# University of California, San Diego
# ECE 45 Spring 2014
# Final Examination

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**Print your name**: ____________________________________

**Student ID Number:** _________________________________

- No Books, No Notes, No calculators allowed

| Question | Score |
|---|---|
| 1 | /35 |
| 2 | /40 |
| 3 | /50 |
| 4 | /30 |
| 5 | /45 |
| Total: | /200 |

## Problem 1) [35 pts]

Determine the minimum sampling frequency $f_s$ in $Hz$ needed to sample the following signals without aliasing:

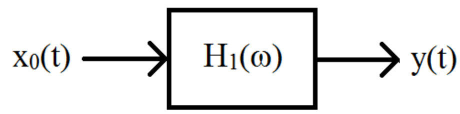
a. **[5 pts]** $f(t) = \cos(6000\,\pi\, t) + 2$

b. **[10 pts]** $h(t) = f(t)\,\left(\frac{\sin(1000\,\pi\, t)}{\pi t}\right)$

c. **[10 pts]** $x(t) = \left(\frac{\sin(1000\,\pi\, t)}{\pi t}\right) * \left(\frac{\sin(1000\,\pi\, t)}{\pi t}\right)$

d. **[10 pts]** Determine the energy of the signal $h(t)$ i.e. $\int_{-\infty}^{\infty} |h(t)|^2 dt$

**Problem 2) [40 pts]** Suppose you are trying to play your favorite song for your friend who lives very far away. Unfortunately, your friend does not have a computer, so we are forced to send the analog, continuous time signal of the song, $x_0(t)$. We are able to model the connection to your friend as the LTI system $H_1(\omega)$. In order for your friend to be able to correctly hear your song, we must have $y(t) = x_0(t)$ for the system below.

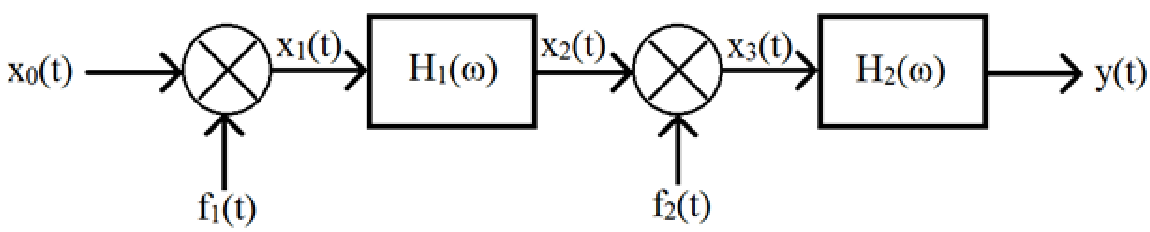



where $FT\big(x_0(t)\big) = X_0(j\omega) \neq 0$ when $|\omega| < 2\pi\ 10^4$ and $X_0(j\omega) = 0$ when $|\omega| > 2\pi\ 10^4$

and $H_1(\omega) = \begin{cases} 1 & 4\,\pi\ 10^4 \leq \omega \leq \pi\ 10^5 \\ 0 & else \end{cases}$

a. **[10 pts]** Show that with this set up, your friend will not correctly hear your song.

b. **[30 pts]** Luckily, you have taken ECE45 and propose that, with some modifications to our system, your friend can hear your song. You claim that we can select $f_1(t)$, $f_2(t)$, and $H_2(\omega)$ such that $y(t) = x_0(t)$. What should these functions be? Show that your friend can correctly hear your song.
(Hint: consider using modulation/frequency shifting)




Note: $x_1(t) = x_0(t)\ f_1(t)$ and $x_3(t) = x_2(t)\ f_2(t)$

## Problem 3) [50 pts]

a. **[35 pts]** Compute and sketch $x(t) = f(t) * g(t)$ where

$$g(t) = \begin{cases} t & -1 \le t \le 1 \\ 0 & else \end{cases} \quad \text{and} \quad f(t) = \begin{cases} 1 & 0 \le t < 2 \\ 0 & else \end{cases}$$

b. **[15 pts]** Determine $y(t) = f(t) * h(t)$ where

$$h(t) = \begin{cases} t - 1 & 0 \le t \le 2 \\ 3 - t & 2 < t \le 4 \\ 0 & else \end{cases}$$

(Hint: this can be done without any additional computation. Try to write $h(t)$ in terms of $g(t)$ and use the result from part a.)

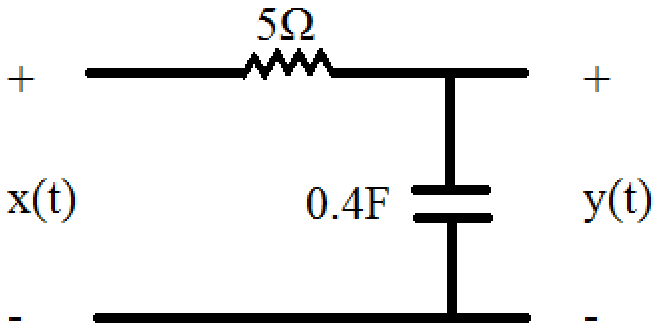
## Problem 4) [30 pts]

a. **[20 pts]** Write $y(t)$ in its simplest form, where $y(t) = x(t) * (u(t+1) - u(t-1))$ and $FT\big(x(t)\big) = X(j\omega) = \pi\,[\delta(\omega - 2\pi) + \delta(\omega - 1) + \delta(\omega + 1)]$

b. **[5 pts]** Is $x(t)$ periodic? If so, what is its period?

c. **[5 pts]** Is $y(t)$ periodic? If so, what is its period?

**Problem 5) [45 pts]**




a. **[10 pts]** Determine the impulse response $h(t)$ of the above circuit

b. **[15 pts]** What is $y(t)$ when the input to is $x(t) = e^{-2t}\, u(t)$

c. **[20 pts]** If the output of a (different) system is $y(t) = e^{-2(t-1)}u(t-1)$ when the input is $x(t) = u(t)$, find the transfer function $H(\omega)$

## TABLE 4.1 PROPERTIES OF THE FOURIER TRANSFORM

| Section | Property | Aperiodic signal | Fourier transform |
|---|---|---|---|
| | | $x(t)$<br>$y(t)$ | $X(j\omega)$<br>$Y(j\omega)$ |
| 4.3.1 | Linearity | $ax(t)+by(t)$ | $aX(j\omega)+bY(j\omega)$ |
| 4.3.2 | Time Shifting | $x(t-t_0)$ | $e^{-j\omega t_0}X(j\omega)$ |
| 4.3.6 | Frequency Shifting | $e^{j\omega_0 t}x(t)$ | $X(j(\omega-\omega_0))$ |
| 4.3.3 | Conjugation | $x^*(t)$ | $X^*(-j\omega)$ |
| 4.3.5 | Time Reversal | $x(-t)$ | $X(-j\omega)$ |
| 4.3.5 | Time and Frequency Scaling | $x(at)$ | $\frac{1}{\|a\|}X\left(\frac{j\omega}{a}\right)$ |
| 4.4 | Convolution | $x(t)*y(t)$ | $X(j\omega)Y(j\omega)$ |
| 4.5 | Multiplication | $x(t)y(t)$ | $\frac{1}{2\pi}\int_{-\infty}^{+\infty}X(j\theta)Y(j(\omega-\theta))d\theta$ |
| 4.3.4 | Differentiation in Time | $\frac{d}{dt}x(t)$ | $j\omega X(j\omega)$ |
| 4.3.4 | Integration | $\int_{-\infty}^{t}x(t)dt$ | $\frac{1}{j\omega}X(j\omega)+\pi X(0)\delta(\omega)$ |
| 4.3.6 | Differentiation in Frequency | $tx(t)$ | $j\frac{d}{d\omega}X(j\omega)$ |
| 4.3.3 | Conjugate Symmetry for Real Signals | $x(t)$ real | $\begin{cases} X(j\omega)=X^*(-j\omega) \\ \mathcal{R}e\{X(j\omega)\}=\mathcal{R}e\{X(-j\omega)\} \\ \mathcal{I}m\{X(j\omega)\}=-\mathcal{I}m\{X(-j\omega)\} \\ \|X(j\omega)\|=\|X(-j\omega)\| \\ \sphericalangle X(j\omega)=-\sphericalangle X(-j\omega) \end{cases}$ |
| 4.3.3 | Symmetry for Real and Even Signals | $x(t)$ real and even | $X(j\omega)$ real and even |
| 4.3.3 | Symmetry for Real and Odd Signals | $x(t)$ real and odd | $X(j\omega)$ purely imaginary and odd |
| 4.3.3 | Even-Odd Decomposition for Real Signals | $x_e(t)=\mathcal{E}v\{x(t)\}$ [$x(t)$ real]<br>$x_o(t)=\mathcal{O}d\{x(t)\}$ [$x(t)$ real] | $\mathcal{R}e\{X(j\omega)\}$<br>$j\mathcal{I}m\{X(j\omega)\}$ |

4.3.7 Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty}|x(t)|^2dt=\frac{1}{2\pi}\int_{-\infty}^{+\infty}|X(j\omega)|^2d\omega$$

**TABLE 4.2** BASIC FOURIER TRANSFORM PAIRS

| Signal | Fourier transform | Fourier series coefficients (if periodic) |
|---|---|---|
| $\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$ | $2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$ | $a_k$ |
| $e^{j\omega_0 t}$ | $2\pi\delta(\omega - \omega_0)$ | $a_1 = 1$<br>$a_k = 0$, otherwise |
| $\cos \omega_0 t$ | $\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$ | $a_1 = a_{-1} = \frac{1}{2}$<br>$a_k = 0$, otherwise |
| $\sin \omega_0 t$ | $\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$ | $a_1 = -a_{-1} = \frac{1}{2j}$<br>$a_k = 0$, otherwise |
| $x(t) = 1$ | $2\pi\delta(\omega)$ | $a_0 = 1, \quad a_k = 0, \ k \neq 0$<br>(this is the Fourier series representation for any choice of $T > 0$) |
| Periodic square wave<br>$x(t) = \begin{cases} 1, & \|t\| < T_1 \\ 0, & T_1 < \|t\| \le \frac{T}{2} \end{cases}$<br>and<br>$x(t + T) = x(t)$ | $\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$ | $\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$ |
| $\sum_{n=-\infty}^{+\infty} \delta(t - nT)$ | $\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$ | $a_k = \frac{1}{T}$ for all $k$ |
| $x(t) \begin{cases} 1, & \|t\| < T_1 \\ 0, & \|t\| > T_1 \end{cases}$ | $\frac{2 \sin \omega T_1}{\omega}$ | — |
| $\frac{\sin Wt}{\pi t}$ | $X(j\omega) = \begin{cases} 1, & \|\omega\| < W \\ 0, & \|\omega\| > W \end{cases}$ | — |
| $\delta(t)$ | $1$ | — |
| $u(t)$ | $\frac{1}{j\omega} + \pi\delta(\omega)$ | — |
| $\delta(t - t_0)$ | $e^{-j\omega t_0}$ | — |
| $e^{-at}u(t), \ \Re e\{a\} > 0$ | $\frac{1}{a + j\omega}$ | — |
| $te^{-at}u(t), \ \Re e\{a\} > 0$ | $\frac{1}{(a + j\omega)^2}$ | — |
| $\frac{t^{n-1}}{(n-1)!} e^{-at}u(t)$,<br>$\Re e\{a\} > 0$ | $\frac{1}{(a + j\omega)^n}$ | — |

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\omega)e^{j\omega t}d\omega \quad \leftrightarrow \quad X(j\omega) = \int_{-\infty}^{\infty} x(t)e^{-j\omega t}dt$$

$$f(t) * g(t) = \int_{-\infty}^{\infty} f(\tau)g(t-\tau)\, d\tau$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$u(t) = \begin{cases} 0, & t < 0 \\ 1, & t \geq 0 \end{cases}$$

$$\delta(t) = \begin{cases} 0, & t \neq 0 \\ +\infty, & t = 0 \end{cases}$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0}$$