University of California, San Diego
ECE 45
Midterm Exam

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Student ID Number :

- No Books, No Notes, No calculators allowed

Question	Score
1	
2	
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Problem 1) [15 pts]

Assume that you measured the response y1(t) of an LTI system to the signal x1(t) indicated below.

It is now late into the night, you are far from the lab, and just realized that to complete your project you also need to know the responses to signals x2(t), x3(t), x4(t).

Your collaborators panic, but you have taken ECE45 and claim that without making any additional measurements you can write these responses in terms of y(t).

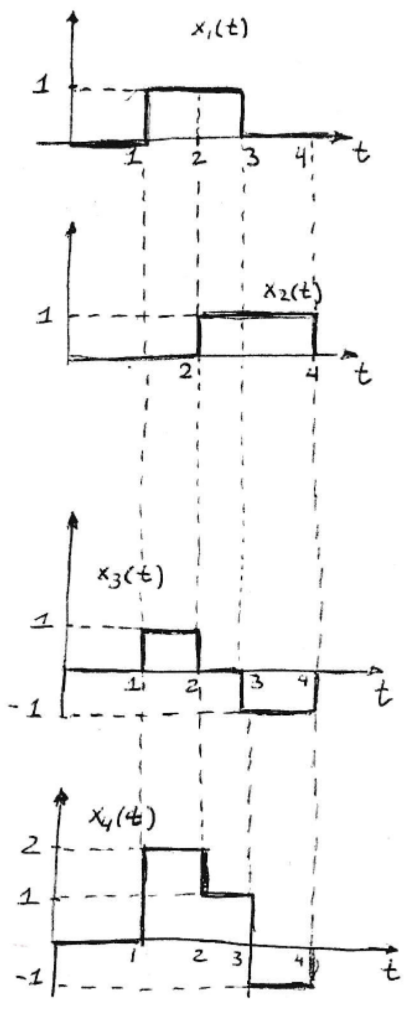
Prove that this is the case by writing the response to x2(t), x3(t), x4(t) in terms of the signal y1(t).

$$x_i(t) \longrightarrow \boxed{H(\omega)} \longrightarrow y_i(t)$$

$$\begin{aligned} x_2(t) &= x_1(t-1) \\ x_3(t) &= x_1(t) - x_1(t-1) \\ x_4(t) &= 2\cdot x_1(t) - x_1(t-1) \end{aligned}$$

System is LTI $\longrightarrow$

$$\boxed{\begin{aligned} y_2(t) &= y_1(t-1) \\ y_3(t) &= y_1(t) - y_1(t-1) \\ y_4(t) &= 2y_1(t) - y_1(t-1) \end{aligned}}$$



$$x_1(t) = \begin{cases} 1 & \text{for } t \in [1,3] \\ 0 & \text{otherwise} \end{cases}$$

$$x_2(t) = \begin{cases} 1 & \text{for } t \in [2,4] \\ 0 & \text{otherwise} \end{cases}$$

$$x_3(t) = \begin{cases} 1 & \text{for } t \in [1,2] \\ -1 & \text{for } t \in [3,4] \\ 0 & \text{otherwise} \end{cases}$$

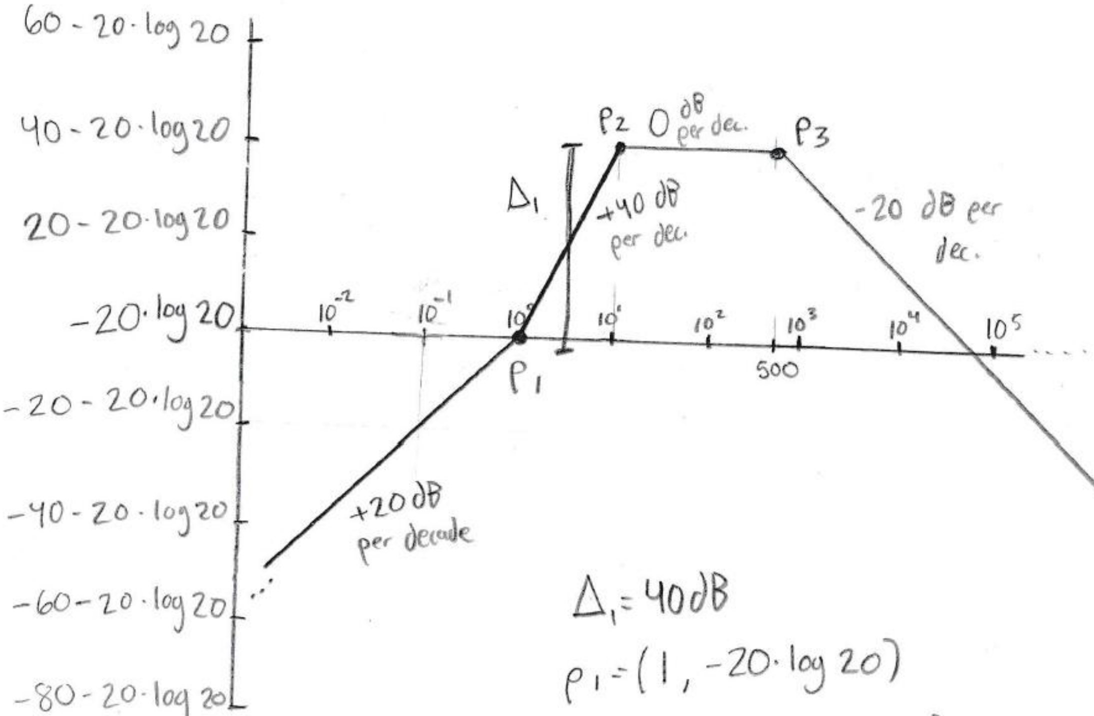
$$x_4(t) = \begin{cases} 2 & \text{for } t \in [1,2] \\ 1 & \text{for } t \in [2,3] \\ -1 & \text{for } t \in [3,4] \\ 0 & \text{otherwise} \end{cases}$$

Problem 2) [30 pts]
Draw the Bode plots (amplitude and phase) for the following frequency response

$$H(w) = 2500\frac{(jw)^2 + jw}{(jw+10)^2(jw+500)} = \frac{2500\cdot jw\cdot(1+jw)}{\left(10\cdot\left(1+j\frac{w}{10}\right)\right)^2\cdot 500\cdot\left(1+j\frac{w}{500}\right)}$$

$$= \frac{25\cdot 10^2\cdot jw(1+jw)}{10^2\cdot 5\cdot 10^2\cdot\left(1+j\frac{w}{10}\right)^2\left(1+j\frac{w}{500}\right)} = \underbrace{\frac{1}{20}\cdot\frac{jw\cdot(1+jw)}{\left(1+j\frac{w}{10}\right)^2\left(1+j\frac{w}{500}\right)}}_{\text{standard form}}$$

$$20\cdot\log|H(w)| = 20\cdot\log w + 20\log\sqrt{1+w^2}$$
$$- 20\cdot\log 20 - 40\cdot\log\sqrt{1+\frac{w^2}{10^2}} - 20\cdot\log\sqrt{1+\frac{w^2}{500^2}}$$



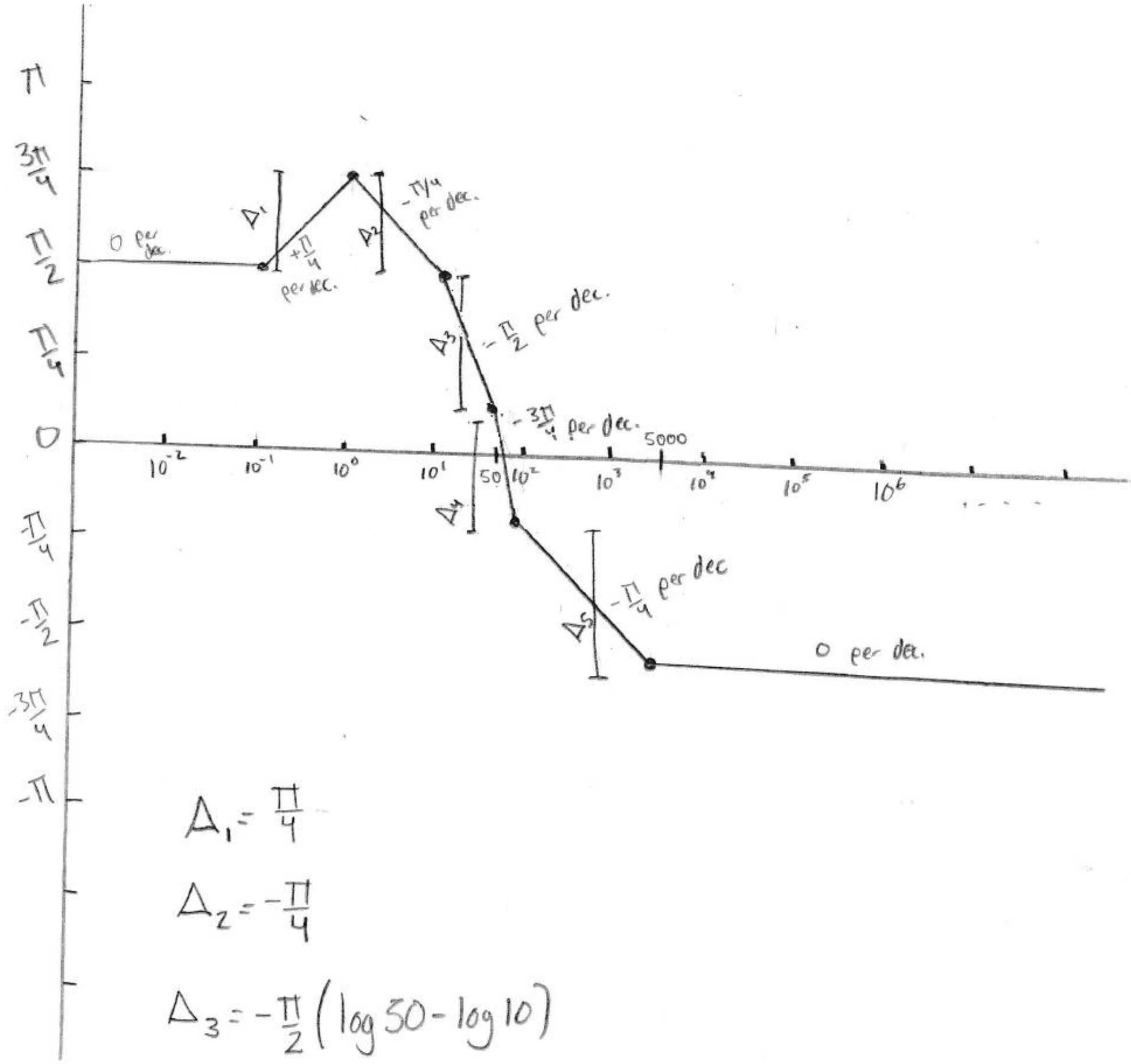
$\Delta_1 = 40\text{dB}$

$P_1 = (1, -20\cdot\log 20)$

$P_2 = (10, 40 - 20\cdot\log 20)$

$P_3 = (500, 40 - 20\cdot\log 20)$

$$\angle H(\omega) = \frac{\pi}{2} + \tan^{-1}(\omega) - 2 \cdot \tan^{-1}\left(\frac{\omega}{10}\right) - \tan^{-1}\left(\frac{\omega}{500}\right)$$



$$\Delta_1 = \frac{\pi}{4}$$

$$\Delta_2 = -\frac{\pi}{4}$$

$$\Delta_3 = -\frac{\pi}{2}(\log 50 - \log 10)$$

$$= -\frac{\pi}{2} \cdot \log 5$$

$$\Delta_4 = -\frac{3\pi}{4} \cdot (\log 100 - \log 50)$$

$$= -\frac{3\pi}{4} \cdot \log 2$$

$$\Delta_5 = -\frac{\pi}{4}(\log 5000 - \log 100)$$

$$= -\frac{\pi}{4} \cdot \log 50$$

$$= -\pi/4\,(1 - \log 5)$$

Problem 3) [55 pts]

a) [10 pts] Compute the average power of the periodic signals x1(t) and x2(t) depicted below.

b) [20 pts] Determine the Fourier series coefficients for x1(t).
Hint: Recall the integration by parts formula:

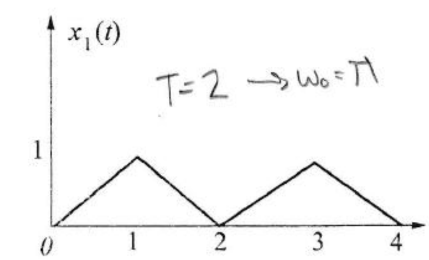
$$\int u(x)v'(x)dx = u(x)v(x) - \int u'(x)v(x)dx$$

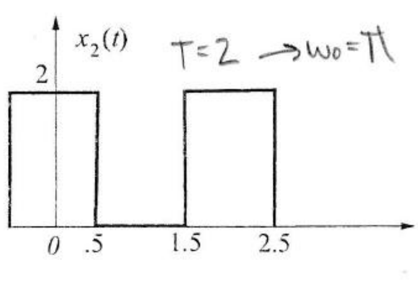
Rather than using the formula twice, you may use superposition and write the signal as the sum of two signals

c) [15 pts] Without performing the computation again, but using the properties of the Fourier series on page 2 of the exam, also determine the coefficients for x2(t).

d) [10pts] Write the response to signals x1(t) and x2(t) of an LTI system defined by

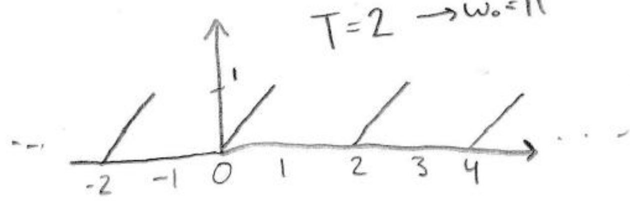
$$H(w) = 20\frac{(j\omega)}{(1+j\omega)} = \frac{20\omega}{\sqrt{1+\omega^2}} \cdot e^{j(\pi/2 - \tan^{-1}\omega)}$$



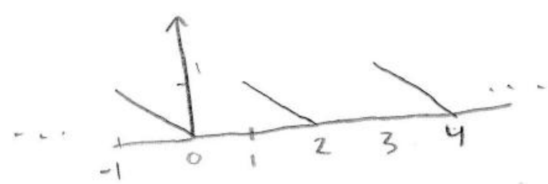


Let $f(t) =$

$T=2 \rightarrow \omega_0=\pi$



$f(-t) =$



Note: $x_1(t) = f(t) + f(-t)$

$x_2(t) = \frac{d x_1(t+1/2)}{dt} + 1$

a. $P_{avg}^{(x_1)} = \frac{1}{2}\int_0^2 |x_1(t)|\, dt = \frac{1}{2}\int_0^1 t^2\, dt + \frac{1}{2}\int_1^2 (2-t)^2\, dt = \frac{1}{2}\int_0^1 t^2\, dt + \frac{1}{2}\int_1^2 4-4t+t^2\, dt$

$$= \frac{1}{2}\left(\left[\frac{t^3}{3}\right]_{t=0}^{1} + \left[4t-2t^2+\frac{t^3}{3}\right]_{t=1}^{2}\right) = \frac{1}{2}\left(\cancel{\tfrac{1}{3}} + \cancel{8} - \cancel{8} + \frac{8}{3} - 4 + 2 - \cancel{\tfrac{1}{3}}\right)$$

$$= \frac{1}{2}\left(\frac{8}{3} - \frac{6}{3}\right) = \boxed{1/3}$$

$$P_{avg}^{(x_2)} = \frac{1}{2}\int_{-1/2}^{3/2} |x_2(t)|^2\, dt = \frac{1}{2}\int_{-1/2}^{1/2} 2^2\, dt = \frac{1}{2}\left[4(1/2 + 1/2)\right] = \boxed{2}$$

b. $x_1(t) = f(t) + f(-t) \longrightarrow X_{1n} = F_n + F_{-n}$

$F_0 = \frac{1}{2}\int_0^1 t\, dt = \frac{1}{4}$

For $n \neq 0$

$$F_n = \frac{1}{2}\int_0^1 t \cdot e^{-j\pi n t}\, dt$$

let $u = t$ $\quad dv = e^{-j\pi n t}$
$du = dt$ $\quad v = -\frac{e^{-j\pi n t}}{j\pi n}$

$$= \frac{1}{2}\left(\left[t \cdot \frac{e^{-j\pi n t}}{-j\pi n}\right]_{t=0}^{1} + \int_0^1 \frac{e^{-j\pi n t}}{j\pi n}\, dt\right) = \frac{1}{2}\left(\frac{j \cdot e^{-j\pi n}}{\pi n} - \left[\frac{e^{-j\pi n t}}{(j\pi n)^2}\right]_{t=0}^{1}\right)$$

$$= \frac{1}{2}\left(\frac{j\pi n \cdot e^{-j\pi n}}{\pi^2 n^2} + \frac{e^{-j\pi n}}{\pi^2 n^2} - \frac{1}{\pi^2 n^2}\right) = \frac{e^{-j\pi n} + j\pi n \cdot e^{-j\pi n} - 1}{2\pi^2 n^2} = F_n$$

$$F_{-n} = \frac{e^{j\pi n} - j\pi n \cdot e^{j\pi n} - 1}{2\pi^2 n^2}$$

Note: $e^{-j\pi n} = e^{j\pi n} = (-1)^n$
since n is an integer

$$\rightarrow X_{1n} = \frac{e^{j\pi n} - j\pi n \cdot e^{j\pi n} - 1 + e^{-j\pi n} + j\pi n \cdot e^{-j\pi n} - 1}{2\pi^2 n^2} = \frac{(-1)^n - 1}{\pi^2 n^2}$$

$X_{1_0} = 2 \cdot F_0 = 1/2$

$$X_{1n} = \begin{cases} 1/2 & n = 0 \\ \frac{-2}{\pi^2 n^2} & n \text{ is odd} \\ 0 & \text{else} \end{cases}$$

c. $X_2(t) = \dfrac{\partial x_1(t+1/2)}{\partial t} + 1$

$$\rightarrow X_{2_n} = \begin{cases} j\pi n \cdot e^{jn\pi/2} \cdot X_{1_n} & n \neq 0 \\ 1 & n = 0 \end{cases}$$

Note $-j = e^{-j\pi/2}$

$$X_{2_n} = \begin{cases} 1 & n = 0 \\ \frac{-2}{\pi^2 n^2} \cdot j\pi n \cdot e^{jn\pi/2} & n \text{ odd} \\ 0 & \text{else} \end{cases} = \boxed{\begin{cases} 1 & n = 0 \\ \frac{2}{\pi n} \cdot e^{j\pi/2(n-1)} & n \text{ is odd} \\ 0 & \text{else} \end{cases}}$$

d. $|H(\omega)| = \dfrac{20\omega}{\sqrt{1+\omega^2}}$ $\quad \angle H(\omega) = \dfrac{\pi}{2} - \tan^{-1}\omega$

$$Y_{1_n} = |H(n\pi)| \cdot X_{1_n} \cdot e^{j\angle H(n\pi)} = \begin{cases} \frac{-2}{\pi^2 n^2} \cdot \frac{20\pi n}{\sqrt{1+\pi^2 n^2}} \cdot e^{j(\pi/2 - \tan^{-1}\pi n)} & n \text{ is odd} \\ 0 & \text{else} \end{cases}$$

note $-1 = e^{-j\pi}$

$$Y_{1_n} = \begin{cases} \frac{40}{\pi n \sqrt{1+\pi^2 n^2}} \cdot e^{-j(\pi/2 + \tan^{-1}\pi n)} & n \text{ is odd} \\ 0 & \text{else} \end{cases}$$

$$Y_{2n} = |H(n\pi)| \cdot X_{2_n} e^{j\angle H(n\pi)} = \begin{cases} \frac{2}{\pi n} \cdot e^{j\pi/2(n-1)} \cdot \frac{20\pi n}{\sqrt{1+\pi^2 n^2}} \cdot e^{j(\pi/2 - \tan^{-1}\pi n)} & n \text{ is odd} \\ 0 & \text{else} \end{cases}$$

$$Y_{2n} = \begin{cases} \frac{40}{\sqrt{1+\pi^2 n^2}} \cdot e^{j(n\frac{\pi}{2} - \tan^{-1}\pi n)} & n \text{ is odd} \\ 0 & \text{else} \end{cases}$$