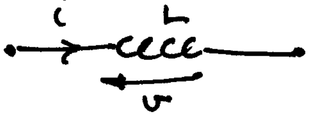
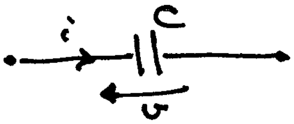
ECE 45
Prof. Franceschetti
Lecture Notes #1

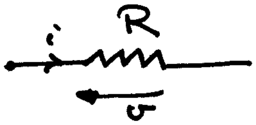
I will assume you are familiar with RLC circuits. In particular recall the fundamental relationships:



$$v(t) = L\frac{di(t)}{dt}$$



$$i(t) = C\frac{dv(t)}{dt}$$



$$v(t) = R\,i(t)$$

Above relationships are <u>linear</u> so they lead to <u>linear</u> circuits, however, as you have seen in ECE 35 to solve these circuits you need to solve differential equations. Now students <u>HATE</u> to solve differential equations in time domain. For your pleasure in this class we will develop alternative methods that allow simpler analysis of linear circuits. The first method is the **PHASOR METHOD**

This is used for analysis of linear circuits under sinusoidal excitation. <u>Warning!</u> The phasor method is simple but nothing comes for free! You will need to know complex numbers.

PHASORS

Let us consider for example a sinusoidal current

$$i(t) = A \cos(\omega t + \phi)$$

amplitude angular frequency phase

This is the real part of a complex number:

$$Re\left[A e^{j(\omega t+\phi)}\right] = Re\left[A \cos(\omega t+\phi) + j \sin(\omega t+\phi)\right]$$
$$= A \cos(\omega t+\phi)$$

Now, any RLC circuit excited by a sinusoid produces only voltages and currents that are at the <u>same frequency</u> of the excitation. The amplitudes and phases only will vary. [<u>QUESTION</u>: do you understand why is this?]

So it is reasonable that in the analysis of these circuits we "forget" about the frequency and keep track of the changes in the amplitude and phase. We do this by introducing the concept of phasor:

$$A \cos(\omega t + \phi) \longleftrightarrow A e^{j\phi}$$

real domain sinusoid complex domain phasor

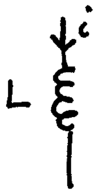
Notice to go back from the complex phasor domain to the real domain we just need to multiply by $e^{j\omega t}$ and take the real part:

$$Re\left[A e^{j\phi} e^{j\omega t}\right] = A \cos(\omega t + \phi)$$

So, what is this good for?

The idea of the phasor method is to Transform all the quantities in our circuit into the complex domain, then solve the circuit using complex numbers, and finally obtain the real solution by multiplying by $e^{j\omega t}$ and taking the real part.

Let us start with the first step:



$$v(t) = L\frac{d\,i(t)}{dt}$$

let $v(t) = A\cos(\omega t + \phi)$

$$A\cos(\omega t+\phi) = L\frac{d\,i(t)}{dt}$$

$$\frac{A}{L}\int \cos(\omega t+\phi)\,dt = i(t)$$

$$\frac{A}{L\omega}\sin(\omega t+\phi) = i(t)$$

$$\boxed{\frac{A}{L\omega}\cos(\omega t+\phi-\pi/2) = i(t)}$$

amplitude is scaled by $L\omega$

phase is shifted by $\pi/2$

Let's write this result using phasors:

$$\underline{V} = A e^{j\phi}$$

$$\underline{I} = \frac{A}{L\omega} e^{j(\phi-\pi/2)} = \frac{A}{j\omega L} e^{j\phi}$$

$$\boxed{\underline{V} = j\omega L\ \underline{I}}$$

<u>EXERCISE</u>: show that the same steps lead for the capacitor to the relation:

$$\boxed{\underline{V} = \frac{1}{j\omega C}\underline{I}}$$

and finally for the resistor we have:

$$\boxed{\underline{V} = R\,\underline{I}}$$

Note that in all three cases we have a phasor equation of the form:

(1) $$\boxed{\underline{V} = Z\,\underline{I}}$$

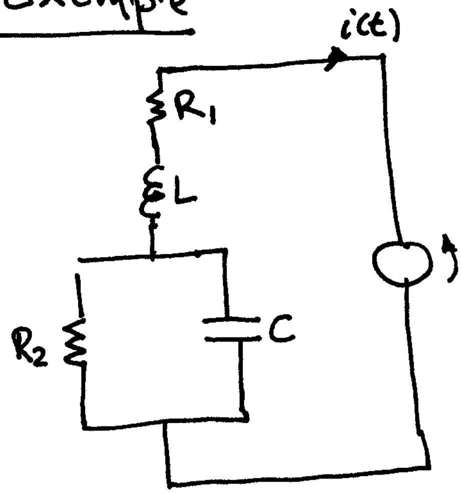
where $Z = R$ for resistor
$Z = \frac{1}{j\omega C}$ for capacitor
$Z = j\omega L$ for inductor

The nice thing is that since (1) has the same form of a voltage drop over a resistor, we can solve easily any circuit in the phasor domain like we solve a pure resistor circuit!
At the end we will only need to multiply our result by $e^{j\omega t}$ take the real part, and we are done, obtaining the final result in time domain.

PHASOR METHOD for the analysis under Sinusoidal excitation

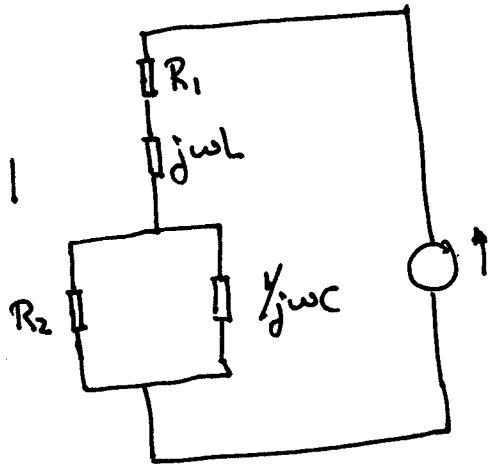
1) Take phasor of the input
2) change all circuit components into impedances Z
3) solve the circuit of impedances like it was a resistor network
4) multiply by $e^{j\omega t}$ the solution
5) take the real part as the final solution.

Example



$v(t) = A\cos(\omega t + \phi)$

Find the current $i(t)$



$\underline{V} = A e^{j\phi}$

$$Z_{eq} = R_1 + j\omega L + \left(R_2 \,/\!/\, \frac{1}{j\omega C} \right)$$

$$= R_1 + j\omega L + \frac{R_2 / j\omega C}{R_2 + \frac{1}{j\omega C}}$$ this is a complex number

$$\underline{V} = Z_{eq}\, \underline{I}$$

$$\underline{I} = \frac{\underline{V}}{Z_{eq}} = \frac{A e^{j\phi}}{Z_{eq}}$$

$$\boxed{i(t) = Re\left[\frac{A e^{j\phi} e^{j\omega t}}{Z_{eq}} \right]}$$

so we need to know how to divide complex numbers and take the real part.