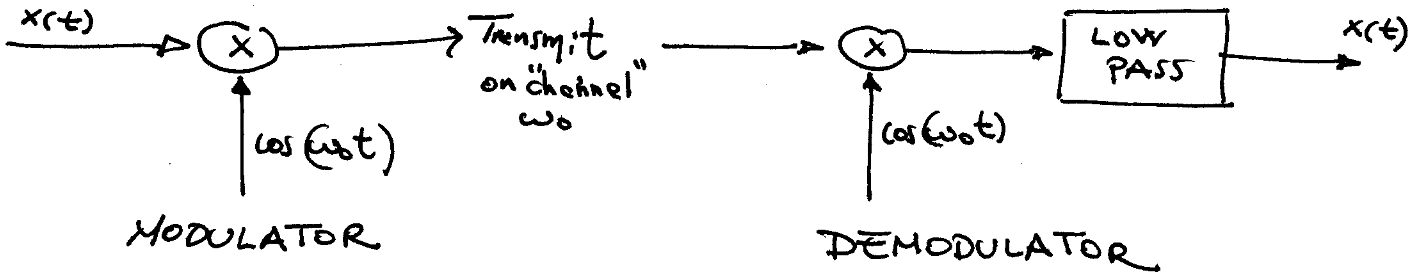
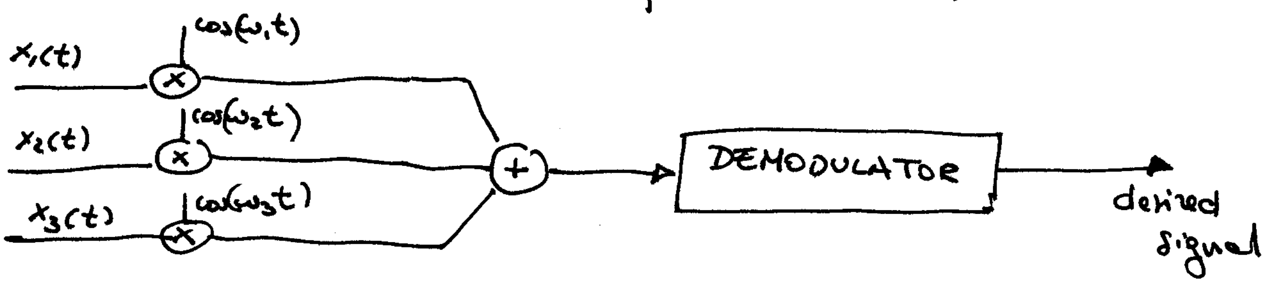
ECE 45 Lecture # 12

Some more on Modulation.

Recall Amplitude Modulation from last lecture.

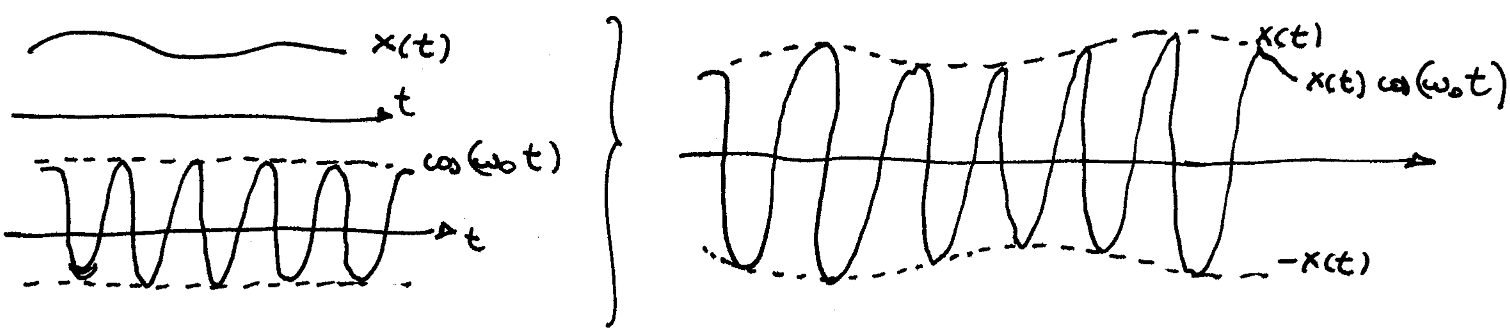


This scheme is used to "multiplex" different channels in frequency (see the example at the end of last lecture.



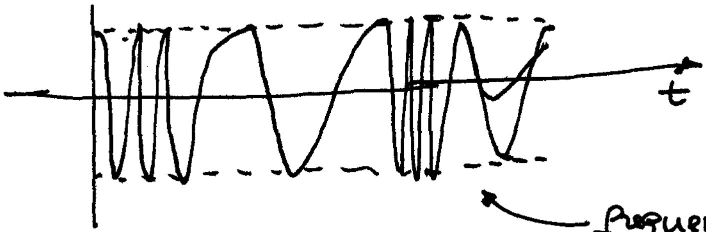
Now, let's give an interpretation of modulation in time-domain.

Consider $x(t)\cos(\omega_0 t)$



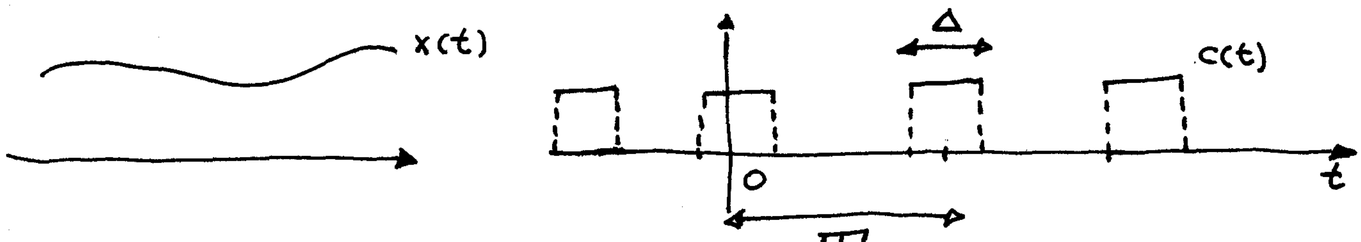
Now we can understand why it is called amplitude modulation. The signal appears in the modulation of the amplitude of the cosine.

A similar idea is performed to implement frequency modulation FM signals. Although we do not discuss this in detail, the main point is that the signal now appears in the modulation of the frequency of the cosine:



PULSE MODULATION

Another type of modulation is obtained by considering a periodic rectangular wave as the carrier.



consider $x(t) \cdot c(t) = y(t)$

By the convolution property, in frequency domain this becomes:

$$Y(j\omega) = \frac{1}{2\pi} X(\circledast C(j\omega) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} X(j\omega')\, C(j(\omega-\omega'))\,d\omega'$$

Now let's recall what is the spectrum of $c(t)$

$c(t)$ is periodic of period T

Therefore the spectrum is a train of δ-functions each weighted by the corresponding coeff. of the Fourier Series of $c(t)$ and multiplied by 2π.

$$C(j\omega) = 2\pi \sum_{k=-\infty}^{+\infty} c_k\, \delta(\omega - k\omega_0) \qquad \boxed{\omega_0 = \frac{2\pi}{T}}$$

See lecture notes #10 for a derivation of this.

$$Y(j\omega) = \sum_{k=-\infty}^{+\infty} \cancel{\frac{2\pi}{2\pi}} \int_{-\infty}^{+\infty} X(j\omega')\ c_k\ \delta(\omega - k\omega_0 - \omega')\, d\omega'$$

$$= \boxed{\sum_{k=-\infty}^{+\infty} c_k\, X(j(\omega - k\omega_0))}$$

So we obtain the spectrum of $x(t)$ replicated in frequency with each replica weighted by the corresponding Fourier coefficient of $c(t)$. Now let's recall what are these Fourier coefficients for the square wave (train of rectangles).

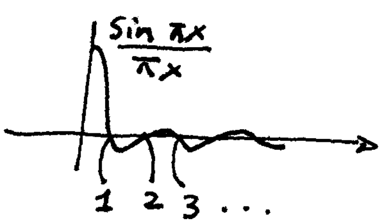
$$\begin{cases} c_0 = \dfrac{\Delta}{T} \\ c_k = \dfrac{\sin(k\omega_0 \Delta/2)}{k\pi} = \dfrac{\sin\left(\pi \frac{k\Delta}{T}\right)}{\frac{k\pi\Delta}{T}} \dfrac{\Delta}{T} = \boxed{\dfrac{\Delta}{T}\, \mathrm{sinc}\left(\dfrac{\pi k \Delta}{T}\right)} \end{cases}$$

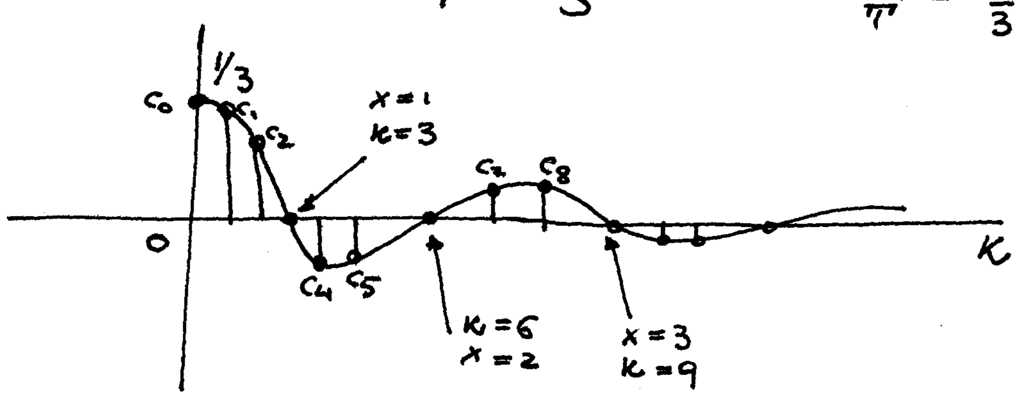
See lecture notes #6 for a derivation of this.

Now let's plot the coefficients (be careful here!)

Let us assume $\Delta = 1$, $T = 3$

$$\frac{\Delta}{T} = \frac{1}{3}, \quad \text{let } k\frac{\Delta}{T} = \frac{k}{3} = x$$

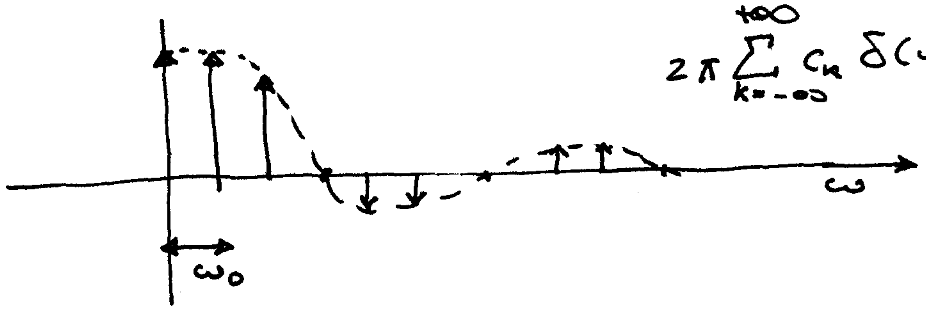




This is the sequence of coefficients $c_0, c_1, c_2 \ldots$

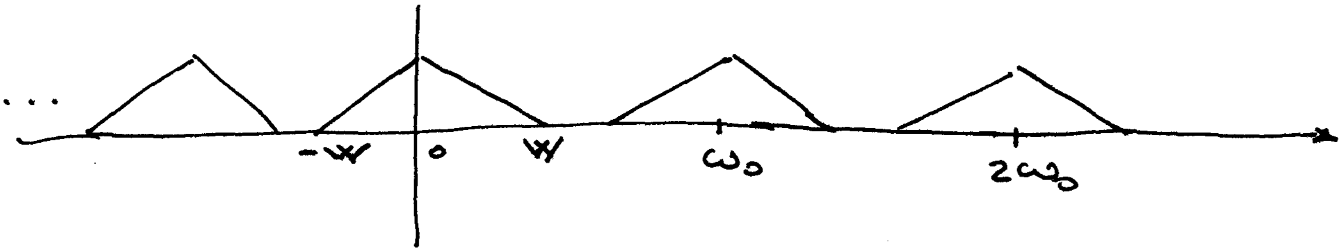
From the sequence of coefficients we now have that the spectrum $C(j\omega)$ is the corresponding sequence of δ-functions:

$$2\pi \sum_{k=-\infty}^{+\infty} c_k\, \delta(\omega - k\omega_0)$$



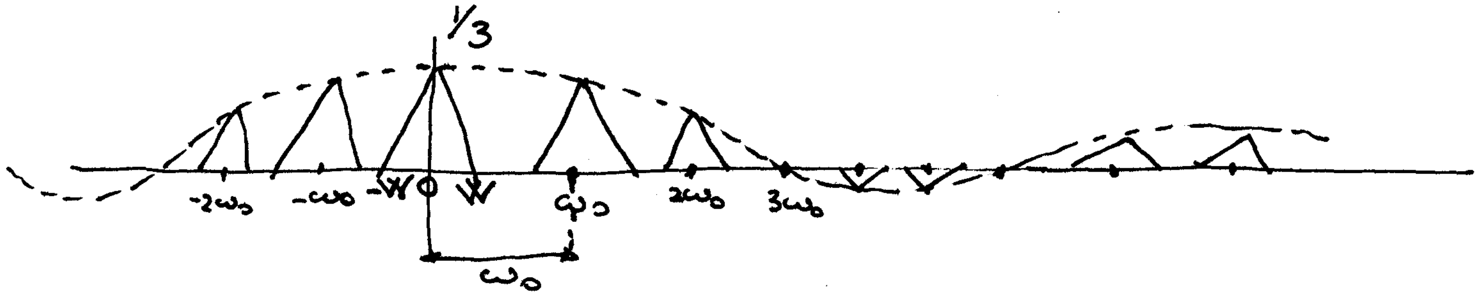
The replicated spectrum of $x(t)$ is

$$\sum_{k=-\infty}^{+\infty} X(j(\omega - k\omega_0))$$



And finally the replicated and modulated spectrum of $x(t)$ is

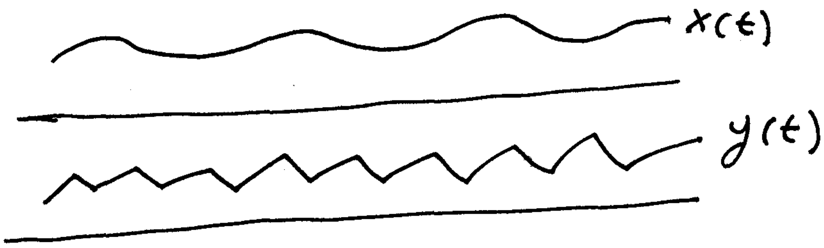
$$\sum_{k=-\infty}^{+\infty} c_k\, X(j(\omega - k\omega_0))$$

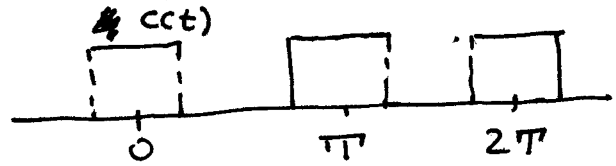


So provided that $\boxed{\omega_0 > 2W}$ we can apply a low-pass filter to de-modulate and re-obtain $x(t)$

What is this good for?

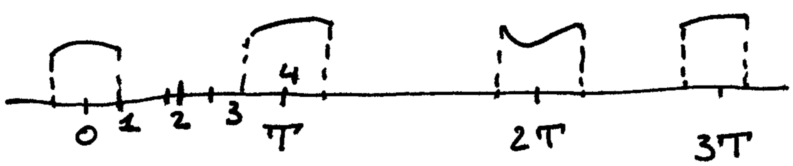
Suppose you have two signals $x(t)$ and $y(t)$





If we multiply $x(t)$ by the square wave we obtain

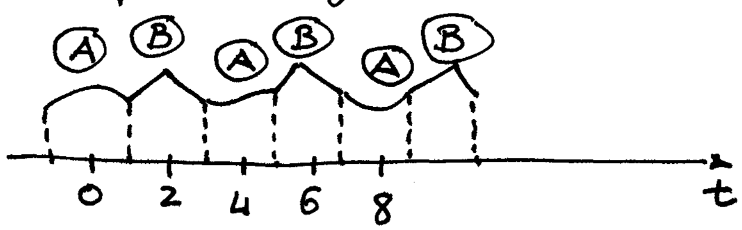
$x(t) \cdot c(t)$



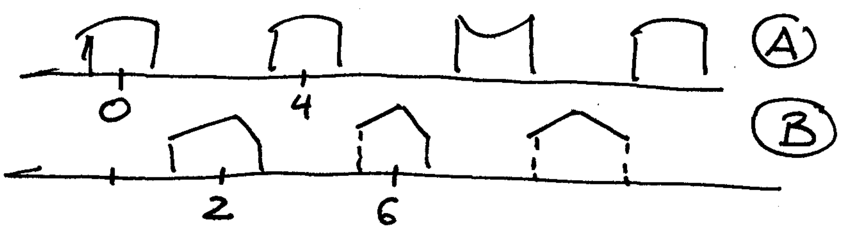
And we know that we can recover $x(t)$ by low-pass filtering

Now say $T = 4$, $\Delta = 2$

Then we have an empty interval $\Delta = 2$ that can be used to represent $y(t)$. So we have dividing time between $x(t)$ and $y(t)$



We can transmit this signal and then recover $x(t)$ at the receiver by low-passing the A components and $y(t)$ by low-passing the B components



<u>Two questions:</u>

1. What else is needed beside low-pass to recover $y(t)$?
2. Can modulation be implemented using a linear system?