PHASOR METHOD

The phasor method can be applied to find the response of a circuit to sinusoidal excitation (AC analysis)

It is based on the following steps:

1. Write the input in the form $A\cos(\omega t+\varphi)$
2. Write the corresponding phasor $Ae^{j\varphi}$
3. Change circuit components into impedences

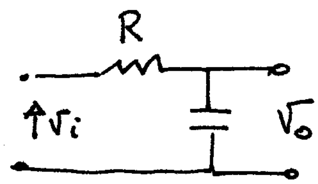
$$R \to R$$
$$C \to 1/j\omega C$$
$$L \to j\omega L$$

4. Solve impedence circuit as it was a resistor network
5. Write the phasor of the solution $Be^{j\alpha}$ (Response in PHASOR Domain)
6. Multiply by $e^{j\omega t}$
7. Take the real part (Response in Time domain)

NOTE: For step 1 if instead than $\cos(\omega t+\varphi)$ you have $\sin(\omega t+\varphi)$ what do you do?

Remember $\sin(\omega t+\varphi) = \cos(\omega t+\varphi-\pi/2)$

How about you have two sinusoids?
We'll see this with an example.



Consider a simple RC circuit where the input is v_i and the output is v_o

Using phasors we can write the following response:

$$V_o = \frac{Z_c}{Z_R + Z_c} V_i = \frac{1/j\omega C}{R + 1/j\omega C} V_i = \frac{1}{1 + j\omega RC} V_i$$

We call:

$$\boxed{H(\omega) = \frac{1}{1 + j\omega RC}}$$

This is an important quantity. It gives the response to an excitation at a given frequency ω.

Assume now the input is given by two sinusoids:

$$v_i(t) = \frac{4}{\pi} \sin(8\pi t) + \frac{2}{\pi} \sin(16\pi t)$$

we can use the principle of superposition and solve separately for the two frequencies and then add the results. Let $\omega_1 = 8\pi$, $\omega_2 = 16\pi$

$$V_1 = \frac{4}{\pi} e^{j0} e^{-j\pi/2} \qquad \text{phasor for } v_1 = \frac{4}{\pi}\sin(8\pi t)$$

$$V_2 = \frac{2}{\pi} e^{j0} e^{-j\pi/2} \qquad \text{phasor for } v_2 = \frac{2}{\pi}\sin(16\pi t)$$

$$H(\omega_1) = \frac{1}{1 + j8\pi RC} \qquad H(\omega_2) = \frac{1}{1 + j16\pi RC}$$

Response to V_1: $\quad V_o^{(1)} = \dfrac{1}{1 + j8\pi RC} \dfrac{4}{\pi} e^{-j\pi/2} = |H(\omega_1)| \dfrac{4}{\pi} e^{j\angle H(\omega_1)} e^{-j\pi/2}$

Response to V_2: $\quad V_o^{(2)} = \dfrac{1}{1 + j16\pi RC} \dfrac{2}{\pi} e^{-j\pi/2} = |H(\omega_2)| \dfrac{2}{\pi} e^{j\angle H(\omega_2)} e^{-j\pi/2}$

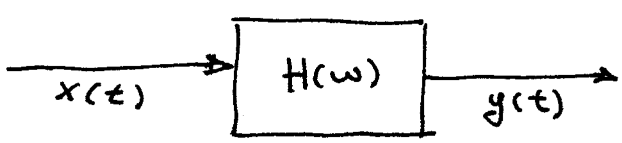
We now go back to time domain multiplying by $e^{j\omega t}$ and taking the real part.

$$Re\left[V_o^{(1)}\, e^{j8\pi t}\right] + Re\left[V_o^{(2)}\, e^{j16\pi t}\right]$$

$$= \frac{4}{\pi}\,|H(8\pi)|\,\sin\left(8\pi t + \angle H(8\pi)\right) + \frac{2}{\pi}|H(16\pi)|\sin\left(16\pi t + \angle H(16\pi)\right)$$

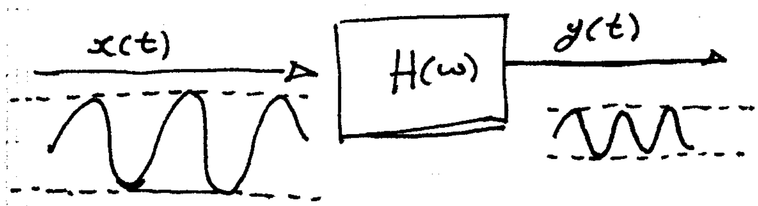
Notice that each sinusoid undergoes a phase shift given by the phase of H at <u>the given frequency</u> and is multiplied by the amplitude of H at <u>the given frequency</u>

The response function obtained writing $\frac{\text{output}}{\text{input}}$ in phasor domain is quite important as it tells us the attenuation and phase shift of ~~any~~ a sinusoid of any given frequency. We can represent this in abstract terms as a system of input $x(t)$ output $y(t)$



in the previous case $x(t)$ was ~~inp~~ voltage V_i and $y(t)$ was voltage V_o on capacitor.

$H(\omega)$ is the response to a sinusoid of frequency ω.
Suppose $|H(\omega)| < 1$ it means the sinusoid is attenuated by the system.

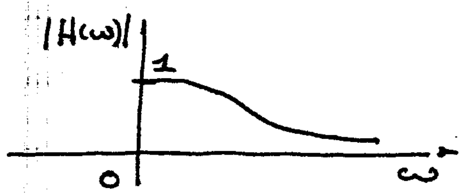


Attenuated by how much?
It depends on the frequency of the sinusoid and on the function $H(\omega)$.
For example:

$$V_o = V_i \frac{1}{1+j\omega RC}$$

$\omega \to 0 \quad V_o = V_i$
$\omega \to \infty \quad V_o = 0$

$$H(\omega) = \frac{1}{\sqrt{1+(\omega CR)^2}} e^{-j \arctan \omega CR}$$

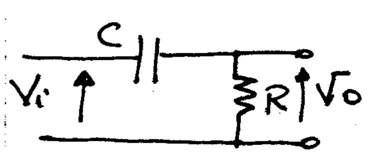


This shows that high frequency sinusoids are attenuated and low frequency sinusoids are not.

Since H attenuates only high frequencies, it is called <u>LOW-PASS</u> filter.

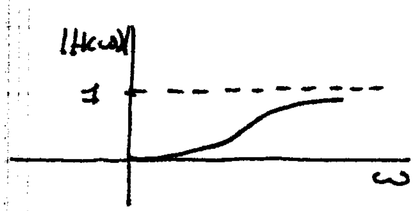
$$\frac{\omega CR\, e^{j\pi/2}}{\sqrt{1+(\omega CR)^2}\, e^{j \arctan \omega CR}}$$

Consider instead:



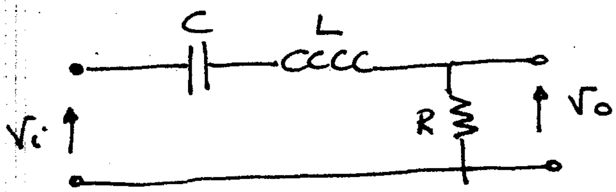
$$V_o = V_i \frac{R}{R+\frac{1}{j\omega C}} = \frac{j\omega C R}{1+j\omega CR} V_i = \frac{\omega CR}{\sqrt{1+(\omega CR)^2}} e^{-j(\arctan \omega CR - \pi/2)}$$

$\omega \to \infty \quad V_o = V_i$
$\omega \to 0 \quad V_o = 0$



<u>HIGH-PASS</u> filter
because it attenuates only the low frequencies

BAND-PASS filter



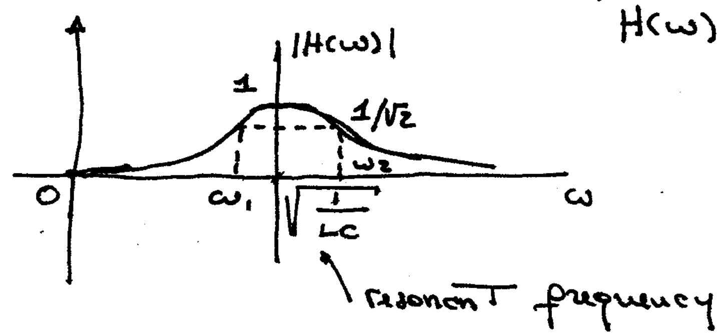
$$V_o = V_i \frac{R}{R + \frac{1}{j\omega C} + j\omega L} = \underbrace{\frac{j\omega C R}{j\omega C R + 1 + (j\omega)^2 LC}}_{H(\omega)} V_i$$

$\omega = \sqrt{\frac{1}{LC}}$ $\quad |H(\omega)| = 1$

$\omega \to \infty$ $\quad |H(\omega)| = 0$

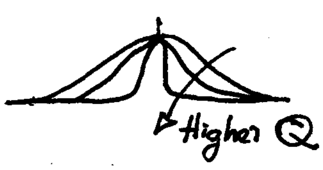
$\omega \to 0$ $\quad |H(\omega)| = 0$

$\omega_2 - \omega_1$ is the BAND of the filter



$Q = \frac{1}{R}\sqrt{\frac{L}{C}}$ is the quality factor of the filter.

Q increases the sharper is the resonance effect and the filter becomes more frequency selective



How about the phase?

Remember that
1. Any complex number is characterized by **magnitude** AND **phase**
2. Response $H = |H|\, e^{j\angle H}$

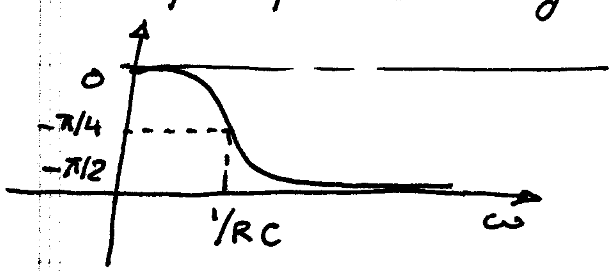
multiplication factor for amplitude sinusoid at the input

phase factor to be added to the sinusoid

LOW PASS FILTER

We have seen $H = \dfrac{V_o}{V_i} = \dfrac{1}{\sqrt{1+(\omega CR)^2}}\, e^{-j \arctan \omega CR}$

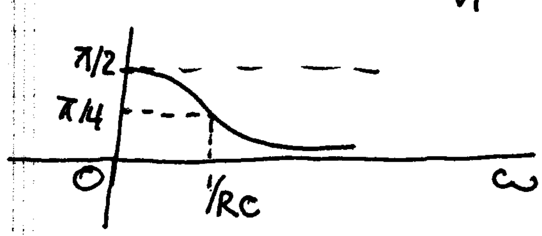
The phase plot is $-\arctan \omega CR$



Gives a negative shift in frequency to the applied sinusoid

HIGH PASS FILTER

We have seen $H = \dfrac{V_o}{V_i} = \dfrac{\omega CR}{\sqrt{1+(\omega CR)^2}}\, e^{+(-j \arctan \omega CR + \pi/2)}$



Gives a positive shift in frequency to the applied sinusoid