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ECE 45 DC Circuits Review

Circuits Concepts:

- *Sources:* voltages in series can be combined; currents in parallel can be combined.

  The power supplied/dissipated by a component is the current multiplied by the voltage drop. If current flows from higher to lower (lower to higher) voltage, power is dissipated (supplied).
- A *branch* is two components (source, resistor, capacitor, inductor, etc.) connected end to end.
- A *node* is a connection of multiple branches.

  Nodes have a particular voltage, and branches have a particular current.
- **Kirchhoff's Current Law:** A node has zero net current, i.e. the incoming current of a node equals the outgoing current of a node.
- **Kirchhoff's Voltage Law:** The net voltage in a closed loop in a circuit is zero.
- These laws can be used to set up systems of equations which can be used to solve for specific voltages and currents in the circuit.
- **KCL (Node Voltage):** (Used by SPICE)
  - Assign a node to be the reference ground, 0V.
  - Assign a voltage V_i to all additional nodes, where V_i is the voltage with respect to ground.
  - Decide on arbitrary directions for the currents in each branch. It helps to write equations for each current in terms of the voltages V_i ahead of time for consistency.
  - For each node, the net current is zero, so write an equation $\sum I_{in} = \sum I_{out}$.
  - Solve the system of equations for desired values.
- **KVL (Mesh Current):** (Generally easier if fewer loops than nodes.)
  - Assign each independent loop a current I_i.
  - Write a KVL equation for each loop such that $\sum V = 0$. It helps to write equations for each voltage in terms of the currents I_i ahead of time.
  - If multiple loops include a branch, the current in that branch is the sum of the loops which include that branch.
  - Solve the system of equations.
- **Power:** The power generated or dissipated by a component is proportionate to the current through the component and the voltage across the component. $P = VI$.

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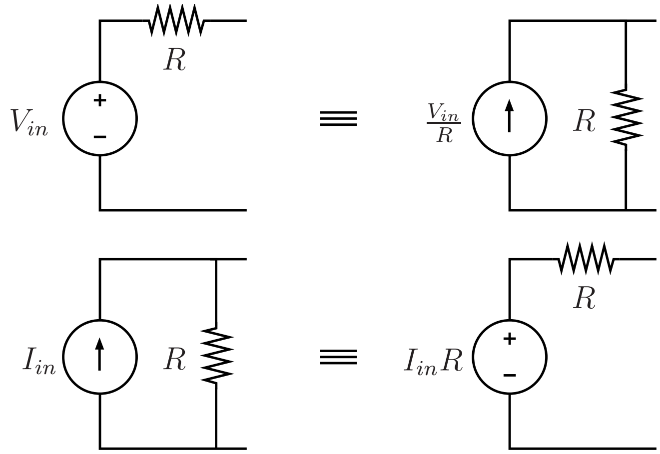
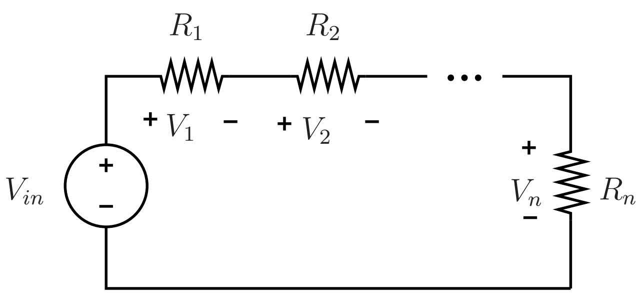
- **Source Transformation:** Replace a current/voltage source with a voltage/current source:



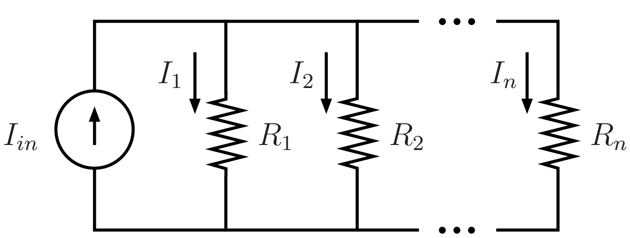
- Voltage/Current Divider: Application of KCL/KVL that allows for quick calculation of a voltage/current. For any $n \geq 2$.



Voltage Divider:

$$V_k = V_{in}\,\frac{R_k}{R_1 + R_2 + \cdots + R_n}$$

for each $k = 1, 2, \ldots, n$



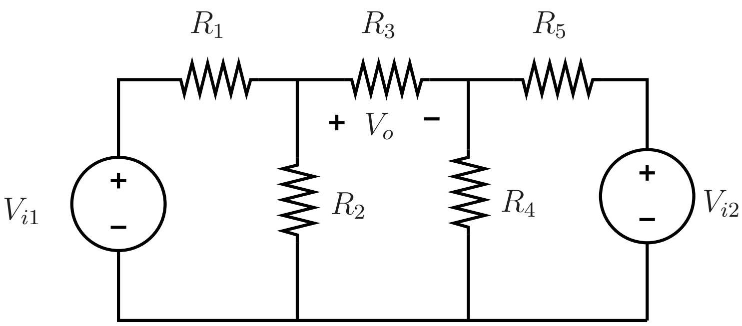
Current Divider:

$$I_k = I_{in}\,\frac{\frac{1}{R_k}}{\frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_n}}$$

for each $k = 1, 2, \ldots, n$

Circuit Examples:

1. *Find the output voltage of the following circuit:*



$V_{i1} = 20V$

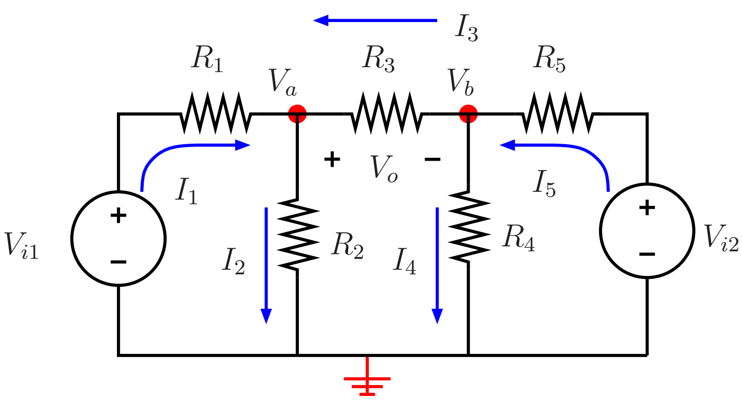
$V_{i2} = 5V$

$R_1 = R_2 = R_5 = 10\Omega$

$R_3 = R_4 = 5\Omega$

Solutions

Labeling currents and voltages gives us:



By KCL:	By Ohm's Law:
$\sum I_{in} = \sum I_{out}$	$I_1 = (V_{i1} - V_a)/R_1$
$a)\ I_1 + I_3 = I_2$	$I_2 = V_a/R_2$
$b)\ I_5 = I_3 + I_4$	$I_3 = (V_b - V_a)/R_3$
	$I_4 = V_b/R_4$
	$I_5 = (V_{i2} - V_b)/R_5$

Substituting expressions for $I_1, ..., I_5$ into $a)$ and $b)$ gives us:

$$a) \quad \frac{V_1}{R_1} = V_a\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) - V_b\frac{1}{R_3}$$

$$b) \quad \frac{-V_2}{R_5} = V_a\frac{1}{R_3} - V_b\left(\frac{1}{R_3} + \frac{1}{R_4} + \frac{1}{R_5}\right)$$

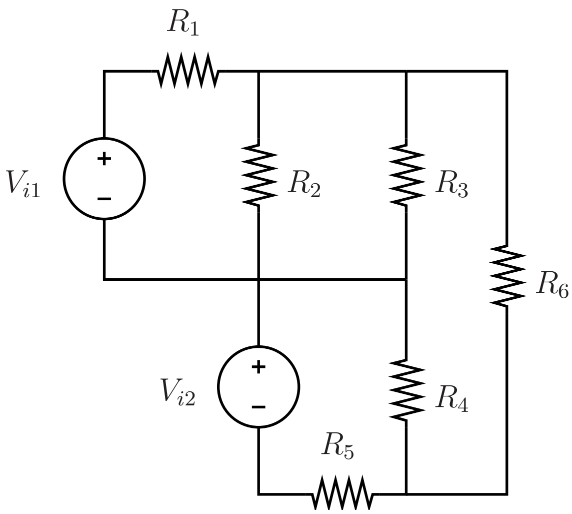
Plugging in numerical values and subtracting $a)$ from $b)$ gives us

$$V_b\frac{4}{5} = 3 \ \rightarrow V_b = \frac{15}{4}V \text{ and } V_a = \frac{55}{8}V$$

and so

$$V_o = V_a - V_b = \frac{25}{8}V$$

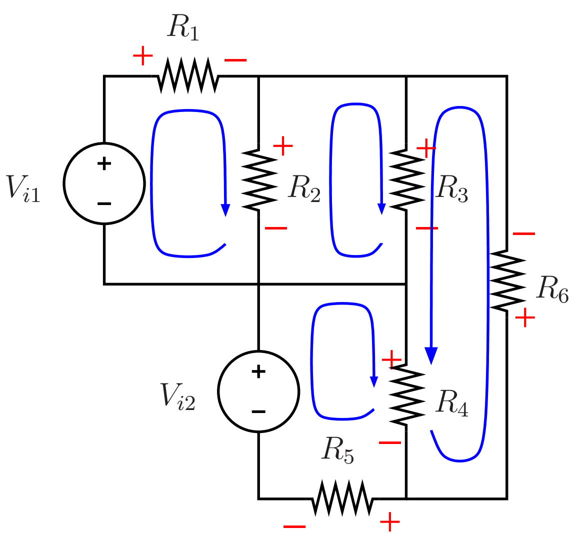
2. *Find the power dissipated by* R_6



$$
\begin{aligned}
V_1 &= 1V \\
V_2 &= 2V \\
R_1 = R_4 &= 2\Omega \\
R_2 &= 1\Omega \\
R_3 = R_5 &= 4\Omega \\
R_6 &= 8\Omega
\end{aligned}
$$

Solutions

Labeling currents and voltages gives us:



By KVL:	By Ohm's Law:
$\sum V_{gain} = \sum V_{drop}$	$V_{R_1} = R_1\, I_1$
1) $V_1 = V_{R_1} + V_{R_2}$	$V_{R_2} = R_2\,(I_1 - I_2)$
2) $V_{R_2} = V_{R_3}$	$V_{R_3} = R_3\,(I_2 + I_4)$
3) $V_2 = V_{R_4} + V_{R_5}$	$V_{R_4} = R_4\,(I_3 + I_4)$
4) $0 = V_{R_3} + V_{R_4} + V_{R_6}$	$V_{R_5} = R_5\, I_3$
	$V_{R_6} = R_6\, I_4$

Plugging in numerical values and substituting the expressions for $V_{R_1}, ..., V_{R_6}$ into $1), .., 4)$ yields:

$$
\begin{array}{ll}
1) & 1 = 3\,I_1 - I_2 \\
2) & 0 = I_1 - 5\,I_2 - 4\,I_4 \\
3) & 2 = 6\,I_3 + 2\,I_4 \\
4) & 0 = 4\,I_2 + 2\,I_3 + 14\,I_4
\end{array}
\longrightarrow
\begin{pmatrix} 3 & -1 & 0 & 0 \\ 1 & -5 & 0 & -4 \\ 0 & 0 & 6 & 2 \\ 0 & 4 & 2 & 14 \end{pmatrix}
\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{pmatrix}
=
\begin{pmatrix} 1 \\ 0 \\ 2 \\ 0 \end{pmatrix}
\longrightarrow
\begin{array}{rr}
I_1 = & 5/13A \\
I_2 = & -2/13A \\
I_3 = & 19/52A \\
I_4 = & -5/52A
\end{array}
$$

Now that we know the values of all of the currents in the circuit, we can solve for the current through R_6 and the voltage across R_6, and so:

$$
P_{R_6} = V_{R_6}\, I_{R_6} = R_6\,(I_{R_6})^2 = \frac{25}{338}\,W
$$