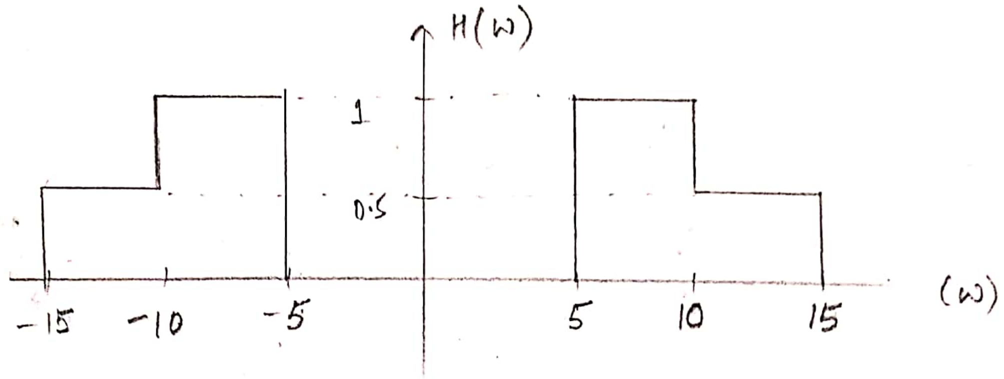
1.

a)



b) i) $X_1(\omega) = \begin{cases} \pi & |\omega| \leq 10 \\ 0 & \text{elsewhere} \end{cases}$.

Hence $X_1(\omega) \cdot H(\omega) \neq X_1(\omega) \Rightarrow$ 'Distorted signal'.

ii) $X_2(\omega) = [\delta(\omega-8) + \delta(\omega+8)] + \frac{1}{j}[\delta(\omega-8) + \delta(\omega+8)]$

$\Rightarrow$ 'Pass through Undistorted'.

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iii) $X_3(\omega) = 2\pi\delta(\omega) \Rightarrow$ 'Completely filtered out'.

iv) $X_4(\omega) = 1 \Rightarrow$ 'Distorted'.

2.) $$b\,y(t) - c\frac{dy(t)}{dt} - \frac{d^2y(t)}{dt^2} = x(t) + \frac{dx(t)}{dt}.$$

By taking Fourier transform of both sides of the differential equation,

$$bY(\omega) - c\,j\omega\,Y(\omega) - (j\omega)^2\,Y(\omega) = X(\omega) + j\omega X(\omega)$$

$$Y(\omega)\left[b - cj\omega - (j\omega)^2\right] = X(\omega)\left[1 + j\omega\right]$$

$$H(\omega) = \frac{Y(\omega)}{X(\omega)} = \frac{(1+j\omega)}{b - cj\omega - (j\omega)^2}$$

$$= \frac{(1+j\omega)}{3 - 2j\omega - (j\omega)^2}.$$

3)

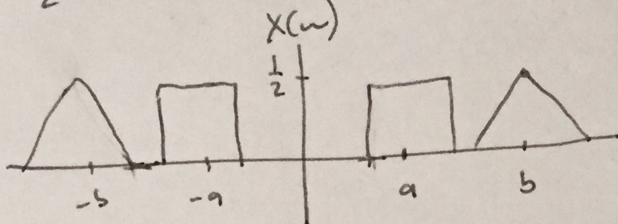
a) $x(t) = \cos(at) \cdot f(t) + \cos(bt) \cdot g(t)$

$$x(t) = \frac{e^{jat} + e^{-jat}}{2} \cdot f(t) + \frac{e^{jbt} + e^{-jbt}}{2} g(t)$$

$$x(t) = \frac{e^{jat}}{2} f(t) + \frac{e^{-jat}}{2} f(t) + \frac{e^{jbt}}{2} g(t) + \frac{e^{-jbt}}{2} g(t)$$

$$x(t) \longleftrightarrow X(\omega)$$

$$X(\omega) = \frac{1}{2} F(\omega - a) + \frac{1}{2} F(\omega + a) + \frac{1}{2} G(\omega - b) + \frac{1}{2} G(\omega + b)$$



b) $Y(\omega) = X(\omega) \cdot H(\omega)$

$Y(\omega) = \frac{1}{2} F(\omega - a) + \frac{1}{2} F(\omega + a)$

because $\omega_1 > a+1$, $\omega_1 < b-1$, so $G(\omega)$ term filtered out.

c) $y(t) = \mathcal{F}^{-1}(Y(\omega))$

Find $f(t)$ and apply Frequency shift property

$$F(\omega) = \begin{cases} 1 & -1 \le \omega \le 1 \\ 0 & \text{else} \end{cases} \longleftrightarrow \frac{\sin(t)}{\pi t} = \text{sinc}(t)$$

$$y(t) = \frac{1}{2} e^{jat} \text{sinc}(t) + \frac{1}{2} e^{-jat} \text{sinc}(t)$$

$$y(t) = \cos(at) \cdot \text{sinc}(t)$$