University of California, San Diego
ECE 45 Spring 2014
Midterm Exam 2

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Print your name: ____

Student ID Number: ____

- No Books, No Notes, No calculators allowed

Question	Score
1	/25
2	/40
3	/20
4	/15
Total:	/100

TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

Section	Property	Aperiodic signal	Fourier transform
		$x(t)$ $y(t)$	$X(j\omega)$ $Y(j\omega)$
4.3.1	Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
4.3.2	Time Shifting	$x(t - t_0)$	$e^{-j\omega t_0}X(j\omega)$
4.3.6	Frequency Shifting	$e^{j\omega_0 t}x(t)$	$X(j(\omega - \omega_0))$
4.3.3	Conjugation	$x^*(t)$	$X^*(-j\omega)$
4.3.5	Time Reversal	$x(-t)$	$X(-j\omega)$
4.3.5	Time and Frequency Scaling	$x(at)$	$\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$
4.4	Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
4.5	Multiplication	$x(t)y(t)$	$\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$
4.3.4	Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
4.3.4	Integration	$\int_{-\infty}^{t} x(t)dt$	$\frac{1}{j\omega}X(j\omega) + \pi X(0)\delta(\omega)$
4.3.6	Differentiation in Frequency	$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
4.3.3	Conjugate Symmetry for Real Signals	$x(t)$ real	$X(j\omega) = X^*(-j\omega)$ $\mathcal{Re}\{X(j\omega)\} = \mathcal{Re}\{X(-j\omega)\}$ $\mathcal{Im}\{X(j\omega)\} = -\mathcal{Im}\{X(-j\omega)\}$ $\|X(j\omega)\| = \|X(-j\omega)\|$ $\sphericalangle X(j\omega) = -\sphericalangle X(-j\omega)$
4.3.3	Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
4.3.3	Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
4.3.3	Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ [$x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ [$x(t)$ real]	$\mathcal{Re}\{X(j\omega)\}$ $j\mathcal{Im}\{X(j\omega)\}$

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

TABLE 4.2 BASIC FOURIER TRANSFORM PAIRS

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	a_k
$e^{j\omega_0 t}$	$2\pi\delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$, otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$, otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$, otherwise
$x(t) = 1$	$2\pi\delta(\omega)$	$a_0 = 1, \quad a_k = 0, \; k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$)
Periodic square wave $x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & T_1 < \|t\| \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all k
$x(t)\begin{cases} 1, & \|t\| < T_1 \\ 0, & \|t\| > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, & \|\omega\| < W \\ 0, & \|\omega\| > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi\delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at}u(t), \mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!}e^{-at}u(t)$, $\mathcal{R}e\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ +\infty, & t = 0 \end{cases}$$

Problem 1) [25 pts]

a. [20 pts] Determine and plot the magnitude and the phase of the Fourier Transform of the function:

$$h(t) = \delta(t) - \left(\cos\left(t\ \frac{\pi}{2}\right) - j\sin\left(t\ \frac{\pi}{2}\right)\right)\frac{\sin\left(t\frac{\pi}{4}\right)}{t\ \pi}$$

b. [5 pts] Assume $h(t)$ is the impulse response to an LTI system. Determine the output for the following inputs:

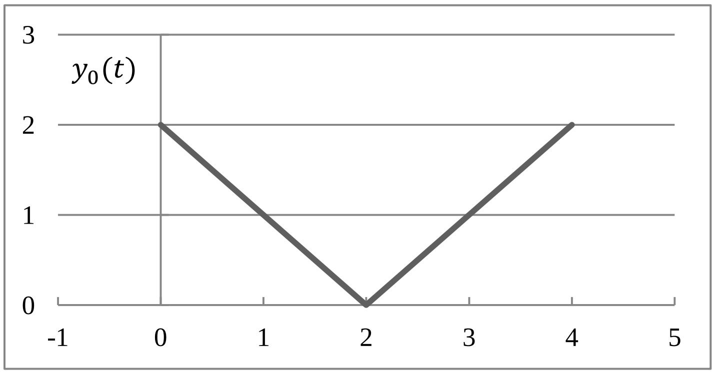
i. $x_1(t) = 2\cos\left(t\ \frac{2\pi}{3}\right)$

ii. $x_2(t) = \sin(4t) + 1$

Problem 2) [40 pts]

a. [20 pts] When the signal $x(t)$ is sent into the LTI system with transfer function $H_0(\omega)$, the output is $y_0(t)$. Determine the transfer function $H_0(\omega)$

$$x(t) = u(t+2) - 2u(t) + u(t-2)$$



b. [5 pts] Evaluate

$$\left(\int_{-\infty}^{\infty} |X(j\omega)|^2 \, d\omega\right)^2$$

c. [15 pts] $x(t)$ is sent into a new system with impulse response $h_1(t) = \delta(t-1) + \delta(t+1)$

Determine and plot the output $y_1(t)$

Problem 3) [20 pts]

a. [10 pts] If $F(j\omega) = B(j\omega)C(j\omega)$ and $G(j\omega) = \frac{e^{j\omega}}{2} B\left(\frac{j\omega}{3}\right) C\left(\frac{j\omega}{3}\right)$

Write $g(t)$ in terms of $f(t)$

b. **[10 pts]** Give an example of an input $x(t)$, a system $H(\omega)$, and an output $y_1(t)$ such that:

$$x(t) \rightarrow H(\omega) \rightarrow y_1(t) \quad \text{and} \quad x(2t) \rightarrow H(\omega) \rightarrow y_2(t) \neq y_1(2t)$$

(i.e. show that it is not generally true that the input $x(2t)$ has the output $y_1(2t)$)

Problem 4) [15 pts]

Suppose the signal $g(t)$ is sent into a system $H(\omega) = (2 + j\omega)$

Unfortunately, we don't know what $g(t)$, but we do know that

$$G(j\omega) = j\frac{d}{d\omega}F(-j\omega) \quad \text{where } F(j\omega) = \frac{1}{2-j\omega}$$

Determine the output, $y(t)$, of the system
(Hint: You can do this without performing calculus)

Bonus: [5 pts] Determine the Fourier transform of

$$x(t) = \frac{1}{2 - jt}$$