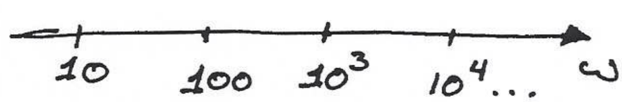
ECE 45
Prof. Franceschetti
Lecture notes #3

FREQUENCY RESPONSE BODE PLOTS

Last lecture we have focused on the Frequency Response $H(\omega)$ which is a complex quantity that describes the behavior of the system under sinusoidal excitation.
Bode Plots are a convenient graphical representation of the frequency response which makes it possible to appreciate exceedingly small values of the response (see attached figures)

The trick is to represent the response on a logarithmic scale rather than on a linear scale.
We need to plot $|H(\omega)|$ an $\angle H(\omega)$ separately
We let ω vary on a logarithmic scale, which means that on the ω-axis we report decades



Then for $\angle H(\omega)$ we report the angle radians (or degrees), while for $|H(\omega)|$ we report dB's which is $20\log_{10}|H(\omega)|$

- Let us start with our favorite low-pass filter:

$$\frac{1}{1+j\omega RC} = H(\omega)$$

$$|H(\omega)| = \frac{1}{\sqrt{1+(\omega RC)^2}}$$

$$20 \log |H(\omega)| = -20 \log \sqrt{1+(\omega RC)^2}$$

Let $\frac{1}{RC} = \omega_0$ Then we have

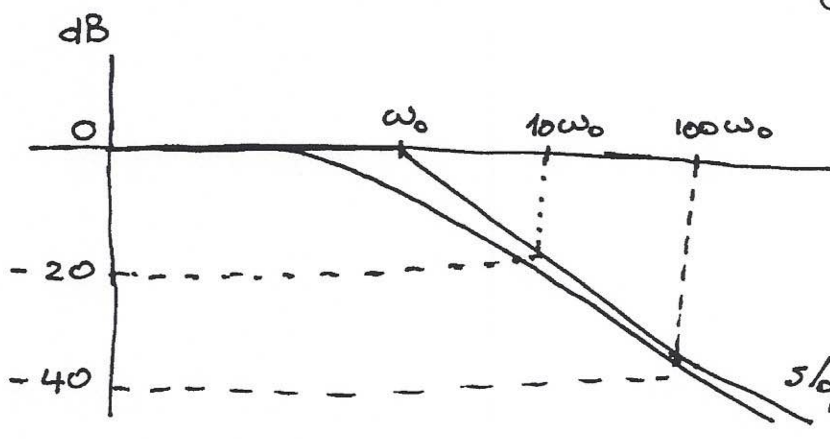
$$20 \log |H(\omega)| = -20 \log \sqrt{1+(\omega/\omega_0)^2}$$

We study Two <u>asymptotic regimes</u>

(1) $\omega \ll \omega_0 \Rightarrow -20 \log |H(\omega)| \approx -20 \log 1 = 0\,dB$

(2) $\omega \gg \omega_0 \Rightarrow -20 \log |H(\omega)| \approx -20 \log \sqrt{\left(\frac{\omega}{\cancel{\omega_0}}\right)^2} = -20 \log \omega$

which is a line of constant slope -20 dB/decade in dB scale



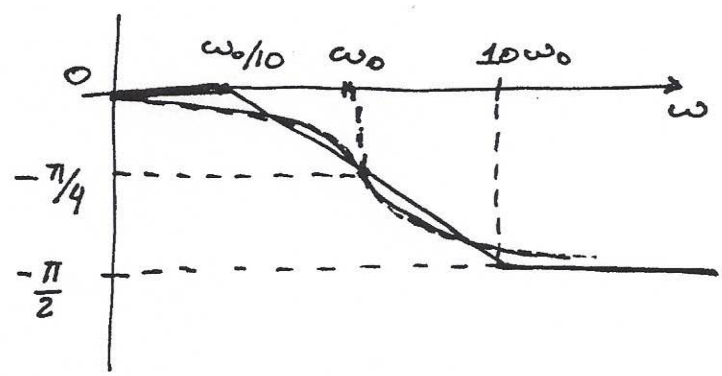
slope is -20 dB/decade

<u>Question</u>: what happens when $\omega = \omega_0$?

$$-20 \log \sqrt{2} = -10 \log_{10} 2 \approx -3\,dB$$

The plot falls 3dB below maximum value, This identifies Bandwidth of filter

The situation is analogous in the study of the phase.
The trick is always to identify the behavior in the asymptotic regime.
From last lecture we know that the phase shift will be close to zero for $\omega \ll \omega_0$ and will tend to $-\pi/2$ and for $\omega = \omega_0$ the phase is $-\pi/4$



So approximately we say that the phase starts shifting <u>one decade before</u> ω_0 and has shifted by $-\pi/2$ <u>one decade after</u> ω_0

SUMMARY

For magnitude $|H(\omega)|$
- identify the "breakpoint" ω_0
- until ω_0 the magnitude is 0 dB
- after ω_0 the magnitude is a line of constant slope 20 dB/deca

For phase $\angle H(\omega)$
- identify the "breakpoint" ω_0
- The phase shift starts one decade before ω_0
- the phase shift is practically complete one decade after ω_0

Now, we can generalize this asymptotic method to more complicated expressions of the frequency response. Let's look at an example.

$$H(\omega) = \frac{0.1\,j\omega + 20}{2\cdot 10^{-5}(j\omega)^3 + 0.1002\,(j\omega)^2 + j\omega}$$

First, we write the equation in standard form.

$$H(\omega) = \frac{0.1\,j\omega + 20}{2\cdot 10^{-5}(j\omega)^3 + 0.1002\,(j\omega)^2 + j\omega} = $$

$$= \frac{20\,(j\omega/200 + 1)}{j\omega\left(\frac{j\omega}{10} + 1\right)\left(\frac{j\omega}{5000} + 1\right)}$$

SPECIAL FACTOR

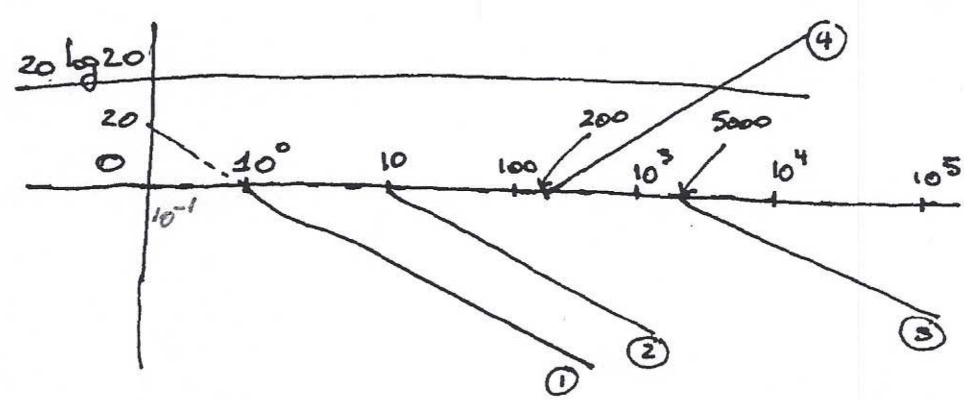
$$\left|\frac{1}{j\omega}\right|_{dB} = \left|e^{-j\pi/2}\,\frac{1}{\omega}\right|_{dB} = \left|\frac{1}{\omega}\right|_{dB}$$

$-20 \log \omega$

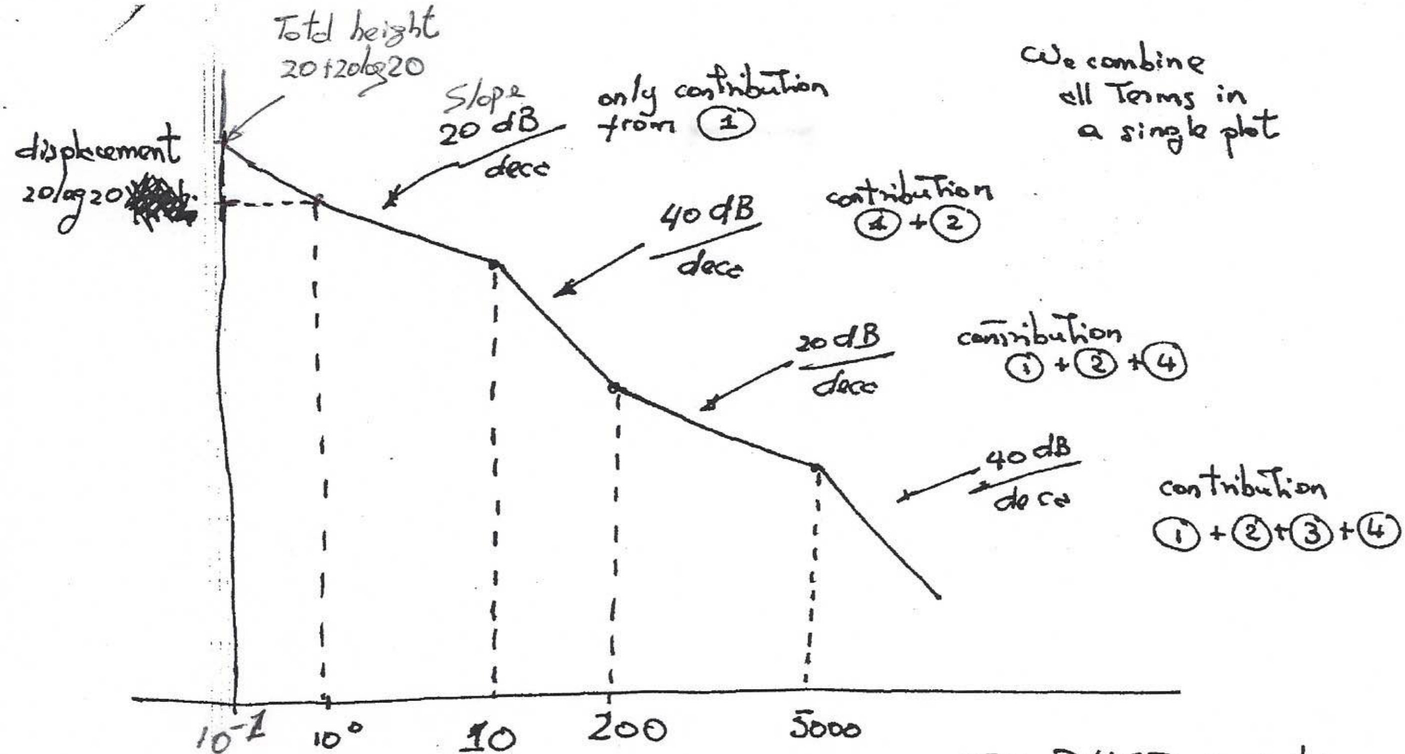
Now we consider all the factor separately. This is possible because:

$$20\log_{10} H(\omega) = 20\log 20 + 20\log\left(1 + \frac{j\omega}{200}\right) - 20\log j\omega - 20\log\left(1 + j\omega/10\right) - 20\log\left(1 + j\omega/5000\right)$$

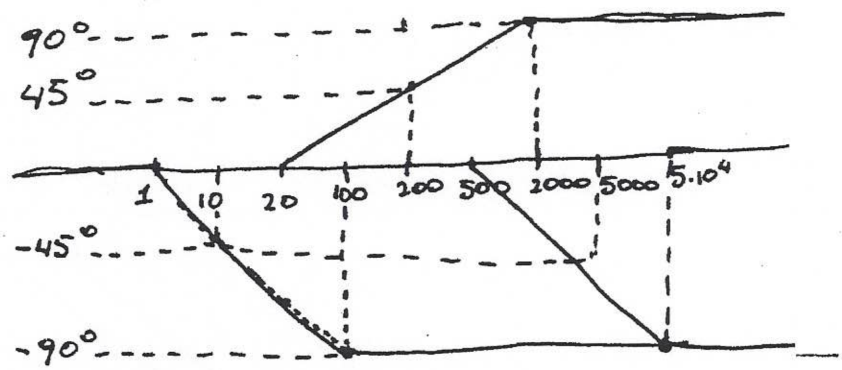
For each of those terms we write an asymptotic Bode plot like the one we have seen for the low-pass filter and then at the end we sum all of them. Let's look at the magnitude first



We combine all Terms in a single plot



FOR PHASE is similar



First find points

10 → -45° (A)

200 → +45° (B)

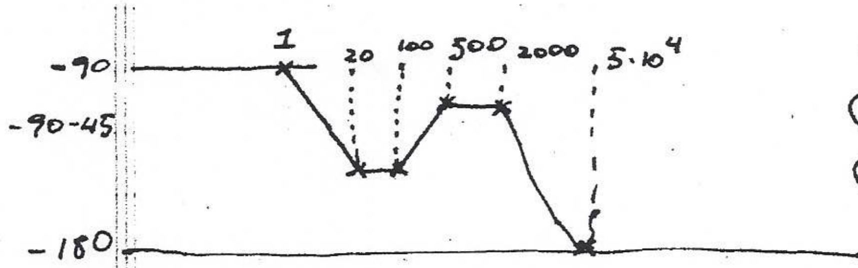
5000 → -45° (C)

The phase shift starts one decade before and ends one decade after points (A) (B) and (C)

(A) 10 → -45° [1, 100]

(B) 200 → +45° [20, 2000]

(C) 5000 → -45° [500, 5×10^4]



Intervals of interest are

[1 - 20] [20, 100] [100 500]

[500 2000] [2000, 5×10^4]

[1, 20] → (A) ~~[illegible]~~ decrease

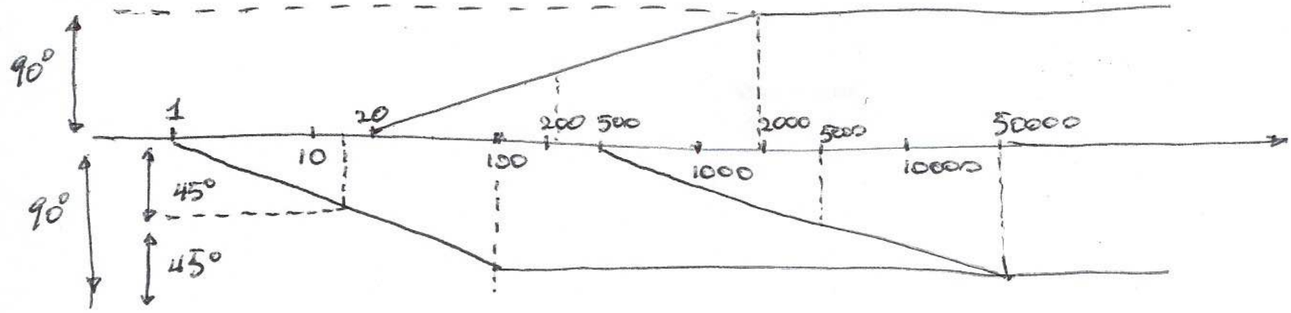
[20, 100] → A+B constant

[100 - 500] → (B) increase

[500 2000] → B+C constant

[2000 5×10^4] → (C) decrease

Now, we have to work a little bit harder to find the correct displacement intervals for the phase

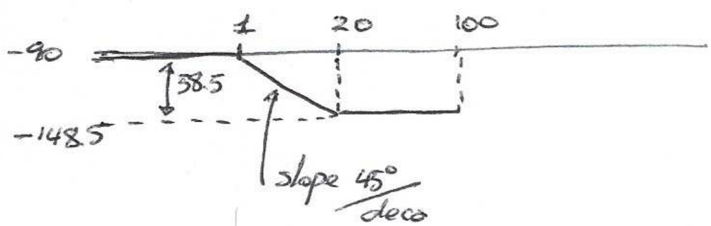


The special factor $\frac{1}{j\omega} = \frac{e^{-j\pi/2}}{\omega}$ gives a constant shift of $-\pi/2$, so the whole graph is shifted down by 90°

Now note that:

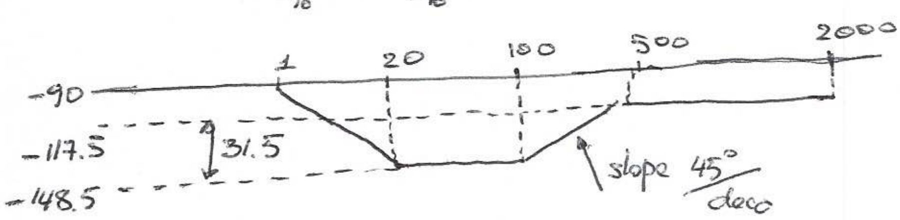
$$\frac{45}{1\ \text{deco}} = \frac{x}{\log_{10} 20 - \log_{10} 1} \Rightarrow x = 58.5$$ so that starting from −90° the graph

starts decreasing at 1 deco reaching the value $-90 - 58.5 = -148.5$ at $\log 20 - \log 1 = 1.3$ deco



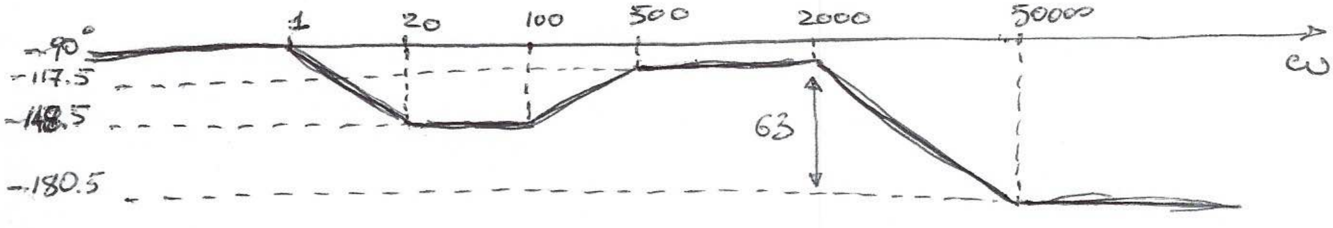
Similarly, to compute the increase of the function between 100 and 500, we have

$$\frac{45}{1} = \frac{x}{\log_{10} 500 - \log_{10} 100} = \frac{x}{0.7} \Rightarrow x = 31.5$$



And finally

$$\frac{45}{1} = \frac{x}{\log 50000 - \log 2000} = \frac{x}{1.4} \Rightarrow x = 63$$ So the final plot is:



$$H(\omega) = \frac{0.1\, j\omega + 20}{2 \cdot 10^{-5} (j\omega)^3 + 0.1002 (j\omega)^2 + j\omega}$$

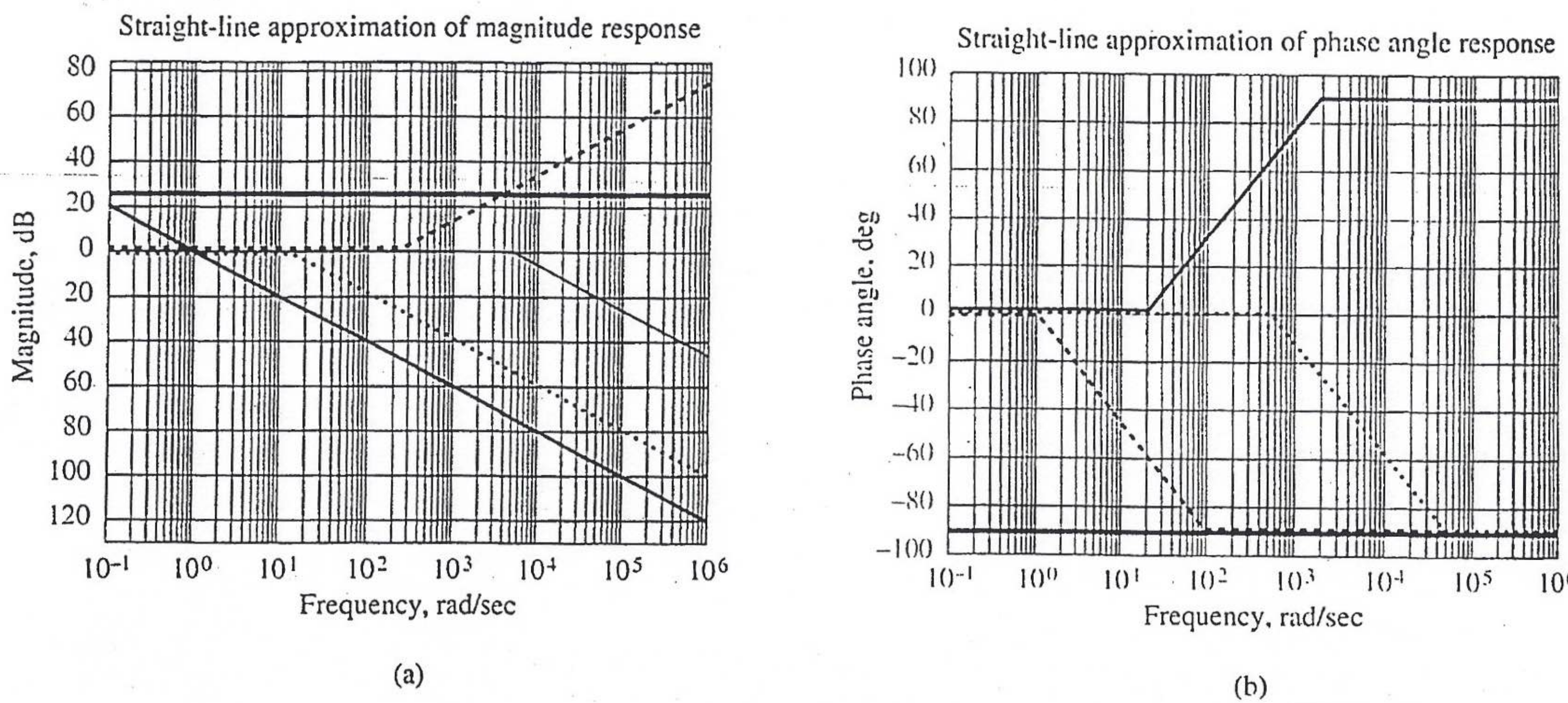
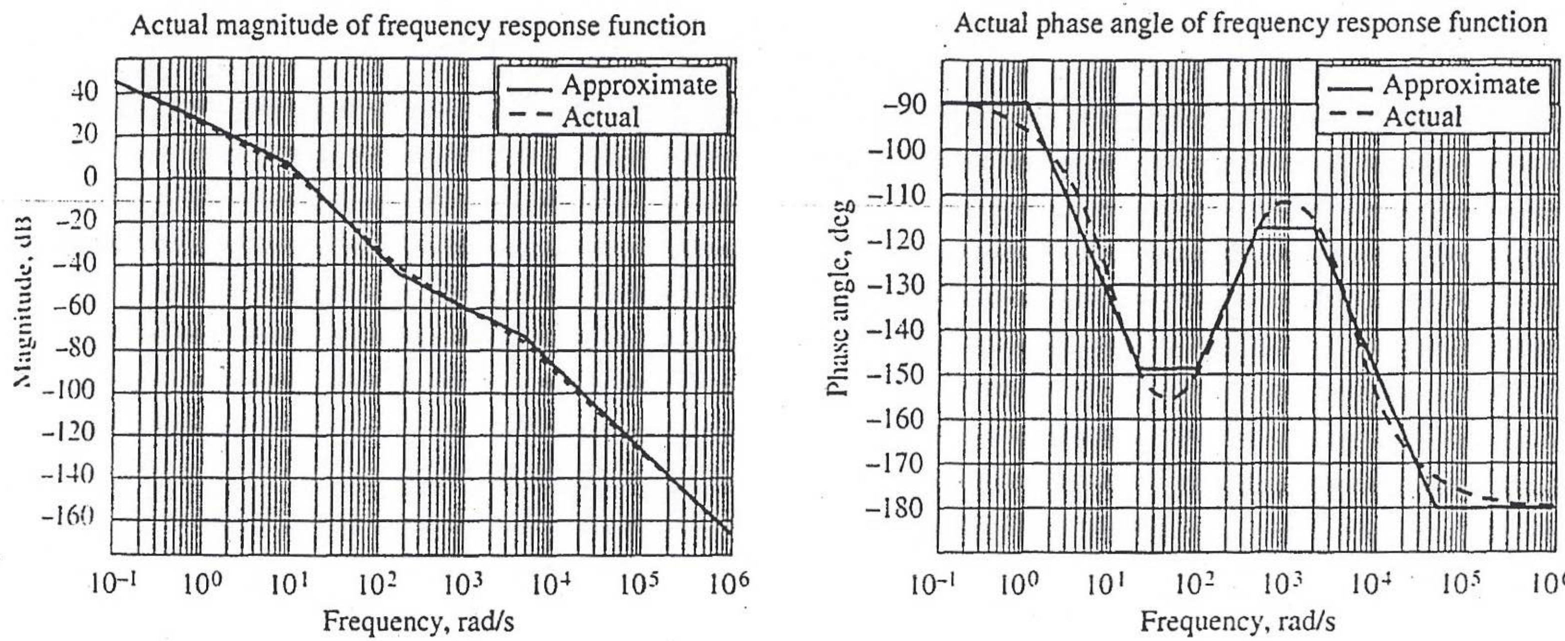


Figure 6.41 Approximate (asymptotic) frequency response of individual first-order terms. (a) Magnitude; (b) phase

$$H(\omega) = \frac{0.1\, j\omega + 20}{2 \cdot 10^{-5}\,(j\omega)^3 + 0.1002\,(j\omega)^2 + j\omega}$$



(a) (b)

Figure 6.42 Comparison of approximate and exact frequency response. (a) Magnitude; (b) phase

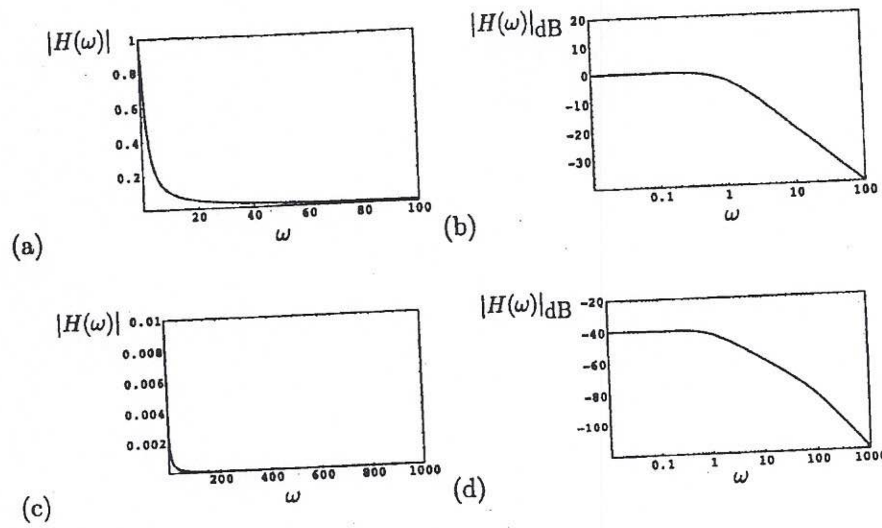


Figure 5.7: Linear (a, c) and decibel or dB (b, d) plots of the amplitude response of systems $H(\omega) = \frac{1}{1+j\omega}$ (a, b) and $H(\omega) = \frac{1}{(1+j\omega)(100+j\omega)}$ (c,d). Note that in dB plots a logarithmic scale is used for the horizontal axes following a common engineering practice.

Using Bode Plots
It is possible to appreciate exceedingly small values of the response -