longleftrightarrow F(j\omega)$

First we determine $H(\omega)\, X(j\omega)$

$$= j\omega\, F(j\omega)\, H(\omega)$$

$$= j\omega\, H(\omega)\, F(j\omega)$$

$$= Y(j\omega)$$

Now we notice $j\omega\, H(j\omega) \longleftrightarrow \frac{d}{dt} h(t) = \frac{d}{dt} u(t) = \delta(t)$

$FT(\delta(t)) = 1$

so that we have

$j\omega\, H(j\omega) = 1$

$Y(j\omega) = F(j\omega)$

$$\boxed{y(t) = f(t)}$$