University of California, San Diego
ECE 45 Spring 2019
FINAL EXAM

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Print your name:

Student ID Number:

Note: No books, calculators, or other electronic devices allowed.

Question	Score
Problem 1	/16
Problem 2	/25
Problem 3	/20
Problem 4	/17
Problem 5	/22
Total	/100

Signal	Fourier Transform
$x(t)$	$X(j\omega)$
$y(t)$	$Y(j\omega)$
$ax(t)+by(t)$	$aX(j\omega)+bY(j\omega)$
$x(t-t_0)$	$e^{-j\omega t_0}X(j\omega)$
$e^{j\omega_0 t}x(t)$	$X(j(\omega-\omega_0))$
$x^*(t)$	$X^*(-j\omega)$
$x(-t)$	$X(-j\omega)$
$x(at)$	$\frac{1}{\lvert a\rvert}X\left(\frac{j\omega}{a}\right)$
$x(t)*y(t)$	$X(j\omega)Y(j\omega)$
$x(t)y(t)$	$\frac{1}{2\pi}X(j\omega)*Y(j\omega)$
$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
$tx(t)$	$j\frac{d}{d\omega}X(j\omega)$
$X(t)$	$2\pi x(-j\omega)$
1	$2\pi\delta(\omega)$
$\delta(t)$	1
$u(t)$	$\frac{1}{j\omega}+\pi\delta(\omega)$
$\cos\omega_0 t$	$\pi[\delta(\omega-\omega_0)+\delta(\omega+\omega_0)]$
$\sin\omega_0 t$	$\frac{\pi}{j}[\delta(\omega-\omega_0)-\delta(\omega+\omega_0)]$
$e^{-at}u(t)$, Re $\{a\}>0$	$\frac{1}{a+j\omega}$
$\begin{cases}1, & \lvert t\rvert\le T\\ 0, & \lvert t\rvert>T\end{cases}$	$\frac{2\sin\omega T}{\omega}$
$\frac{\sin Wt}{\pi t}$	$\begin{cases}1, & \lvert\omega\rvert\le W\\ 0, & \lvert\omega\rvert>W\end{cases}$

Some Useful Formulas:

$$x(t)=\frac{1}{2\pi}\int_{-\infty}^{\infty}X(j\omega)e^{j\omega t}\,d\omega$$
$$X(j\omega)=\int_{-\infty}^{\infty}x(t)e^{-j\omega t}\,dt$$

$$u(t)=\begin{cases}1, & t\ge 0\\ 0, & t<0\end{cases}$$
$$\delta(t)=\frac{d}{dt}u(t)$$

$$e^{j\theta}=\cos\theta+j\sin\theta$$
$$\cos\theta=\frac{e^{j\theta}+e^{-j\theta}}{2}$$
$$\sin\theta=\frac{e^{j\theta}-e^{-j\theta}}{2j}$$

$$\int_{-\infty}^{\infty}\lvert x(t)\rvert^2\,dt=\frac{1}{2\pi}\int_{-\infty}^{\infty}\lvert X(j\omega)\rvert^2\,d\omega$$

$$e^{j\omega_0 t}\to H(\omega)\to H(\omega_0)e^{j\omega_0 t}$$

$$f(t)*g(t)=\int_{-\infty}^{\infty}f(\tau)\,g(t-\tau)\,d\tau$$

$$f(t)*g(t)=\int_{-\infty}^{\infty}f(\tau)\,g(t-\tau)\,d\tau=\int_{-\infty}^{\infty}g(\tau)\,f(t-\tau)\,d\tau=g(t)*f(t)$$

If $y(t)=x(t)*h(t)$ then $\frac{dx(t)}{dt}*h(t)=x(t)*\frac{dh(t)}{dt}=\frac{dy(t)}{dt}$

If $y(t)=x(t)*h(t)$ then $x(t-t_0)*h(t)=x(t)*h(t-t_0)=y(t-t_0)$

$$f(t)*(g(t)+h(t))=f(t)*g(t)+f(t)*h(t)$$

Sampling Perfect Reconstruction: $w_s>2\times w_m$

1) Determine if the following signals can be perfectly reconstructed with sampling frequency $w_s = 50\ Hz$. (4 pts each)

   a) $f(t) = sinc(10t)$

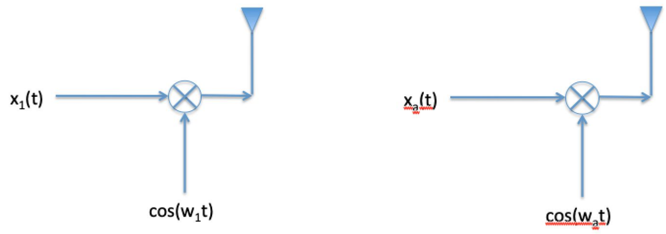
   b) $g(t) = sin^2(20t)$

   c) $h(t) = f(t) \times cos(20t)$

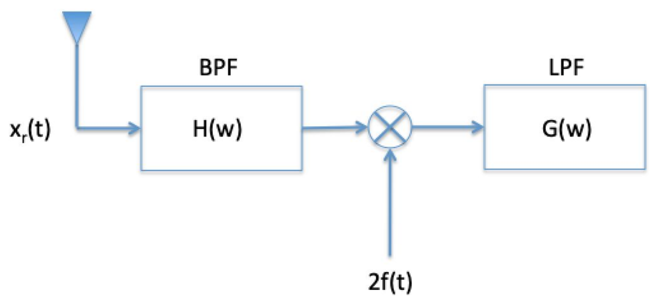
   d) $l(t) = f(t) \times \delta(t - 10)$

2) The company Massimo Networks has hired you (Congrats!) to design a system to receive a signal from JJ Networks. The transmitter is made at JJ Networks, so you will NOT change that. Also, AJ Networks sends signals too, but you do not want to hear AJ Network's signal. JJ Network's signal is $x_1(t)$ and AJ Network's signal is $x_a(t)$.

Transmitters:



Receiver:



Received signal: $x_r(t) = x_1(t) \times cos(w_1 t) + x_a(t) \times cos(w_a t)$

$w_a = 100$, $w_1 = 80$

Assume the spectrum below:

$$X_a(w) = \begin{cases} |w| & -10 \leq w \leq 10 \\ 0 & else \end{cases} \quad X_1(w) = \begin{cases} 1 & -5 \leq w \leq 5 \\ 0 & else \end{cases}$$

a) Design H(w) to only capture the signal from JJ Networks. Justify AND give numerical values for frequencies. Explain the filter's purpose. (7 points)
b) Design f(t). Justify. Explain its purpose. (5 points)
c) Design G(w) so that any high frequency terms are eliminated. Justify AND give numerical values for frequencies. Explain the filter's purpose. (5 points)
d) If JJ Networks desires to only transmit in the range of 1000-1100 Hz, explain the change you would make on the transmitter and provide a numerical answer. (5 points)
a) Design the receiver to recover only $x_1(t)$ without the BPF, H(w). (3 points)

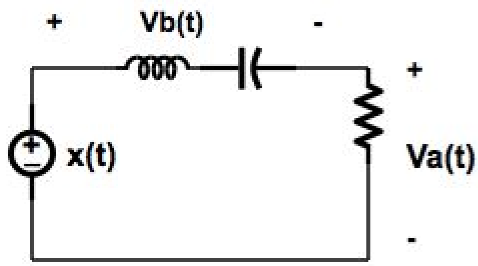
3) a. Convolution. Compute and sketch x(t) = h(t) $*$ g(t) where (15 points)

$$h(t) = \begin{cases} -t & |t| \leq 1 \\ t-2 & 1 \leq t \leq 2 \\ t+2 & -1 \leq t \leq -2 \\ 0 & elsewhere \end{cases}$$

g(t)= u(t)-u(t-3)

b. Determine: $y(t) = \dfrac{dh(t)}{dt} * g(t)$ (5 points)

4) The following circuit is a band-stop filter. It is the opposite of a band-pass filter, which passes most frequencies unaltered, but attenuates those in a specific range to very low levels.



where R=1 ohm, L=2/3H, C=3F.

a. Determine the $H_1(jw)=V_a(jw)/X(jw)$ and $H_2(jw)=V_b(jw)/X(jw)$ for the above circuit. (10 points)

b. What is $v_a(t)$ when the input is $x(t) = e^{-3t}u(t)$. (7 points)

5) Simplify the following expressions as much as possible:

a) Given $x(t)$ (periodic frequency w_o) and its Fourier Series Coefficients, X_n, find the Fourier Series Coefficients, G_n, for $g(t) = x(-2t-4)$. (6 points)

b) Given $y(t)$ and its Fourier Series Coefficients, Y_n, find the Fourier Series Coefficients, H_n, for $h(t) = \frac{d}{dt}(y(t-1))$. (6 points)

c) $c(t) = \int_{6}^{\infty}(\tau^2+5)\delta(\tau-5)d\tau$ (5 points)

d) Determine the energy of the signal $x(t) = \frac{2}{jt+4}u(t)$. (5 points)